ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

DAMAGE DETECTION IN BEAMS USING TIME-FREQUENCY ANALYSIS OF TRANSIENT FLEXURAL WAVES

A. Apostoloudia[1], E. Douka[2], L. J. Hadjileontiadis[3], I. T. Rekanos[1], and A. Trochidis*[1]

[1]Physics Division, Faculty of Engineering, Aristotle University of Thessaloniki
GR-54124 Thessaloniki, Greece
[2]Mechanics Division, Faculty of Engineering, Aristotle University of Thessaloniki
GR-54124 Thessaloniki, Greece
[3]Department of Electrical and Computer Engineering, Faculty of Engineering
Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece
trohidis@gen.auth.gr (e-mail address of lead author)

Abstract

In this paper, a methodology for locating and determining the size of transverse open cracks in beams based on time-frequency analysis of transient flexural waves is presented. A beam with a transverse open crack modeled as a rotational spring is considered. The wave field along the beam generated by an impact force is calculated and expressions for the reflection coefficients as a function of frequency and crack depth are derived. Simulated signals for the acceleration response of the beam for different crack depths are calculated and further analysed employing time-frequency analysis to extract the dispersive characteristics. The location of the crack is estimated using the arrival time of reflected waves. The ratio of the reflected wave energy to the incident wave energy is calculated and used as an indicator of crack size. The estimated reflection coefficients from the simulated signals are in excellent agreement with theory. To validate the theoretical predictions, a series of experiments on a cracked brass beam were carried out. The measured signals were analysed and both the location and size of the crack were estimated. The experimental results verify the effectiveness of the proposed methodology.

INTRODUCTION

Damage detection in structures is a problem of practical importance that has received considerable amount of interest the last decades. Among the existing methods, wave-based techniques gained ground. The basic concept, in most of these techniques, relies on the fact that a propagative wave will be reflected and partially transmitted when it encounters a defect. Measurements of the reflected and/or transmitted waves can provide significant information about the damage location and size. Unlike ultrasonic waves, middle frequency waves are not only sensitive to damage but also decay slowly; therefore, they are suitable for global inspection of structures. Since flexural waves are dispersive, a key issue in wave-based precise damage identification is the use of time-frequency analysis allowing the description of the time variation of each frequency component.

So far, the continuous wavelet transform (CWT) [7] has been adopted for the analysis of guided waves because of its local and self-adaptive time-frequency resolution properties. The CWT has been applied to analyse simulated flexural waves in an Euler-Bernoulli beam, whereas it has been experimentally verified in estimating the group velocity and impact site [3, 5]. Furthermore, the CWT has been employed to determine the location of a crack in a Timoshenko beam through flexural waves analysis [10]. Recently, the CWT has been utilised to identify both the location and depth of a crack in a cantilever beam [4, 6].

In the present work, a method for estimating both the location and size of a crack in a beam by analysing transient flexural waves is presented. Four different time-frequency representations, namely the CWT, the smoothed pseudo-Wigner distribution (SWPD) [1], the Hilbert-Huang transform (HHT) [2], and the smoothed pseudo-Wigner trispectrum distribution (SPWTD) [8] are investigated. The location of the crack is estimated by calculating the arrival times of incident and reflected waves with different group velocities. The ratio of the reflected wave energy to the incident wave one is used as an estimator of crack size. Experimental results in a brass beam have shown good agreement with predictions justifying the accuracy of the proposed method. The advantages and shortcomings of each time-frequency representation employed are compared and discussed.

VIBRATION MODEL OF CRACKED INFINITE BEAM

A beam of infinite length along the x axis, with uniform rectangular cross-section $w \times w$, which has a transverse surface crack located at the origin of the x axis (Fig. 1), is considered. The beam is excited by a force whose Fourier transform is $F(\omega)$. The force acts at a distance L from the crack position. For each frequency ω the harmonic solution of the displacement $n(x, \omega)$ for the three regions defined along the beam is given by

$$\begin{aligned} R_1 : x \leq -L \qquad & n_1(x,\omega) = A_1^-(\omega)\mathrm{e}^{jkx} + B_1^-(\omega)\mathrm{e}^{kx} \\ R_2 : -L \leq x \leq 0 \qquad & n_2(x,\omega) = A_2^+(\omega)\mathrm{e}^{-jkx} + B_2^+(\omega)\mathrm{e}^{-kx} + A_2^-(\omega)\mathrm{e}^{jkx} + B_2^-(\omega)\mathrm{e}^{kx} \\ R_3 : 0 \leq x \qquad & n_3(x,\omega) = A_3^+(\omega)\mathrm{e}^{-jkx} + B_3^+(\omega)\mathrm{e}^{-kx} \end{aligned} \tag{1}$$

In (1), the A and B coefficients are related to travelling and evanescent waves, respectively. The subscripts refer to the corresponding regions, whereas the plus and minus superscripts denote propagation along the positive and negative x direction, respectively. The wavenumber k is derived by the Euler-Bernoulli model for a beam with mass density ρ, cross-sectional area S, Young's modulus E, and moment of inertia I, i.e., $k = [\omega^2 \rho S/(EI)]^{1/4}$.

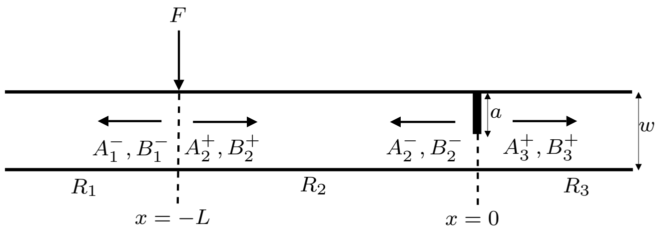

Figure 1: Geometry of the cracked infinitely long beam under study.

The A and B coefficients are derived from the fulfilment of boundary conditions at the locations of the impact and the crack. At the impact ($x = -L$) the continuity of displacement ($n_1 = n_2$), gradient of displacement ($\partial_x n_1 = \partial_x n_2$), moment ($\partial_x^2 n_1 = \partial_x^2 n_2$) and shear ($\partial_x^3 n_2 - \partial_x^3 n_1 = F/(EI)$) is imposed. Also, at the crack ($x = 0$) the continuity of displacement ($n_2 = n_3$), moment ($\partial_x^2 n_2 = \partial_x^2 n_3$), and shear ($\partial_x^3 n_2 = \partial_x^3 n_3$) is imposed. The crack can be modelled by a massless rotational spring, whose bending stiffness K_T is given by [9]

$$K_T = 0.1871 \frac{EI}{wJ(a/w)} \tag{2}$$

where a is the depth of the crack and $J(a/w)$ is the local compliance function given by

$$\begin{aligned} J(u) \quad = \quad & 1.98u^2 - 3.277u^3 + 14.43u^4 - 31.26u^5 + 63.56u^6 \\ & -103.36u^7 + 147.52u^8 - 127.69u^9 + 61.5u^{10}. \end{aligned} \tag{3}$$

Under this assumption, a boundary condition, which is related to the equilibrium between transmitted bending moment and spring rotation, is imposed at the crack location, i.e.,

$$-EI\partial_x^2 n_2(0, \omega) = K_T[\partial_x n_2(0, \omega) - \partial_x n_3(0, \omega)]. \tag{4}$$

By solving the system of the eight equations derived, the A and B coefficients are evaluated. In particular, the coefficients related to the wave travelling toward the crack are given by

$$A_2^+(\omega) = -j\frac{F}{4EIk^3}\mathrm{e}^{-jkL}, \quad B_2^+(\omega) = -\frac{F}{4EIk^3}\mathrm{e}^{-kL}, \tag{5}$$

while the coefficients associated with the reflected wave are related to the incident wave ones through the reflection matrix $\mathbf{R}(\omega)$, i.e.,

$$\begin{bmatrix} A_2^-(\omega) \\ B_2^-(\omega) \end{bmatrix} = \mathbf{R}(\omega) \begin{bmatrix} A_2^+(\omega) \\ B_2^+(\omega) \end{bmatrix} \tag{6}$$

The reflection matrix, which depends on the properties of the crack is given by

$$\mathbf{R}(\omega) = -\frac{\tilde{k}}{\tilde{k}(1-j) - 4j} \begin{bmatrix} 1 & -1 \\ j & -j \end{bmatrix} \tag{7}$$

where $\tilde{k} = EIk/K_T$. As a result, according to (1), $n_2(x, \omega)$ is determined in region R_2, while the associated acceleration $\gamma_2(x, t)$ is obtained as

$$\gamma_2(x, t) = -\frac{1}{2\pi} \int_{-\infty}^{+\infty} \omega^2 n_2(x, \omega) \mathrm{e}^{j\omega t} d\omega. \tag{8}$$

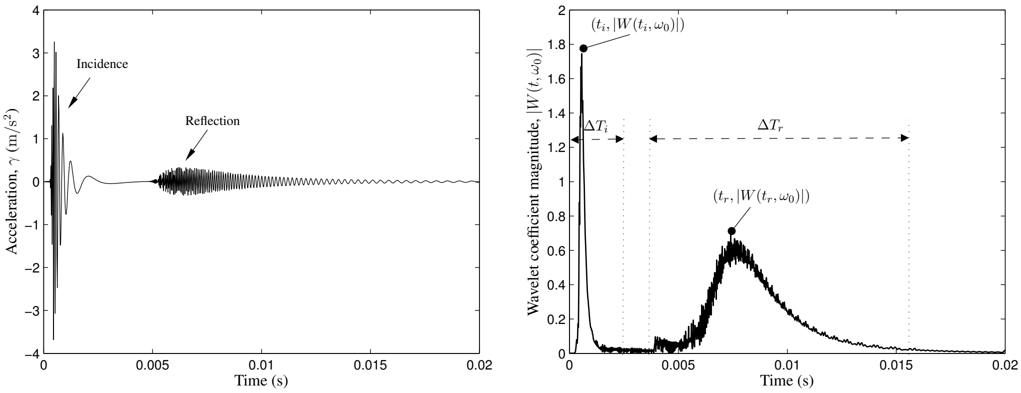

Figure 2: (a) Typical simulated acceleration signal. (b) Time variation of the wavelet coefficient magnitude, for frequency ω_0.

From the above analysis, it is evident that the characteristics of the crack, i.e., location and size, are introduced through the elements of the reflection matrix (7). Furthermore, since the evanescent waves related to the B coefficients do not significantly contribute to the acceleration, the magnitude of the (1,1) element of the reflection matrix, which is considered in the following as the *reflection coefficient magnitude*, can estimated by

$$|\hat{\mathbf{R}}_{11}(\omega)| = [E_r(\omega)/E_i(\omega)]^{1/2} \tag{9}$$

where $E_i(\omega)$ and $E_r(\omega)$ are the incident and the reflected energy from the crack, respectively, at frequency ω. This energy-based approach is adopted to compensate for the dispersion effect, since the energy associated with each distinct frequency is spread in time.

Let us consider again the cracked infinite beam is impacted by delta Dirac force and the acceleration is evaluated at a position in region R_2 placed at a distance L_M from the crack. A typical simulated acceleration signal is illustrated in Fig. 2(a) where the parts of the signal related to the incident and the reflected waves are pointed with arrows. If the CWT is applied, the estimate of the reflection coefficient magnitude at frequency ω_0 is derived from the time variation of the wavelet coefficient magnitude, $|W(t, \omega_0)|$ (Fig. 2(b)). According to (9), the estimate of the reflection coefficient magnitude is given by

$$|\hat{\mathbf{R}}_{11}(\omega_0)| = \left(\frac{\int_{\Delta T_r} P(t, \omega_0) dt}{\int_{\Delta T_i} P(t, \omega_0) dt} \right)^{1/2} \tag{10}$$

where P is the power distribution ($P(t, \omega) = |W(t, \omega)|^2$ for the CWT case), while ΔT_i and ΔT_r are the durations of the incident and the reflected wave, respectively. The time instances $t_i(\omega_0)$ and $t_r(\omega_0)$ of maximum values $|W(t_i, \omega_0)|$ and $|W(t_r, \omega_0)|$ represent the arrival times of the incident and the reflected waves, respectively. Thus, if the group velocity $c_g(\omega_0)$ is known, the estimate of the distance L_M is given by

$$\hat{L}_M = \frac{t_r(\omega_0) - t_i(\omega_0)}{2c_g(\omega_0)}. \tag{11}$$

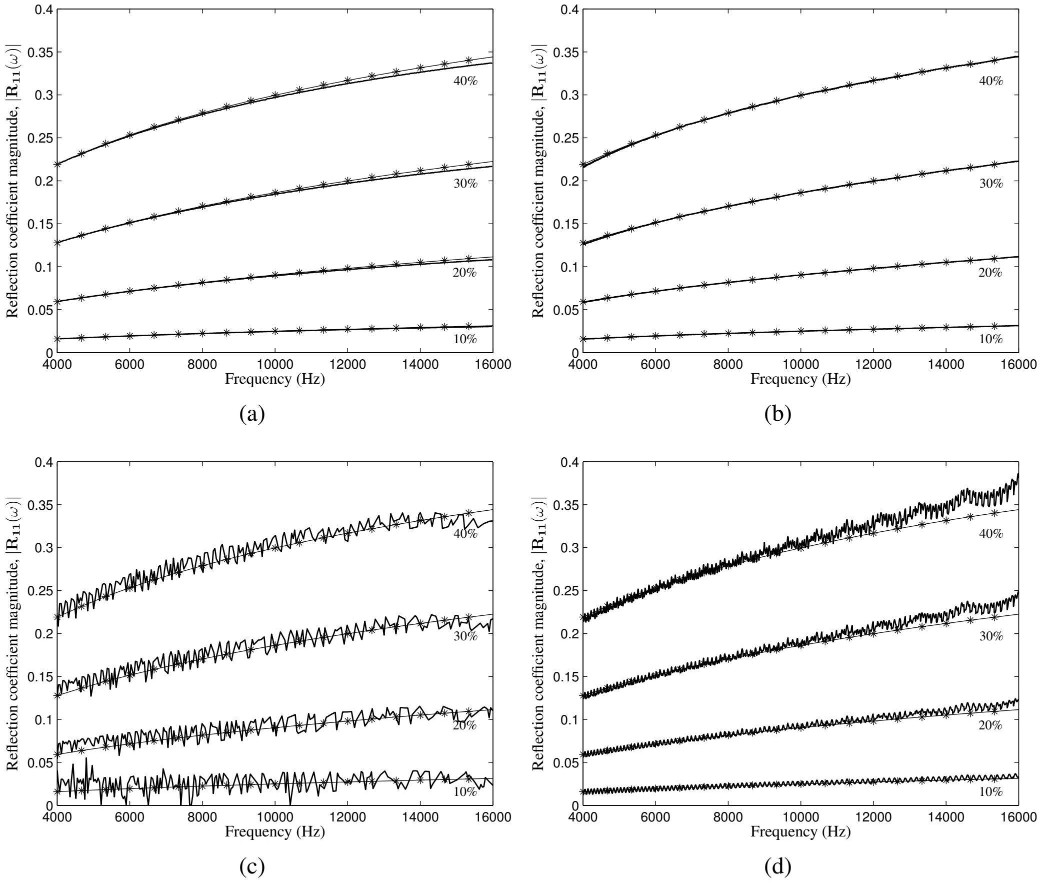

Figure 3: Estimated reflection coefficient magnitude vs. frequency based on simulated signals for relative crack depths of 10%, 20%, 30%, and 40%. Results derived by (a) CWT, (b) SPWD, (c) HHT, and (d) SPWTD. Lines with asterisks stand for the theoretical curves.

SIMULATED RESULTS

An infinite brass beam of rectangular cross-section $S = 0.016 \times 0.016\ \mathrm{m}^2$ with modulus of elasticity $E = 97.66$ GPa and mass density $\rho = 8500\ \mathrm{kg/m}^3$ is considered. A crack of varying depth is introduced and the beam is impacted by a delta Dirac force at a distance $L = 8$ m from crack location. The acceleration response (8) is evaluated at a position $L_M = 7$ m from the crack. Simulated acceleration signals are calculated for relative crack depths of 10%–40% in steps of 10% using a sampling frequency of 51200 Hz. Each case is analysed using four methods, namely CWT, SPWD, HHT and SPWTD. The curves of the reflection coefficient magnitude, for all crack depths considered, are predicted in the [4–16 kHz] frequency range by using (10). The results for all methods examined are shown in Fig. 3. For comparison reasons, the theoretical curves of the reflection coefficient derived from (7) are also depicted. The reflection coefficients predicted through the CWT analysis (Fig. 3(a)) follow the theoretical curves consistently for all crack depths and over the entire frequency range involved. The estimated reflection coefficient magnitude derived by the SPWD is in excellent agreement with theory (Fig. 3(b)). Similar results are obtained by the HHT (Fig. 3(c)) as well as by the SPWTD (Fig. 3(d)). The reflection coefficient magnitude derived by the HHT (Fig. 3(c)), although it follows the trend of the theoretical curves, exhibits an evident fluctuation, appearing stronger in the case of a relative crack depth of 10%. The reason for this fluctuation is that the HHT time-frequency distribution appears in distinct point form resulting in low resolution of power with respect to frequency. For the SPWTD (Fig. 3(d)), the predicted curves coincide with the theoretical ones up to 12 kHz, after which, a small divergence appears.

EXPERIMENTAL RESULTS

The experimental arrangement consisted of a brass beam of total length $L = 3$ m, while the rest of its properties were identical to those used in the numerical simulations. The beam was suspended horizontally by thin threads at a distance $L/10$ away from the ends to approximate free-free boundary conditions. A saw cut was introduced at $x_c = 1.01$ m from the left end to simulate the transverse crack. A miniature accelerometer was mounted at $x_a = 0.01$ m from the left free end to measure the response of the beam. The beam was excited by the impact of a steel ball (diameter 6 mm) dropped from a constant height at the left free end. Experiments were performed for three different relative crack depths, namely 10%, 20% and 30% and the corresponding spectra were obtained by the application of the four time-frequency methods.

Following the procedure described in the simulated results the reflection coefficients are estimated. It can be observed from Figs. 4(a) and 4(b) that both CWT and SPWD give similar results, with the SPWD curves being slightly closer to the theoretical ones (for relative crack depths of 20% and 30%). The predicted curves seem to deviate from the theoretical ones, for all crack depths, mainly because the theoretical reflection coefficient curves are derived from the Euler-Bernoulli beam theory that is an approximation model. The HHT (Fig. 4(c)) and SPWTD (Fig. 4(d)) seem to be more sensitive to the above-mentioned imponderable factor. More specifically, although the SPWTD result for a 10% crack proves to be quite accurate, the reflection coefficients for larger crack depths exhibit a considerable deviation from theory. This deviation is also observed for the case of HHT. Furthermore, the HHT has failed to give a prediction for the case of a 10% crack depth.

The frequency dependent group velocity of the specific beam is obtained experimentally by exciting a heathy beam of the same characteristics and is determined as $c_g(\omega) = 2L_B/[t_b(\omega) - t_i(\omega)]$ where L_B is the distance between the measurement position and the boundary of the beam, while t_b and t_i denote the arrival times of the reflected and incident waves, respectively. Then $c_g(\omega)$ is utilised in (11) to estimate the crack location. Table 1 presents the mean and the standard deviation of the absolute error of the estimated crack location ($L_M = 1$ m is the real location). From these results, we conclude that the SPWD performs better than any of the other methods. The CWT and HHT perform equally well in estimating the crack location, whereas the SPWTD exhibits considerable errors.

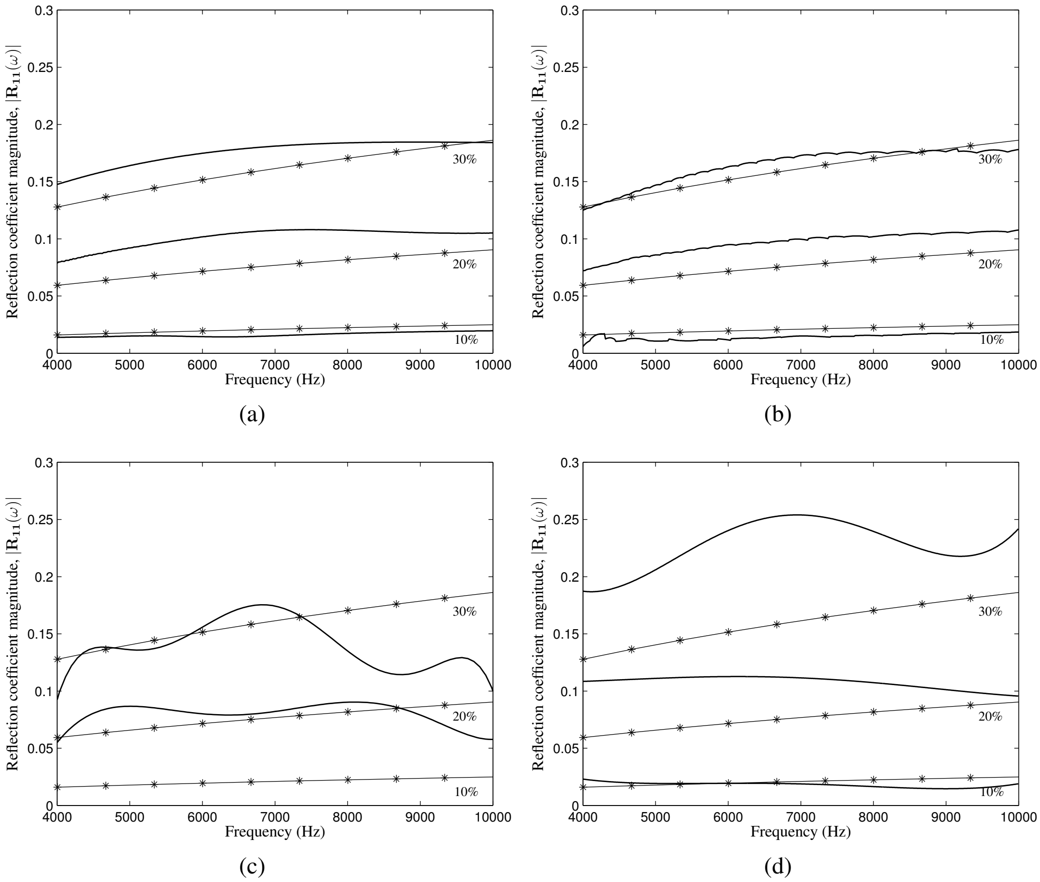

Figure 4: Estimated reflection coefficient vs. frequency based on experimental signals for relative crack depths of 10%, 20%, and 30%. Results derived by (a) CWT, (b) SPWD, (c) HHT, and (d) SPWTD. Lines with asterisks stand for the theoretical curves.

Table 1: Mean value $\bar{\epsilon}$ and standard deviation σ_ϵ of the absolute error of the estimated crack location derived by the four methods examined based on experimental signals.

Crack Depth	10%		20%		30%	
	$\bar{\epsilon}$	σ_ϵ	$\bar{\epsilon}$	σ_ϵ	$\bar{\epsilon}$	σ_ϵ
CWT	0.0211	0.0147	0.0175	0.0127	0.0155	0.0115
SPWD	0.0069	0.0048	0.0064	0.0045	0.0063	0.0044
HHT	0.0180	0.0137	0.0166	0.0118	0.0261	0.0185
SPWTD	0.0548	0.0308	0.0508	0.0431	0.1228	0.0432

CONCLUSIONS

In this paper, four time-frequency representation methods (CWT, SPWD, HHT, and SPWTD) were employed for detecting cracks in beams. The performance of each method was investigated by means of both simulated and experimental signals. Results indicate that the SPWD provides the best performance in accurately estimating both the reflection coefficient magnitude and the location of the crack. The results derived by the CWT were also reliable. However, the SPWD outperforms the CWT, because the SPWD results in finer resolution in the time-frequency plane. Concerning the HHT, it exhibited significant deviation in estimating the reflection coefficient magnitude when experimental signals were analysed. This could be attributed to fact that the time-frequency power distribution in the HHT appears in the form of distinct points. The HHT is also computationally demanding, because it employs empirical mode decomposition of the original signal into intrinsic mode functions. Finally, the SPWTD, compared to the other methods, gave the less accurate results when applied to experimental signals. Its shortcoming could be attributed to the fact that the SPWTD, despite the smoothing procedure that involves, it is still affected by the cross-terms in the time-frequency plane. Concluding, among the methods examined, the SPWD is the most accurate and reliable one for crack detection based on wave propagation of transient flexural waves. Also, the CWT performance could be further improved by appropriate selection of the mother wavelet used.

References

[1] L. Cohen, “Time-frequency distributions – A review”, Proc. IEEE, **77**, 941-981 (1989)

[2] N. E. Huang, Z. Shen, S. R. Long, M. L. C. Wu, H. H. Shih, Q. N. Sheng, N. C. Yen, C. C. Tung, H. H. Liu, “The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis”, Proc. of the Royal Soc. London Ser. A - Math. Phys. Eng. Sci., **454**, 903-995 (1998)

[3] H. Inoue, K. Kishimoto, T. Shibuya, “Experimental wavelet analysis of flexural waves in beams”, Exp. Mech., **36**, 212-217 (1996)

[4] I. K. Kim, Y. Y. Kim, “Damage size estimation by the continuous wavelet ridge analysis of dispersive bending waves in beam”, J. Sound Vibr., **287**, 707-722 (2005)

[5] K. Kishimoto, H. I. M. Hamada, T. Shibuya, “Time-frequency analysis of dispersive waves by means of wavelet transform”, J. Appl. Mech., Trans. ASME, **62**, 841-846 (1995)

[6] Z. Li, S. Xia, J. Wang, X. Su, “Damage detection of cracked beams based on wavelet transform”, Int. J. Impact Eng., **32**, 1190-1200 (2006)

[7] S. Mallat, *Wavelet Tour of Signal Processing*. (Academic Press, New York, 1998)

[8] C. Nikias, A. Petropulu, *Higher-Order Spectra Analysis: A Nonlinear Signal Processing Framework*. (Prentice-Hall, Englewood Cliffs, 1993)

[9] H. Okamura, H. W. Liu, C.-S. Shu, H. Liebowitz, “A cracked column under compression”, Eng. Fract. Mech., **1**, 547-564 (1969)

[10] J. Y. Tian, A. Li, X. Su, “Crack detection in beams by wavelet analysis of transient flexural waves”, J. Sound and Vibr., **261**, 715-727 (2003)