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ANALYSIS OF A HYBRID CONTROL ALGORITHM FOR DISTURBANCE REJECTION AND ITS APPLICATION ON AN ACTIVE ISOLATION SYSTEM

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Abstract

In this paper, hybrid control architecture is proposed which integrates FX-LMS (Filtered-x Least Mean Square) adaptive feedforward control with FSCF-LQG (Frequency Shaping Cost Functional Linear Quadratic Gaussian) feedback control into one design. Since one of the major disadvantages of the adaptive feedforward controllers is its poor transient response due to their long learning process, the transient performance can be significantly improved with the proposed hybrid control. Root locus technique is applied to provide more physical interpretation. Analytical results demonstrate that the proposed method can achieve better transient and steady state responses as compared to single feedforward or feedback design. It is found that the FSCF-LQG controller with the FX-LMS algorithm should be designed simultaneously to achieve better performance. Finally, the hybrid control is applied to an active-passive isolation system with white noise disturbance input.

INTRODUCTION

For the semiconductor manufacturing and high precision systems, vibrations caused by ground disturbances might affect their performance significantly. To reduce unwanted vibration effect, passive, active or active-passive isolation systems have been developed to protect high-precision equipments from external disturbances [1, 2]. Passive systems have the advantages of low cost, easy maintenance, and high stability. Active controls normally can achieve better performance than passive systems, but it might results in instability if there is high uncertainty payload. To overcome this problem, active-passive systems have been developed to obtain both advantages of the passive and active systems Beard[1994] where the active layer consists of piezoelectric actuators and a middle mass, and the passive is consists of elastomer and an upper mass.

The active layer is responsible for low frequency isolation, and the passive layer can provide high attenuation around high frequencies.

There were many control techniques proposed for vibration isolation systems such as classical control [1, 3], adaptive control [4], optimal LQG control [2], and neural network [5]. The optimal control can provide a systematic approach for designing controller, but the uncertainties issues are not well addressed with this approach. Adaptive control can be adapted to the changes of the system dynamics. However, the transient response of the control can be a significant problem to this approach. In this paper, we propose a hybrid control technique which integrates FX-LMS (Filtered-x Least Mean Square) adaptive feedforward control with FSCF-LQG (Frequency Shaping Cost Functional Linear Quadratic Gaussian) feedback into one design. It will be shown that that the hybrid controls can provide better performance as compared to the individual FSCF-LQG and FX-LMS. Parameter design guidelines will be provided for the hybrid control technique.

FREQUENCY SHAPING COST FUNCTIONALS LQG CONTROL

For the purpose of illustration of the fundamental concepts using hybrid control, a simple active vibration isolation system shown in Fig. 1 is used as a working example. The system dynamics is given as:

$$x_p(s) = \frac{k}{m(s^2 + 2\xi\omega_n s + \omega_n^{\ 2})} d + \frac{1}{M(s^2 + 2\xi\omega_n s + \omega_n^{\ 2})} F_c \tag{1}$$

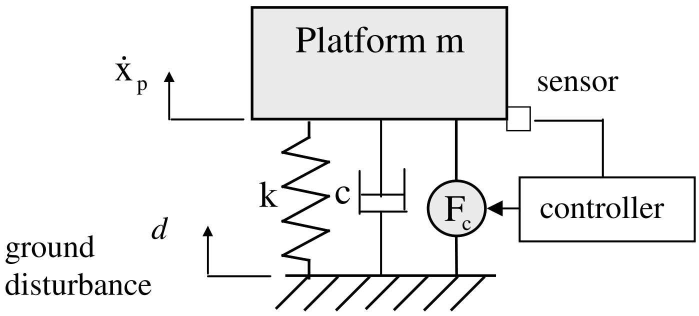

Figure 1 The active vibration isolation system

Equation (1) can be transformed into a state space model given as the following:

$$\begin{aligned} \dot{x} &= Ax + Bu + B_d d \\ y &= y_m + \theta = Cx + \theta \end{aligned} \tag{2}$$

where x are the states, u is the control input , d is the disturbance, y_m is the sensor output which is equal to $\dot{x}_p$, and θ is the sensor noise. If one is interested in controlling a particular frequency band, the output y_m can be weighted by a frequency function given as:

$$F(s)=\frac{y_f}{y_m}=C_f(sI-A_f)^{-1}B_f=\frac{s}{s^2+2\xi_f\omega_f s+\omega_f{}^2} \tag{3}$$

By transforming Eq. (2) into a state space form which is given as:

$$\begin{aligned}\dot{z}&=A_f z+B_f y_m\\ y_f&=C_f z\end{aligned} \tag{4}$$

By combining Eq. (2) and (3), the augmented plant is formed as:

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}A & 0\\ B_s C_m & A_s\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}u+\begin{bmatrix}B_d\\ 0\end{bmatrix}d+\begin{bmatrix}B_f\\ 0\end{bmatrix}F_d \quad (4)$$

$$\begin{aligned}y&=\begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}\\ y_f&=\begin{bmatrix}0 & C_s\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}\end{aligned} \tag{5}$$

The frequency shaping cost functional LQG control is to find the optimal gain K and observer gain L such that the following cost function is minimized:

$$J=\int_0^{\infty}(y_f^T y_f+u^T Ru)dt \tag{6}$$

Here R is the weighting function on the control input. By solving the two Riccati equations, one can obtain the optimal gain K and observer gain L [6]. The observations on the weighting function $F(s)$ is that the smaller ξ_F, the deeper attenuation on the selected frequency. Also, the smaller R, the wider band of the disturbance the controller will achieve.

ADAPTIVE FEEDFORWARD FILTERED-X LMS CONTROL

In this section, the FX-LMS control algorithm is introduced. The FX-LMS algorithm can be adaptive to environmental changes and thus is more robust than the conventional feedforward controller [7]. However, the transient performance of the FX-LMS is strongly dependent on the convergence rate of the control parameters and the filtered-x signal. To achieve high disturbance rejection on both transient and steady states, more analysis should be performed to investigate the behaviour of the FX-LMS algorithm.

Given a block diagram shown in Fig. 2, the plant is given as G(z) and H(z) is the secondary path from the input u to the output y. The error is given as:

$$e(k)=d(k)-\sum_{i=0}^{n}h(i)\sum_{j=0}^{m}w(j)x(k-i-j)=d(k)-W_k^T fx(k) \tag{7}$$

$$fx(k)=\sum_{i=0}^{n}h(i)x(k-i),Fx_k=\begin{bmatrix}fx(k) & fx(k-1) & \cdots & fx(k-m)\end{bmatrix} \tag{8}$$

where $fx(k)$ is the signal after the convolution of $x(k)$ with $h(k)$ and it is so-called the filtered-x signal. $d(k)$ is the disturbance, and $e(k)$ is the error.

The parameters of the controller W(z) are updated according the LMS algorithm on the gradient of the square of the errors [7].

$$W_{k+1} = W_k - \mu\hat{\nabla}_k = W_k + 2\mu e(k)Fx_k \tag{9}$$

$$\hat{\nabla}_k = \begin{bmatrix} \frac{\partial e^2(k)}{\partial w(0)} \\ \vdots \\ \frac{\partial e^2(k)}{\partial w(m)} \end{bmatrix} = 2e(k)\begin{bmatrix} \frac{\partial e(k)}{\partial w(0)} \\ \vdots \\ \frac{\partial e(k)}{\partial w(m)} \end{bmatrix} = -2e(k)Fx_k \tag{10}$$

where $W_k = [w_k(0) \;\; w_k(1) \;\; \ldots w_k(m)]^T$ $W_{k+1} = [w_{k+1}(0) \;\; w_{k+1}(1) \ldots w_{k+1}(m)]^T$ are the current and one step control parameters. μ is the chosen convergent constant. By taking the expected value of Eq. (9), the above equation can be represented by the following:

$$\mathrm{E}\{\mathrm{W}_{k+1}\} = \mathrm{E}\{\mathrm{W}_k\} + 2\mu\mathrm{E}\{\mathrm{e}(k)\mathrm{Fx}_k\} = (\mathrm{I} - 2\mu\mathrm{R})\mathrm{E}\{\mathrm{W}_k\} + 2\mu\mathrm{RW}^* \tag{11}$$

Where $\mathrm{E}\{\mathrm{Fx}_k\mathrm{Fx}_k^T\} = \mathrm{R}$，$\mathrm{E}\{\mathrm{d}(k)\mathrm{Fx}_k\} = \mathrm{P}$，$\mathrm{W}^* = \mathrm{R}^{-1}\mathrm{P}$ is the optimal control parameter. By letting $\mathrm{V} = \mathrm{W} - \mathrm{W}^*$, Eq. (11) can be represented as:

$$\mathrm{E}\{\mathrm{V}_{k+1}\} = (\mathrm{I} - 2\mu\mathrm{R})\mathrm{E}\{\mathrm{V}_k\} \tag{12}$$

The covariance matrix R is can be diagonalized to obtain the following expression:

$$\mathrm{E}\{\mathrm{V}'_{k+1}\} = (\mathrm{I} - 2\mu\Lambda)\mathrm{E}\{\mathrm{V}'_k\} \tag{13}$$

The iterative process gives the final form as

$$\mathrm{E}\{\mathrm{V}'_k\} = (\mathrm{I} - 2\mu\Lambda)^k \mathrm{V}'_0 \tag{14}$$

The sufficient condition for μ to achieve a stable convergence is given as:

$$0 < 2\mu < \frac{1}{\lambda_{max}}, \lambda_{max} \le trace(\Lambda) = trace(R) \tag{15}$$

where $\mathrm{trace(R)}$ is the sum of the power of the filtered-x signal.

$$0 < 2\mu < \frac{1}{(m+1)(fx \;\; signal \;\; power)} \tag{16}$$

It is well-known that the maximum μ that we can achieve is bounded by the power of filtered x signal. Since the *fx* signal is the convolution of the input signal x(k) with $h(k)$, it corresponds to the multiplication of $\mathrm{X}(j\omega)$ with $\hat{\mathrm{H}}(j\omega)$. With a given input signal x(k), $\hat{\mathrm{H}}(j\omega)$ becomes a dominant factor in determining the convergence rate. This gives the motivation of investigating how the hybrid control can provide a solution for changing the H(z) by introducing the optimal LQG control. The block diagram of the hybrid control is shown in Fig. 3 where the secondary path has been changed into $\mathrm{H_c(z)} = \frac{\mathrm{H(z)}}{1 + \mathrm{K_c(z)H(z)}}$. The $K_c(z)$ is the LQG controller designed from the previous section.

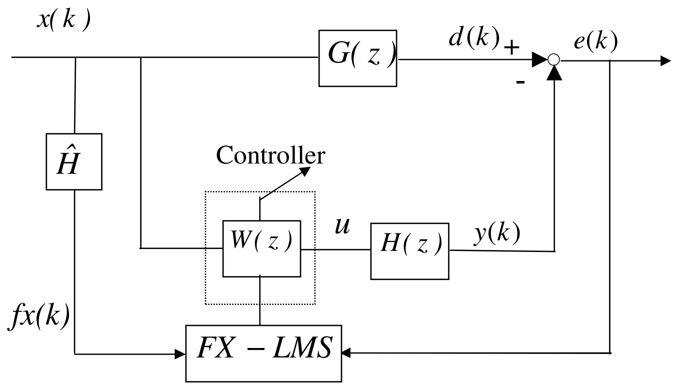

Figure 2 Block diagram of the FX-LMS algorithm

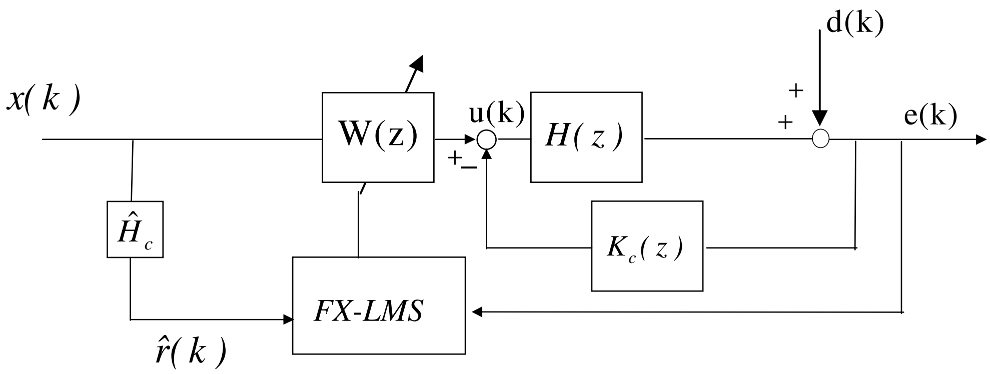

Figure 3 Block diagram of the hybrid control algorithm

HYBRID CONTROL ALOGRITHM

Although equation (12) can provide qualitative descriptions on the convergence rate of the weighting control parameters, the quantitative analysis on how the designed $H_c(z)$ affects the transient response is not an easy task. To investigate the dynamic behaviours, disturbance d is assumed to be a purely sinusoidal signal with frequency equal to ω_0. In such a condition, the secondary plant and filtered signal $\hat{r}(k)$ can be represented as:

$$\hat{H}_c(z)\Big|_{z=e^{j\omega_0}} = Ae^{j\Phi} \tag{17}$$

$$\hat{r}(k) = A\cos(\omega_0 k + \Phi) \tag{18}$$

By following the derivation given in [8], the adaptive feedforward LMS controller can be represented by an equivalent feedback controller K(z).

$$K(z) = \frac{U(z)}{E(z)} = -m\mu A\left[\frac{z\cos(\omega_0 - \Phi) - \cos\Phi}{z^2 - 2z\cos(\omega_0) + 1}\right] = -\beta K^*(z) \qquad (19).$$

The above equation shows that the poles of the K(z) is located in $z = e^{j\omega_0}$ and thus provide a high rejection on single frequency disturbance of frequency ω_0.

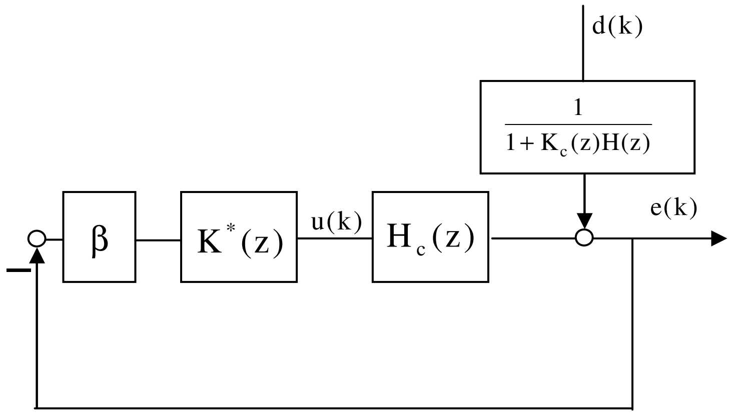

Figure 4 Equivalent block diagram of hybrid control for purely sinusoidal disturbance

The root locus technique in z-domain can be performed to give physical interpretation by varying β from 0→∞. Here the velocity of the platform is chosen as the sensor output for the simulation. Figure 5 shows the root locus of without feedback controller $K_c(z)$ and ξ equal to 0.2 in Eq. (1). The frequency of the disturbance ω_0 is chosen as ω_n. The parameters used in the simulation is ω_n equal to 15 rad/sec, the mass of the platform is 100 Kg. It is shown that there are two open poles (one from $K^*(Z)$ and the other is the open loop H(z)) for this case. The maximum β one can obtain to remain stability is the critical value as one of the root locus crosses the unit circle. The optimal β one can achieve in such a case is that the two roots have almost the same damping ratio of the closed loop poles.

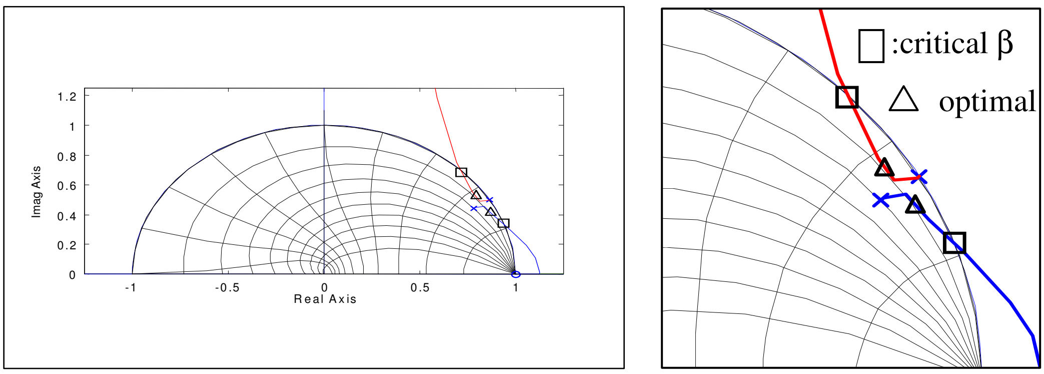

Figure 5 The root locus of the open loop system

To provide quantitative measure of the transient response, the settling time T_s is defined the minimum time such that $e^2(k)$ decaying to the 1% of the magnitude of $d^2(k)$. The optimal β shown in Fig. 5 is defined to have the minimum settling time. It is clear that the system will oscillate if β is chosen to be close to critical value It is also found that as the higher damping ratio of the closed loop poles can be achieved as the damping ratio of the plant ξ becomes higher.

The hybrid control provides a different way of changing the secondary plant $H_c(z)$. By considering the shaping function as Eq. (3), different values ξ_f are tested with a weighting R equal to 1. Since the feedback control produces a pair of complex conjugate zeros on the closed loop $H_c(z)$, and the location of the zeros is dependent on the value of ξ_f. Therefore, the zero might attract the locus and produce better transient results. Figure 6 shows the root locus for ξ_f equal to 0.4. The transient performance becomes better as comparing the case in Fig. 5. The settling time Ts using optimal μ for different ξ_f is shown in Table 1. It is surprised that choosing a smaller ξ_f equal to 0.1 results in the worst performance. It is because that the zero will be very close to the unit circle for ξ_f equal to 0.1, and thus the dominant closed loop pole will be lightly damped.

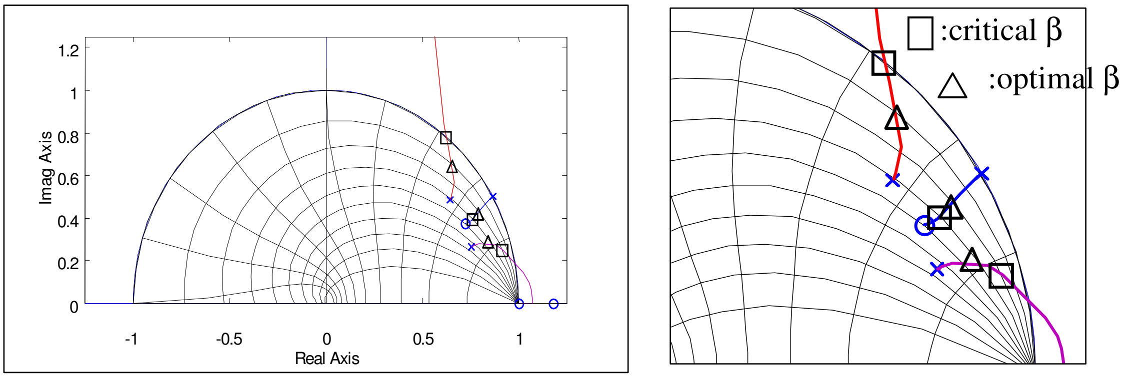

Figure 6 The root locus of the hybrid control system

Table 1 Critical, Optimal values and settling time for different ξ_f

ξ_f	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	Open loop
Optimal μ	0.32	0.14	0.064	0.031	0.017	0.012	0.008	0.007	0.006	0.0024
Critical μ	0.61	0.26	0.15	0.11	0.082	0.065	0.054	0.046	0.041	0.018
Ts (sec) (optimal μ	0.61	0.38	0.32	0.30	0.37	0.41	0.41	0.45	0.48	0.51

By performing the analysis on the weighting R, it is found that the best performance is obtained as R lies between 0.5~1. Smaller R might deteriorate the performance. Again, the results can be expected from root locus technique.

Although the analysis is based on narrow band disturbance, white noise is also tested on a real active-passive system. Figure 7 (a) and (b) show the transient performances for white noise input for the cases of adaptive feedforward LMS control, LQG and hybrid control. At t=0.5 in Fig 7(a), the adaptive LMS controller is applied and the disturbance starts to attenuate. In Fig. 7(b), the LQG control is applied at t=0. At t=0.5, the hybrid control is applied and the disturbance can be rejected at 0.2 second.

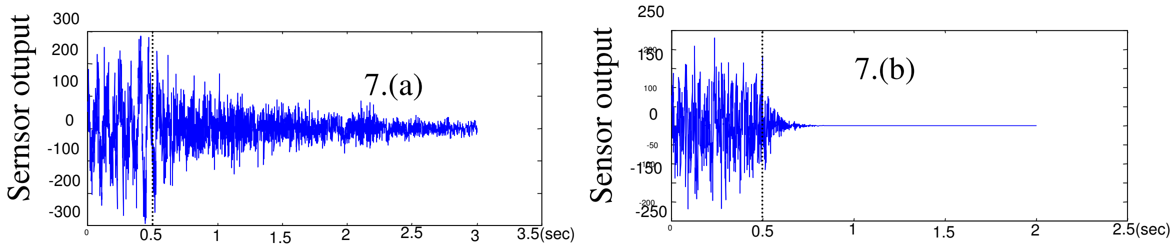

Figure 7 Comparison of different control algorithms for white noise

CONCLUSION

In this paper, hybrid control architecture is proposed which integrates FX-LMS adaptive feedforward control with FSCF-LQG feedback control into one design. By using the root locus technique, it is found that the FSCF-LQG controller with the FX-LMS algorithm should be designed simultaneously to achieve better performance. It is also shown that the hybrid control can achieve better performance in both narrowband and wide band disturbance rejection.

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