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BEM CALCULATION OF ACOUSTIC WAVE LOCALIZED IN 2-DIMENSIONAL RANDOM FIELD

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Abstract

A random structure plays an important role in wave propagations. The Anderson localization is well known in electron transport theories. Such a localization causes divergence of resistivity. In acoustic propagations, localization of sound may be brought or the information of source position may be lost by the localization. These properties of acoustic wave in the random field are investigated through a Boundary Element Method (BEM). The localization of acoustic wave was observed in 2-dimensional random medium by a numerical calculation. A discussion on localization length and mean free path is made by a weak localization theory in quantum transport phenomena.

INTRODUCTION

Properties of wave propagations through random media have been studied extensively. The disorder plays an important role in transport phenomena. The Anderson localization is well known in electron transport theory [1]. Electrons are trapped in a random potential field and the resistivity diverges to infinity. Its critical exponents are predicted by scaling theory [6]. It has been shown experimentally that electro-magnetic wave is trapped in fractal geometry, recently [4], [5]. Also in one dimensional random system, the attenuation of acoustic wave was investigated by Nakasako et al. [3]. The wavelength, randomness and mean free path play important roles in scaling theory.

Here the sound propagations through media with scatterers are investigated by numerical methods. Essential features of wave transport phenomena through random media can be obtained by two-dimensional calculation. In fact, trees in a forest are considered as disordered obstacles to 2-dimensional sound propagation. A model of trees in forest based on an actual

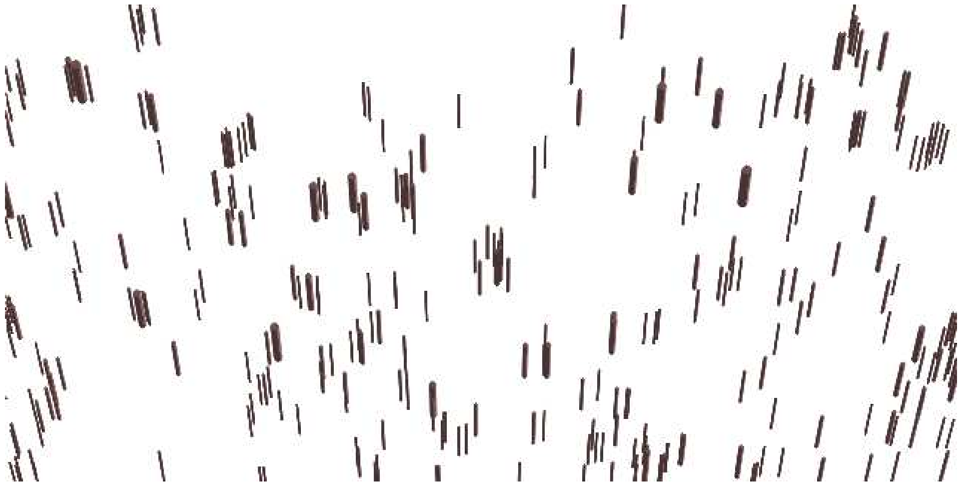

Figure 1: Trees in forest as obstacles for sound propagation

measurement is illustrated in Figure 1. Dense trees scatter sounds.

A Boundary Element Method (BEM) was mainly applied to this problem. Distributions of sound pressure level, acoustic energy and intensity vector have been examined by BEM. The localizations of the acoustic quantities were confirmed numerically. Diverse changes in arrival directions of sound were observed by acoustic intensity mapping. The scaling theory developed in the quantum transport phenomena was applied to acoustic problems and the localization length in random medium was discussed.

BEM CALCULATION

A Boundary Element Method (BEM) is useful for scattering problems. Because a simulation in an unbounded space is easily carried out when a velocity potential ϕ_s of scattering wave satisfies the Sommerfeld radiation condition.

$$\lim_{r\to\infty} r^{\frac{d-1}{2}}(\frac{\partial\phi_s}{\partial n} + ik\phi_s) = 0, \tag{1}$$

where d is a dimensionality. Under this condition, the integration on the infinite domain boundary in BEM vanishes. In order to satisfy the condition above, the fundamental solution $u^*(\boldsymbol{x},\boldsymbol{y})$ represented by the Hankel function of the second kind $H^{(2)}$ is used in 2-dimensional BEM.

$$u^*(\boldsymbol{x},\boldsymbol{y}) = -\frac{i}{4}H_0^{(2)}(kr), \tag{2}$$

where r is the distance between $\boldsymbol{x}$ and $\boldsymbol{y}$.

However, in BEM, a set of linear equations with a dense complex matrix must be solved numerically. The number of matrix elements for 3-dimensional space becomes enormous so it is difficult to carry out practical computations. Nevertheless, BEM in 2-dimension is an efficient method because a number of elements on boundary is not so many and the essential feature of wave propagation can be examined in 2-dimensional calculations. Therefore, the 2-dimensional BEM is applied throughout this report.

The acoustic energy density w and intensity vector (I_x, I_y) are computed by the velocity potential ϕ

$$w = \frac{1}{4}[\frac{k^2}{\rho}|\phi|^2 + \rho(|\frac{\partial \phi}{\partial x}|^2 + |\frac{\partial \phi}{\partial y}|^2)] \tag{3}$$

$$I_n = \frac{1}{2}Re[i\omega\rho\phi(-\frac{\partial \phi}{\partial n})], \quad (n = x, y) \tag{4}$$

Thus, basic acoustic quantities are derived from the velocity potential.

NUMERICAL RESULTS

As mentioned in the previous section, the sound propagations in 2-dimensional media with scatterers were examined by BEM. A basic periodic configuration was made by 13$\times$13 obstacles and intervals of center of mass was set to be 1.5m for both directions. For simplicity, the shape of obstacle was set to be square with 0.25m edges. The random configurations are generated in order that the position of each obstacle is shifted by a uniform random number between $\pm$0.5m. A point source of pure tone sound was set at (0, 100m). The obstacles were considered as completely hard for sound propagation. The length of each element in our simulation model is set to be less than 1/8 of wavelength and 1/5 of the edge of obstacle and the constant element is adopted in which a singularity problem at a vertex point is avoided. The calculations were done up to 800Hz

First, the sound pressure level is shown in Figures 2 and 3. The sound pressure distributions for 250Hz and 500Hz are demonstrated in dB. The left figure and the right one are for periodic and random configurations, respectively. As the sound fields were excited by pure tones, these presentations show strong interference effects. Howerver, a tendencies of weak sound localization can be seen in random configurations.

Figure 4 demonstrates acoustic energy densities for 250Hz. In this figure, levels were also scaled in dB. As the sound pressure and particle velocity supplement with each other, the interference patterns are hidden in the figure and the weak localization of sound wave can be seen well in the disordered configuration .

The sound intensity vector is presented in Figure 5. Here, the length of intensity vector is linearly scaled. In this disordered medium, orientations of intensity vectors rotate around localized positions. From this fact, it is expected that the positional information of sound source may be lost in deep forests where trees are considered as random obstacles. Thus the emergency alarm may not propagate well in a forest filled with dense trees.

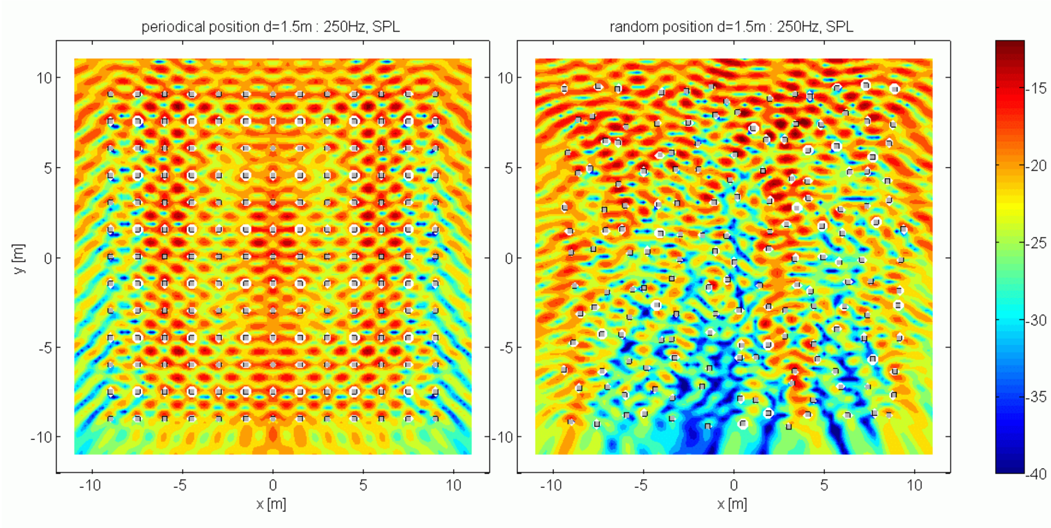

Figure 2: Sound Pressure Distribution 250Hz

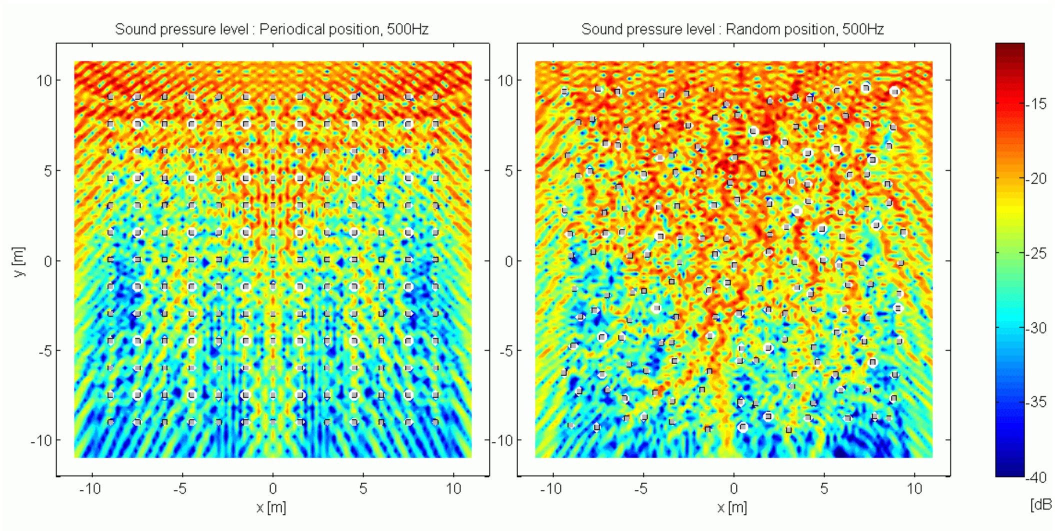

Figure 3: Sound Pressure Distribution 500Hz

Contour profiles parallel to y-axis are plotted in Figure 6. The attenuation and localization of sound in random scatterers are seen clearly through the acoustic energy representation.

We generated 31 configurations of random obstacles. BEM calculations have been carried out for all media and ensemble average of contour profiles along y-axis has been per-

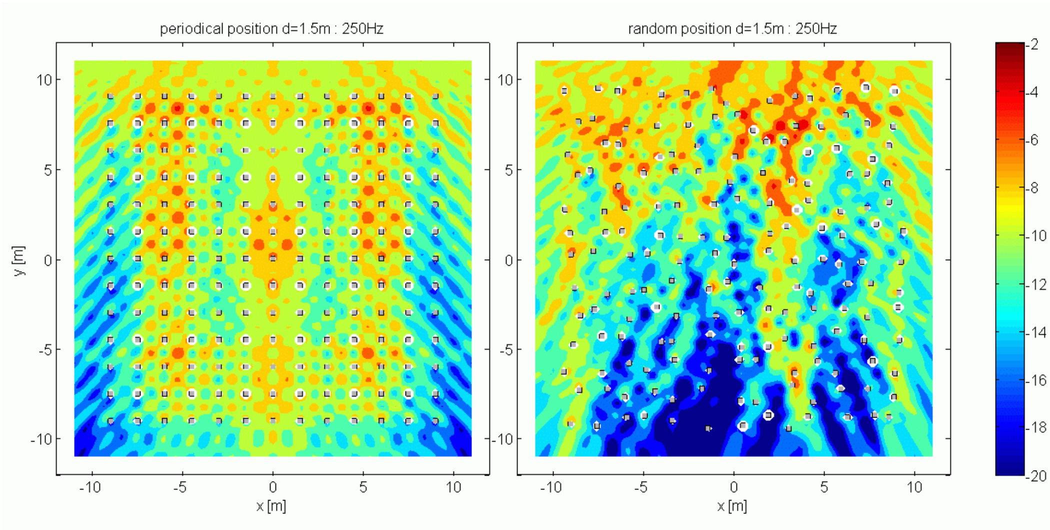

Figure 4: Sound Energy Distribution 250Hz

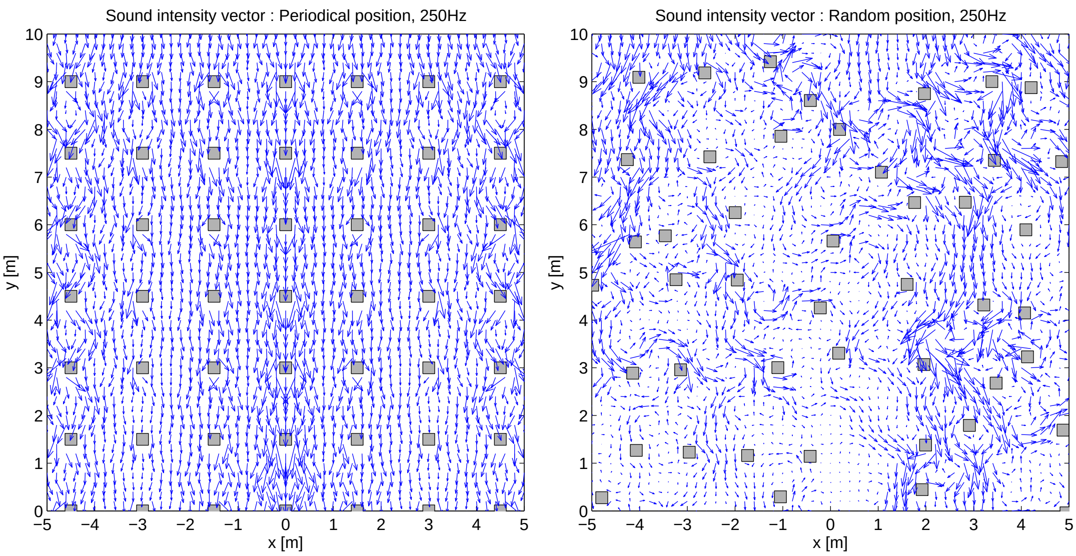

Figure 5: Sound Instensity Vector 250Hz

formed. The averaged profiles are plotted in Figure 7. The exponential damping through the random medium is observed in both figures based on sound pressure and energy. The localization lengths for various frequencies were calculated by taking ensemble averages of acoustic energy along y-axis. The attenuation length is estimated so that the wave amplitude diminishes $1/e$ in the distance. The localization length calculated through BEM for 250Hz is

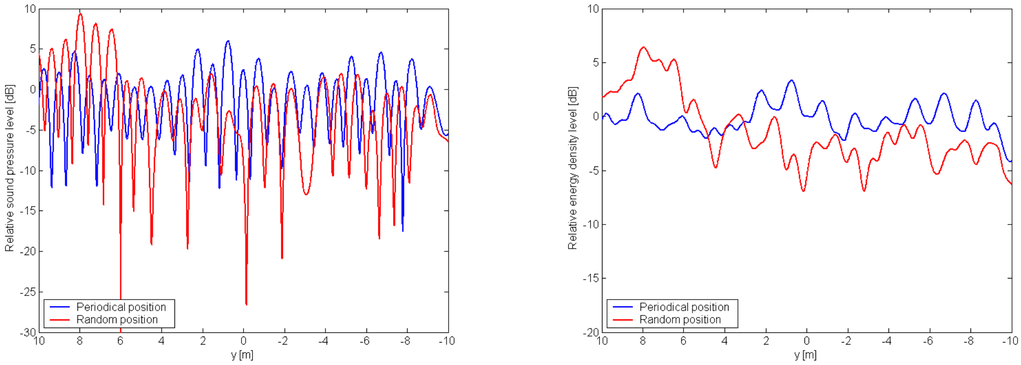

Figure 6: Profile of Pressure and Energy

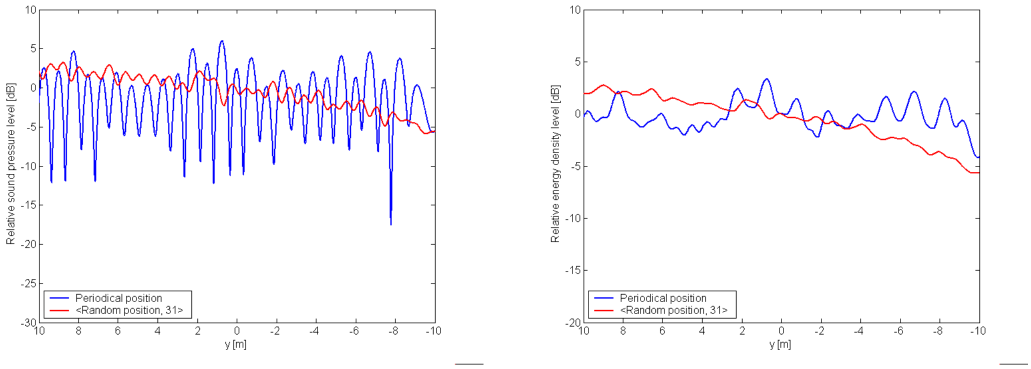

Figure 7: Ensemble Average of Sound Pressure and Energy

estimated to be 22.6m which is equivalent to the system size of our simulation models.

DISCUSSION

According to the electron transport theory, a localization length ξ_{2D} in 2-dimensional space is predicted by

$$\xi_{\mathrm{2D}} = \frac{l}{\sqrt{2}} exp(\frac{\pi^2 l}{\lambda}), \tag{5}$$

where λ is the wavelength and l is the transport mean free path [7]. This may not be our case because the source was situated at the outside the scattering domain and incident wave was a plane wave type. The localization length in 1-dimensional theory [2] is

$$\xi_{\mathrm{1D}} = Nl = \mathrm{Int}\frac{2L_w}{\lambda} l \tag{6}$$

where N and L_w are a number of mode and width of channel respectively.

A mean free path is estimated by the following geometrical calculation. First, a position is chosen in the random medium. (When it happens to be inside an obstacle the point is discarded.) From the selected point, the ray is traced upto higher order reflections. The distance d_i between each reflection point and deflection angle θ_i at each reflection are measured. The mean free path L is estimated by

$$\frac{1}{L} = \frac{1}{N_{tot}} \sum_i (1 - \cos\theta_i) \frac{1}{d_i} \tag{7}$$

where N_{tot} is number of total path segments. This formula brings deflection angle weighted value used in Boltzmann transport theories.

Since it is well known that according to the Bloch's theorem there is no scattering of electron wave by a periodic potential, the transport mean free path is different from that derived by geometrical calculation. The total scattering rate s_{tot} in the geometrical sense is sum of those by periodic potential and the random configuration. When the geometrical mean free path in the periodic structure is l_p, roughly, the mean free path by random origin l_p is estimated by

$$\frac{1}{l_{tot}} = \frac{1}{l_r} + \frac{1}{l_p} \tag{8}$$

The mean free path estimated by geometrical calculation in the medium with random obstacles was l_{tot} = 0.58m. The length for periodic configuration was l_p = 1.01m. Thus, l_r was approximately estimated to be 1.36m. The wavelength λ = 1.36m for 250Hz. The Ioffe-Regel criterion $l_r/\lambda < 1/(2\pi)$ which is not satisfied for this frequency. The localization length estimated by one dimension model is 39m and by two dimensional model 18592m. These arguments neglect wave nature of sound in scattering processes but the estimated localization length by one-dimensional theory is nearer to our numerical estimate. The wave propagation that we have considered may be one dimensional type. However, there still exists a discrepancy between the theoretical and the numerical estimates. One of the reasons of the discrepancy originates from the geometrical estimation of transport mean free path.

A more accurate estimation of the transport mean free path based on diffusion process can be done by wave acoustic calculation. Setting initial distribution of sound pressure as gaussian shape, the diffusion constant D is calculated from the time evolution of the second moments of wave packet.

$$\langle (x - x_0)^2 + (y - y_0)^2 \rangle = 4Dt, \tag{9}$$

where $x_0 =< x >$ and $y_0 =< y >$. The transport mean free path and the diffusion constant D is related by

$$D = \frac{cl}{d} \tag{10}$$

where c and d are sound speed and dimensionality, respectively. Transient simulations of sound propagations based on wave theory may be carried out by finite difference method in time domain. Explicit frequency dependence of mean free path could be computed. The frequency dependent mean free path may compensate the number of mode in channel or wavelength, which can predict the localization length of sound wave correctly.

SUMMARY

The localization of sound wave in random obstacles was confirmed by 2-D BEM calculation and the exponential attenuation was observed. The numerical localization length and that estimated by the weak localization theory were discussed. The mean free path was obtained through geometrical calculations. It was concluded that the transport mean free path should be determined by wave acoustic method. For further studies, 3-dimensional calculation in time domain such as finite difference method could be done and both localization and transport mean free path could be evaluated. In order to research on an influence of quasi randomness on sound waves, sound propagation through fractal geometry is now under study.

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