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IMPEDANCE OF MULTI-LAYERED MATCHING OF ULTRASONIC PIEZO TRANSDUCERS

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Abstract

In recent years, the study of ultrasonic waves has found indispensable applications in the real world. The tremendous impedance mismatch causes highly inefficient energy transformation between acoustical and electrical energies for an air-coupled piezo transducer. Arrangement of matching layers is then necessary for a better acoustic transmission and receiving. In this paper, the acoustic impedance matching result for the piezoceramic transducer and the propagation medium with various numbers of matching layers is presented. By extending the theory of transmission line, the impedance of each layer is obtained in conjunction with the maximally flat-top response. An impedance matrix, based on the piezoelectric wave equation, for the piezoceramic transducer is derived and employed in the transmission matrix method to obtain the mechanical impedances spectra for the piezoceramic transducer. The results of the multilayered matching structure of the transducer are helpful for the design of ultrasonic piezoelectric transducers.

INTRODUCTION

The impedances of piezoceramic patch and air are 30M Rayls and 430 Rayls respectively. The tremendous impedance mismatch causes highly inefficient energy transformation between acoustical and electrical ports for an air-coupled piezoceramic transducer. Arrangement of matching layers is then necessary for a better acoustic transmission and receiving.

Minoru Toda [1] showed that, by tuning the dimension of the air gap, the impedance matching can be adjusted for the air-coupled transducer. Two different structures were discussed in [1]. Tadeusz Gudra et. al. [2] presented the transfer function of multi-layered matching for piezoceramic air-coupled transducer. The transfer function was optimized by choosing the number and impedance values of the matching layers. At the frequency of 100MHz, Tomas E. Gomez et. al. [3] investigated the characteristics of silicon gel as the matching layers for piezo-transducer and proposed the fabrication process.

In this paper, an impedance matrix, based on the piezoelectric wave equation, for the piezoceramic transducer is derived and a numerical model to simulate the structure of multi-layered matching by using transmission matrix method is presented. The impedance spectrum at the mechanical port can be obtained to evaluate the matching condition with the acoustic medium.

STRUCTURE OF TRANSDUCER

An ultrasonic transducer consists of three main components: backing material, active element, and matching layer (Fig. 1). In this paper, the theory of transmission line was employed and the quarter wavelength ($\lambda/4$) matching was effectively equivalent to quarter wavelength transmission line. Every increment of single matching layer is treated as a length increment of transmission line by $\lambda/4$. Because $\lambda/4$ is small, the transmission loss in the matching can be neglected.

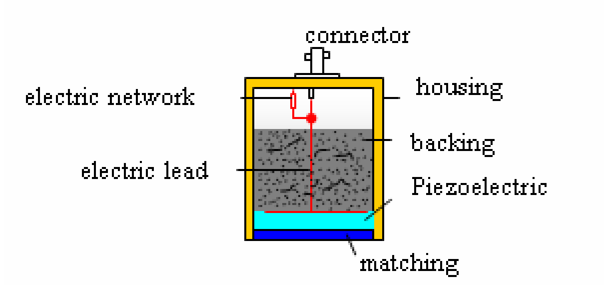

Figure 1 – Schematic diagram of a transducer

EQUIVALENCE OF TRANSMISSION LINE TO MATCHING LAYER

Consider N layers of matching serially connected as shown in Fig. 2. The reflection coefficient at the 1st interface is $\Gamma_0 = (Z_1 - Z_c)/(Z_1 + Z_c)$, at the last ($N^{th}$) interface is $\Gamma_N = (Z_L - Z_N)/(Z_L + Z_N)$, and at the n^{th} interface is $\Gamma_n = (Z_{n+1} - Z_n)/(Z_{n+1} + Z_n)$ where $n = 1 \sim N-1$, Z_c and Z_L are impedances of piezoceramic and load, respectively, and Z_1 to Z_N represent the impedances of 1st to N^{th} matching layers.

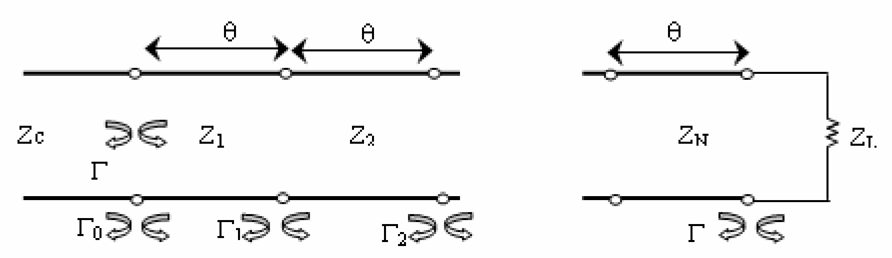

Figure 2 – Transmission line model of matching layers

With equal thickness $\ell = \lambda/4$ for each matching layer, the equivalent electric length $\theta = kl$, as shown in Fig. 2, will be also equivalent. Here, k and l are the wave number and the physical length of the matching layer, respectively. Assuming the magnitude differences of the impedance between each matching layer are small, the total reflection coefficient can be approximated as [4].

$$\Gamma(\theta)=\Gamma_0+\Gamma_1 e^{-2j\theta}+\Gamma_2 e^{-4j\theta}+\cdots\cdots+\Gamma_N e^{-2jN\theta} \tag{1}$$

Theoretically speaking, with fixed number of matching layers, the passband close to the central frequency will be optimally flattened. To obtain the most flat response by using N matching layers, the derivatives of $|\Gamma(\theta)|$ from the first to the $(N\text{-}1)^{\text{th}}$ order with respect to θ at the central frequency f_0 have to be zero, i.e.,

$$\Gamma(\theta)_{\theta=f_0} = \Gamma'(\theta)_{\theta=f_0} = \cdots = \Gamma^{N-1}(\theta)_{\theta=f_0} = 0 \tag{2}$$

Therefore, $\Gamma(\theta)$ can be assumed as $\Gamma(\theta)=A\cdot(1+e^{-2j\theta})^N$ and equal to Eq. (1) as

$$\Gamma(\theta)=A\sum_{n=0}^{N} C_n^N e^{-2jn\theta}=\Gamma_0+\Gamma_1 e^{-2j\theta}+\Gamma_2 e^{-4j\theta}+\cdots\cdots+\Gamma_N e^{-2jN\theta} \tag{3}$$

Therefore $\Gamma_n=A\cdot C_n^N$ and the impedance relationship can be arranged as

$$\ln\frac{Z_{n+1}}{Z_n}=2^{-N}C_n^N\ln\frac{Z_L}{Z_C} \tag{4}$$

TRANSMISSION MATRIX

The equivalent circuit of the piezo material is modelled as a three port electrical element with two acoustic ports and one electric port. The three port equivalent circuit is employed in the transmission matrix method to establish the transmission relation between mechanical and electrical energies. Force and particle velocity are equivalent to electrical voltage and current, respectively. In the form of energy, the transmission matrix relates the multi-layered matching and the transducer by the multiplication of matrices and simplifies the procedures of solving boundaries conditions between layers [5].

Being a three-port element, in addition to the acoustic energy transmission, the electrical current induced from the particle velocity and the applied electrical voltage has to be considered in the piezoelectric transducer. The impedance matrix for a piezo transducer is derived as

$$\begin{bmatrix} F_1 \\ F_2 \\ V \end{bmatrix} = -i \begin{bmatrix} \frac{Z_0}{\tan(kd)} & \frac{Z_0}{\sin(kd)} & \frac{h_{33}}{\omega} \\ \frac{Z_0}{\sin(kd)} & \frac{Z_0}{\tan(kd)} & \frac{h_{33}}{\omega} \\ \frac{h_{33}}{\omega} & \frac{h_{33}}{\omega} & \frac{1}{\omega C_0} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix} \tag{5}$$

The electrical voltage and current are related to the acoustic energy as

$$V_i = \left(W^+ + W^-\right)\sqrt{Z_0} \tag{6}$$

and

$$\mathrm{I_i} = \frac{\left(W^+ - W^-\right)}{\sqrt{Z_0}} \tag{7}$$

where i = 1, 2, Z_0 is the characteristic impedance of the piezo element, W^+ and W^- are the square root of acoustic energies, and the superscript + and – denote the energy flows into and out of the element, respectively. Substituting the voltage and current by the energies transferring along positive and negative directions, a transmission matrix **T,** corresponding to the impedance matrix (Eq. (5)), which relating the acoustic energies and the applied electrical field is obtained as

$$\begin{bmatrix} W^+{}_{i-1} \\ W^-{}_{i-1} \\ I \end{bmatrix} = \begin{bmatrix} \mathrm{T}_{11} & \mathrm{T}_{12} & \mathrm{T}_{13} \\ \mathrm{T}_{21} & \mathrm{T}_{22} & \mathrm{T}_{23} \\ \mathrm{T}_{31} & \mathrm{T}_{32} & \mathrm{T}_{33} \end{bmatrix} \begin{bmatrix} W^+{}_i \\ W^-{}_i \\ V \end{bmatrix} \tag{8}$$

This transmission matrix **T** (Fig. 3) can be used for serial connection with various structures.

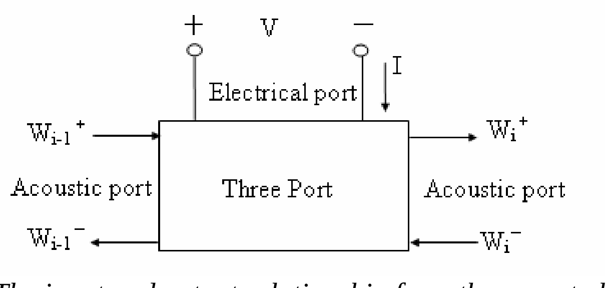

Figure 3 – The input and output relationship for a three-port element

Similarly, the equivalent transmission matrix can be applied for two-port elements (Fig. 4) such as backing and matching materials, without considering the piezoelectric effect. The matrix is expressed as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cos kd & jZ_0 \sin kd \\ j\dfrac{1}{Z_0}\sin kd & \cos kd \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{9}$$

where Z_0 is the characteristic impedance of the element material, k is the propagation constant, and d is the length of the transmission line.

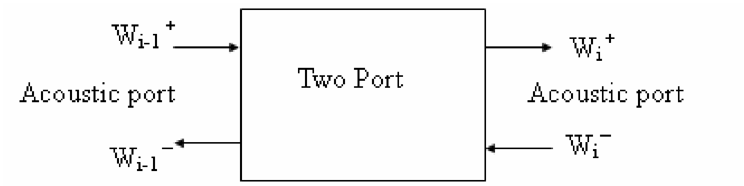

Figure 4 – The input and output relationship for a two-port element

Expressing the electrical voltage and current as the form of energy, Eq. (9) is transformed into

$$\begin{bmatrix} W_{i-1}^{+} \\ W_{i-1}^{-} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2}(A+\dfrac{B}{Z_0}+CZ_0+D) & \dfrac{1}{2}(A-\dfrac{B}{Z_0}+CZ_0-D) \\ \dfrac{1}{2}(A+\dfrac{B}{Z_0}-CZ_0-D) & \dfrac{1}{2}(A-\dfrac{B}{Z_0}-CZ_0+D) \end{bmatrix} \begin{bmatrix} W_i^{+} \\ W_i^{-} \end{bmatrix} \tag{10}$$

where W_{i-1}^{+}、W_{i-1}^{-} and W_i^{+}、W_i^{-} are the acoustic signals, and the element *A*, *B*, *C*, *D* show the relation between the input and output electric signals.

Combining the three-port and two-port transmission matrices and employing the corresponding boundary conditions, the impedance characteristics of transducer with various structures and materials can be analyzed.

The block diagram of the transmission matrices is shown in Fig. 5. *V* and *I* are the applied electrical signals. **B**, **M**, and **P** represent the matrices for backing, matching layer, and piezoelectric material, respectively. The numbers after the matrices denote the order.

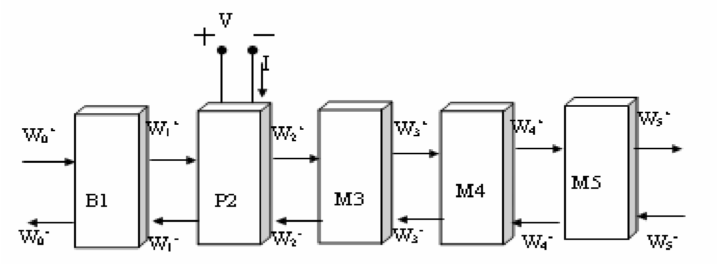

Figure 5 – The block diagram of transmission matrices

By the energy iteration the energy equivalence can be established as

$$\begin{bmatrix} W_0^{+} \\ W_0^{-} \end{bmatrix} = [B1][P2][M3][M4][M5]\begin{bmatrix} W_5^{+} \\ W_5^{-} \end{bmatrix} + [B1]\begin{bmatrix} \mathrm{T}_{13} \\ \mathrm{T}_{23} \end{bmatrix}_2 V \qquad (11)$$

From the **T** matrix for piezo material in Eq. (8), the voltage and current is related as

$$I = [T_{31} \quad T_{32}]_2 [W_2] + V[T_{33}]_2 \qquad (12)$$

If we set the impedance for the electrical port to be 50 Ω and let the impedance of left mechanical port of the piezo element to be Z_A and the impedance of backing to be $Z_{backing}$, respectively, then $V/I = 50$, $W_0^{+} = -REF_A * W_0^{-}$, where $REF_A = \frac{Z_A - Z_{backing}}{Z_A + Z_{backing}}$.
The relationship between [W_2] and [W_5] can then be rearranged as

$$\begin{bmatrix} W_2^{+} \\ W_2^{-} \end{bmatrix} = [M3][M4][M5]\begin{bmatrix} W_5^{+} \\ W_5^{-} \end{bmatrix} \qquad (13)$$

Similarly, the relationship between [W_0] and [W_5] can be determined and, by using Eq. (11), the transmitted energy W_5^{+}, W_5^{-}, W_0^{+}, and W_0^{-} can be calculated. Equation (10) can be used to find the characteristic impedance for each layer. Tables 1-3 show the values of the multi-layered characteristic impedances.

Table 1 Characteristic impedances of 1-5 layered matching for piezo ceramic and air

no of layers	characteristic impedance for each layer (Rayls)				
	1st layer	2nd layer	3rd layer	4th layer	5th layer
single layer	Z_1=120400.9				
2 layers	Z_1=2014699.75	Z_2=7195.3			
3 layers	Z_1=8241393.42	Z_2=120400.9	Z_3=1759		
4 layers	Z_1=16668000	Z_2=996100	Z_3=14522	Z_4=869.66	
5 layers	Z_1=23705000	Z_2=4074700	Z_3=120400.9	Z_4=3557.5	Z_5=611.49

Table 2 Characteristic impedances of 1-5 layered matching for piezo ceramic and water

no of layers	characteristic impedance for each layer (Rayls)				
	1st layer	2nd layer	3rd layer	4th layer	5th layer
single layer	$Z_1=7.11*10^6$				
2 layers	$Z_1=1.55*10^7$	$Z_2=3.26*10^6$			
3 layers	$Z_1=2.28*10^7$	$Z_2=7.11*10^6$	$Z_3=2.21*10^6$		
4 layers	$Z_1=2.78*10^7$	$Z_2=1.28*10^7$	$Z_3=3.97*10^6$	$Z_4=1.82*10^6$	
5 layers	$Z_1=3.06*10^7$	$Z_2=1.89*10^7$	$Z_3=7.11*10^6$	$Z_4=2.69*10^6$	$Z_5=1.65*10^6$

Table 3 Characteristic impedances of 1-5 layered matching for piezo ceramic and steel

no of layers	characteristic impedance for each layer (Rayls)				
	1^{st} layer	2^{nd} layer	3^{rd} layer	4^{th} layer	5^{th} layer
single layer	Z_1=3.72*10^7				
2 layers	Z_1=3.54*10^7	Z_2=3.91*10^7			
3 layers	Z_1=3.46*10^7	Z_2=3.72*10^7	Z_3=4.01*10^7		
4 layers	Z_1=3.41*10^7	Z_2=3.59*10^7	Z_3=3.86*10^7	Z_4=4.06*10^7	
5 layers	Z_1=3.39*10^7	Z_2=3.50*10^7	Z_3=3.72*10^7	Z_4=3.96*10^7	Z_5=3.84*10^7

Fig. 6 shows the frequency response of reflectivity for piezoceramic transducer with multi-layered matching. The acoustic medium is air, water, and steel respectively. It is apparent that the bandwidth of the passband around the normalized central frequency increases with increasing number of layers and the magnitude of reflectivity within the passband approaches 0. It means that the efficiency of energy transfer from piezo transducer to the acoustic media will increase with more layers of matching. Fig. 6(d) compares the results of five layers shown in Fig. 6 (a)-(c). It can be seen that due to the impedances for piezoceramic and steel are closer, matching is more effective.

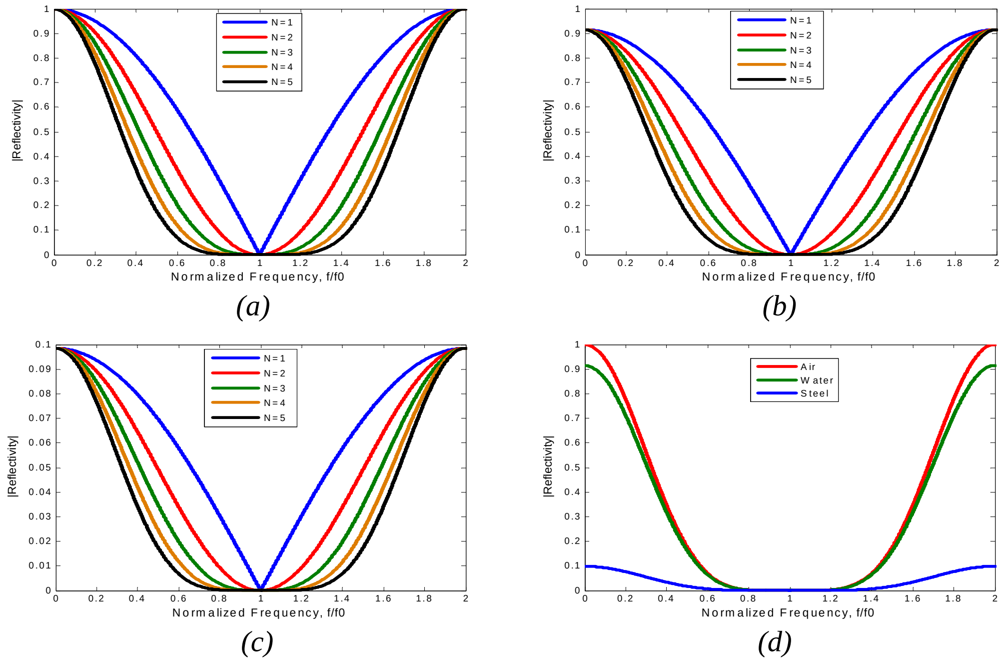

Figure 6 – (a)Frequency response of reflectivity for multi-layered matching with air (N: no of layers). (b)Frequency response of reflectivity for multi-layered matching with water (N: no of layers). (c) Frequency response of reflectivity for multi-layered matching with steel (N: no of layers). (d) Comparison of frequency response of reflectivity for five-layered matching with air, water, and steel.

From the transmission matrix shown as in Eq. (11) and given the material constants for backing B1, piezoceramic patch P2, and matching layers M3, M4, and M5 and the working resonant frequency, the impedance can be calculated. Fig. 7 shows the frequency responses of impedance for transducer w/o backing and matching. The 1 - 3 layers of matching are selected, respectively, from Tables 1-3 for most flat response. The acoustic media are air, water, and steel. It is seen that at the resonant frequency the impedances are matched for the transducer and the acoustic media.

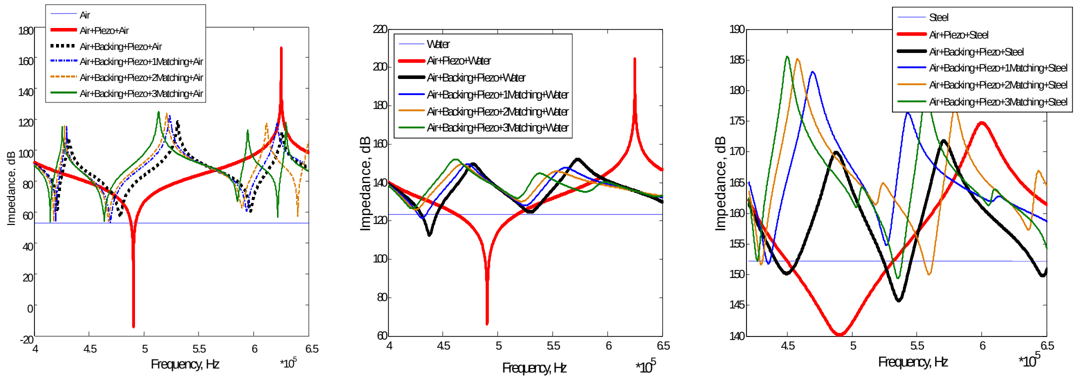

Figure 7 – (a) Frequency response of output impedance for transducer in air. (b) Frequency response of output impedance for transducer in water. (c) Frequency response of output impedance for transducer in steel.

CONCLUSIONS

By using the concept of transmission line, the impedances of multi-layered matching are calculated to increase the bandwidth of the ultrasonic transducers. Besides, the impedance matrix was employed to establish the transmission matrix for the calculation of output mechanical impedance of transducer with multi-layered matching. The optimal impedance for each layer of matching is provided. With the calculated matching impedances the output mechanical impedance of the transducer will match the impedance of the acoustic media, air, water, and steel, perfectly.

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