Linear Acoustic Simulation and Experimental Characterisation of a Modular Automotive Muffler

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Abstract

Development schedules and acoustic performance criteria for automotive mufflers require ever-increasing understanding of the flow phenomena inside the muffler. The ability to predict the effect of changes to the muffler layout quickly and accurately is needed. One option is to use a linear acoustic (frequency domain) approach to simulate the different parts of the muffler. This has the advantage of being extremely quick compared to both non-linear (time domain) simulation methods and experimental techniques. In order to test this approach a modular version of a production automotive muffler consisting of multiple chambers, perforated baffles and pipes with perforated sections is used. The arrangement of the muffler can be changed in terms of porosity of perforated sections, positions of baffles and lengths of pipes. A linear acoustic computer simulation is then used to predict the transmission loss of different configurations of this modular muffler. All parts of the muffler are modelled using fundamental linear acoustic elements such as pipes and quarter wave resonators. The perforated sections are modelled using lumped impedance elements based on theoretical models for the flow through the holes. Measurements of the transmission loss of two different configurations of the modular muffler are used to validate the simulation models. These measurements are done using cold flow on a test bench using the two microphone method.

INTRODUCTION

Linear acoustic simulation is a frequency domain approach to determine the acoustic properties of a system. The transfer matrix method is used to determine pressures and volume flows for each model element in the acoustic circuit of the system. This method uses the relationship between two pairs of state variables coupled by a two-dimensional matrix. This matrix is known as the transfer matrix. The state variables are the temporal Fourier transforms of the sound pressure and sound volume velocity at the inlet and the acoustic pressure and volume flow velocity at the outlet. Provided certain assumptions are valid there exists a complex 2*2 matrix which completely describes the sound transmission within a system at a particular frequency:

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} \tag{1}$$

The main results available are the transmission loss, insertion loss and noise reduction. The major advantages of the linear analysis are the numerical speed and stability entailing from the frequency domain analysis. The disadvantages are the lack of non-linear dissipation and wave steepening effects.

This work uses this linear acoustic approach to model a modular automotive muffler. The modular muffler closely matches a production muffler and consists of five chambers and two pipes with perforated sections. The baffles and the perforated sections of the modular muffler can be moved or closed. This allows a wide range of configurations to be tested. A total of five variations of this muffler were considered but only two arrangements are described in this paper for clarity. Both cases are without flow and cold conditions in order to match the measurement setup.

The paper starts by developing the theory on how the perforated pipes and baffles are modelled. This is then used in the linear acoustic model of the muffler model. The actual measurements on the same muffler arrangements that are modelled are then described and compared to the simulation results.

THEORY

The basic element to be considered is two parallel flow ducts which are joined together by a perforated section of length L and specific acoustic impedance Z_w. The impedance can vary along the perforate. The main idea of the segmentation approach is to divide the perforated section into a number of segments along the length of the ducts. Each segment consists of two parts. The first part is two parallel hard ducts where convective plane wave motion is assumed. The second part is discrete impedance, which can be seen as an open branch representing a parallel coupling of all the holes in the segment.

Consider a short segment Δx of a perforated wall as in Figure 1, the pressure and normal volume velocity on each side are related by

$$U_1 = U_2 \tag{2}$$

$$\frac{p_1 - p_2}{U_1} = Z \tag{3}$$

where p is the acoustic pressure, U is the volume velocity incident on the perforated section, and $Z = Z_w/S_w$ is the segment impedance (S_w is the wall area of the perforated segment), 1 denotes the inlet side and 2 denotes the outlet side. Only plane waves are assumed to propagate on each side of the perforate and parallel to it.

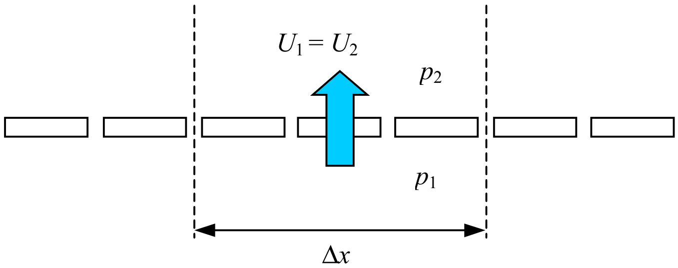

Figure 1 - A segment of the perforated wall.

A code to analyse low frequency sound propagation in complex duct networks is called BOOST-SID. It is based on the representation of a duct network as a network of two-ports. The two-port elements are then joined and analysed using the method described in reference [1]. The present version of SID couples the elements at each node using the continuity of pressure and volume velocity. In order to model perforated tubes using this scheme [2], they are divided into discrete lumped impedance elements separated by hard segments on both sides of the perforate in a similar way to that suggested by Sullivan [3]. The main difference here is that the perforated tube is described by a number of two-port elements instead of the four-port elements used by Sullivan. The representation in the form of two-ports is attractive since two-port codes are commonly used for muffler analysis, and the proposed method makes it possible to model arbitrary complex perforated systems.

Assuming that the perforated section Δx is much shorter than the acoustic wavelength λ, then it can be represented as a lumped element. The two-port of such an element is found by rearranging equations (2) and (3) in matrix form to give

$$\begin{bmatrix} p_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_2 \\ U_2 \end{bmatrix} \tag{4}$$

where Z is the lumped (“point wise”) impedance associated with the two-port element and is given by

$$Z = \frac{p_1 - p_2}{U} = \frac{Z_w}{S_w} \tag{5}$$

where S_w is the wall area of the perforated section.

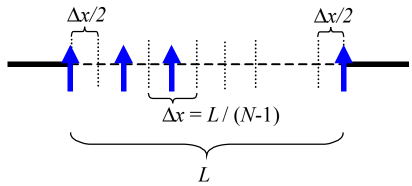

Figure 2 - Location of the lumped elements. Note: A lumped element is put at the beginning and end. From the figure, it follows that Δx = L / (N-1) where N is the number of lumped elements.

The perforated tube is divided into several segments and the perforate effect is lumped at discrete points separated by hard pipes. The number of segments must be large enough to obtain acceptable accuracy. The length of each segment should be much smaller than the wavelength (say less than λ /8). To determine the number of necessary lumped elements, N, we can use the following criteria

$$\Delta x = \frac{L}{N-1} \leq \frac{\lambda}{r} \tag{6}$$

where r is the required resolution and L is the length of the perforated section of the tube. The wavelength will be determined by the maximum frequency of interest, normally determined by the plane wave (1-D) limitation. In terms of frequency, the relation becomes

$$N \geq 1 + \frac{L \cdot r \cdot f}{c} \tag{7}$$

N should be rounded off to the nearest larger integer. It is better to put one lumped element at the beginning and one at the end of the perforate in order to model the lengths of any attached devices accurately. For a perforated section divided into N segments, the lumped elements will be placed as in Figure 2. The area of each element will be the lateral surface area of the corresponding segment ($\pi D L_i$), where Li is equal to the segment length Δx except at the edge elements where it is equal to $\Delta x/2$.

Perforate Impedance Model

The key design parameter of perforates is the acoustic impedance. The impedance determines the efficiency of perforates to absorb sound waves. Several studies have been conducted to develop impedance models, including the effect of different configurations and surrounding conditions. In spite of the large number of published research, a single verified global model does not exist. The objective of an earlier work done at KTH[1] was to review all available models in the literature and include all effects in a single global model. A model based on Crandall's theory[5] was developed for the no flow case. Some parameters were included from later models which were not in the original model. Other parameters were determined from experiments and semi-empirical formulae were derived. Measurements were performed to verify the model. The grazing and bias flow impedance values were calculated according to Bauer[6]. The resistance consists of four terms due to viscous losses inside the hole, radiation resistance to the vibrating piston of air inside the orifice, grazing flow term, and bias flow term. The reactance is assumed to be only caused by the mass plug and any flow effects on the reactance are neglected here. The proposed model can be summarized in the following equations for the normalized perforate resistance and reactance respectively

$$\theta = \mathrm{Re}\left\{\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')}+\frac{\delta_{re}}{F(\mu)}f_{\mathrm{int}}\right]\right\}+\frac{1}{\sigma}\left[1-\frac{2J_1(kd)}{kd}\right]+\frac{0.3}{\sigma}M_g+\frac{1.15}{\sigma C_D}M_b \tag{8}$$

$$\chi = \mathrm{Im}\left\{\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')}+\frac{0.5d}{F(\mu)}f_{\mathrm{int}}\right]\right\} \tag{9}$$

where t is orifice thickness, d is the orifice diameter, σ is the porosity, k is the wavenumber ω/c, c is the speed of sound, C_D is the orifice discharge coefficient, J is the Bessel function, $\upsilon=\tilde{\mu}/\rho_0$ is the kinematic viscosity, ρ is the fluid density, μ is the adiabatic dynamic viscosity, μ'=2.179 μ, M_g is the grazing flow Mach number, and M_b is the bias flow Mach number inside the holes of the perforate. The rest of the parameters are defined as follows

$$K=\sqrt{-\frac{j\omega}{\upsilon}},\; K'=\sqrt{-\frac{j\omega}{\upsilon'}} \tag{10}$$

$$F(Kd)=1-\frac{4J_1(Kd/2)}{Kd\cdot J_0(Kd/2)} \tag{11}$$

$$\delta_{re} = 0.2\,d + 200\,d^2 + 16000\,d^3 \tag{12}$$

$$f_{\mathrm{int}} = 1-1.47\sqrt{\sigma}+0.47\sqrt{\sigma^3} \tag{13}$$

Equation (13) is a correction factor for the orifice interaction effects.

SIMULATION

Figure 3 shows the two configurations considered in this work. A baseline configuration and no perforations on the inlet pipe. There is no mean flow and the pressures and temperature reflect the measurement conditions at room temperature.

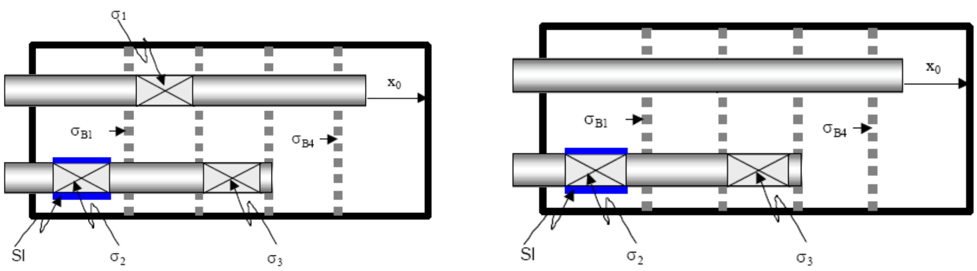

Figure 3 - Muffler configurations

In both cases the muffler is reduced to fundamental linear acoustic modelling components such as pipes and quarter wave resonators. For the perforated sections the theory described earlier is used to determine the lumped impedance values. In order to determine the number of segments for each of the perforated sections, the frequency limit of the model and the length of the perforated part should be considered. The 1D model of the muffler is valid up to approximately 500Hz where the first cross mode inside the muffler starts to propagate. Using the perforated section on the inlet pipe as an example (90mm length) and using a resolution of 8 (r in equation 7) gives 3 elements. This should be sufficient to model the perforated section up to 500Hz.

The perforated baffles are modelled using single lumped impedance elements. The same model is used to model both configurations with the exception that the model of the perforated pipe in the inlet pipe is removed for the second configuration. These two configurations are modelled using the linear acoustics software package [[7]]. The schematics of the simulation model layouts are shown in Figure 4.

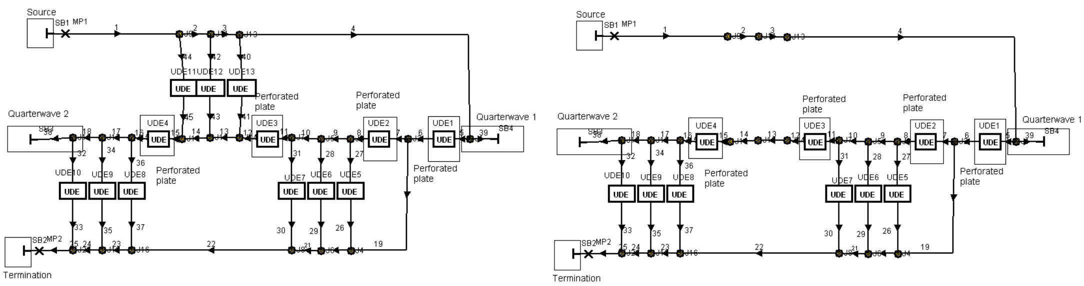

Figure 4 - Model arrangement

MEASUREMENT

In order to identify the actual transfer matrix (T), of the muffler, the two pairs of acoustic state variables (pressure and volume flow) related by T have to be evaluated across the whole frequency band of interest. Therefore, the two microphone multi-load method is used on each side of the muffler. From acoustic pressure measurements at two different locations in a duct one can derive the pair of state variables *(p, v)* at a given point elsewhere in the duct. But for a given frequency, the signal to noise ratio (SNR) of the measurements depends strongly on the configuration of the duct line. Both the duct length and the presence of singularities (changes in diameter, local impedances…) determine the stationary wave configuration in the duct. Therefore, since the positions of the two microphones remain constant for practical matters, multiple line configurations (multi-load) have to be tested in order to get a consistent set of data for each frequency of interest. Two microphones (B&K ¼") are flush mounted on the duct line with a 60 mm spacing at both the inlet and outlet. (The model of the measurement setup is shown in Figure 5: x_1 = 225mm, x_2 = 165mm, x_3 = 135mm, x_4 = 195mm). A number of different duct elements are available for assembling the upstream and downstream lines. They consist of straight tubes with different lengths, curved tubes, or tubes with impedance singularities. Finally, two configurations are selected. The same set is to be used for the different inside arrangements of the modular muffler under test. Only two of these configurations are described in this paper.

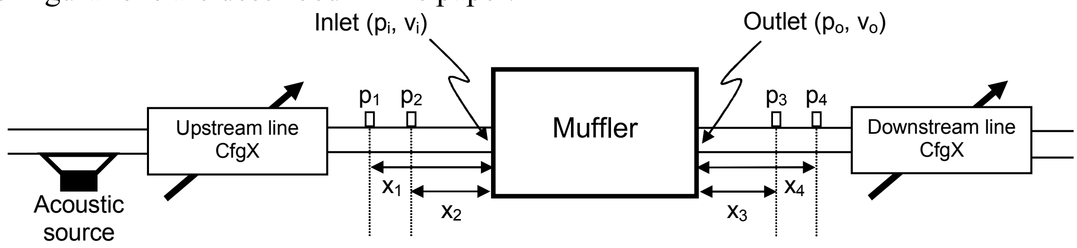

Figure 5 - Measurement Setup

A high power loudspeaker provides the acoustic stationary waves. This is driven with successive harmonic signals from 20Hz to 2kHz. The frequency band is swept using 60 lines per octave logarithmic progression. One measurement provides a phase referenced spectrum for each of the four microphones and for a given configuration of the lines up and downstream. The duration of a measurement for each line configuration is about 10 minutes.

RESULTS

The measured and simulated transmission loss for the two muffler configurations considered in this work are shown in Figure 6 and Figure 7.

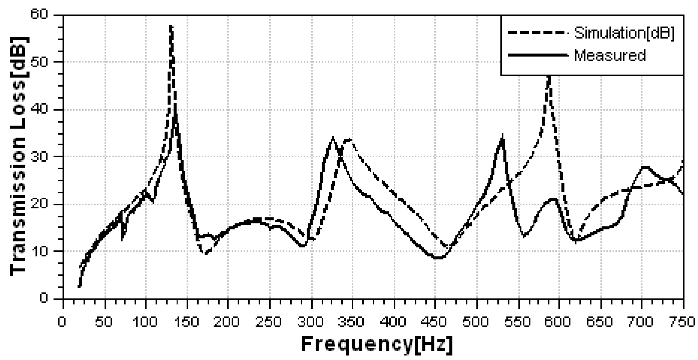

Figure 6 - Comparison of results for muffler with perforation on inlet pipe

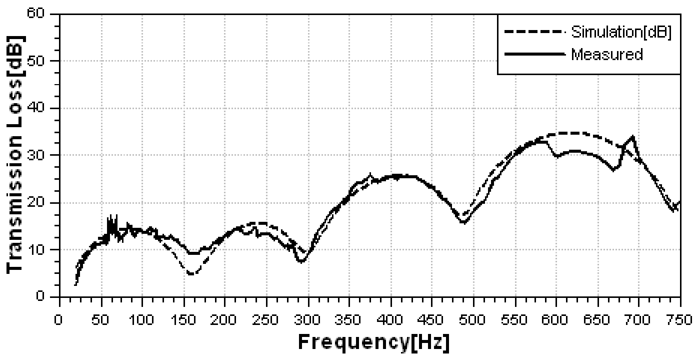

Figure 7 - Comparison of results for muffler without perforation on inlet pipe

These results show that the perforated section of the inlet pipe has a large effect on the transmission loss between 50 and 150 Hz. There are also differences between 300 and 350 Hz. Both these differences are shown in both the measurement and simulation results. The results also show a good overall comparison in both cases between the measured and simulated transmission loss up to the limit of the plane wave range at about 500Hz.

CONCLUSIONS

A modelling theory for the prediction of the acoustic performance of perforates has been developed. This has been applied to the linear acoustic models for two configurations of an automotive muffler. These have been measured experimentally for transmission loss and compared to the model predictions. These show a good comparison in the plane wave region. The simulation closely matches the variation in the measured transmission loss of the muffler caused by a change in one of the perforated sections. This provides some validation for the theory developed for the modelling of perforates. This opens the possibility to model a realistic muffler configuration and predict the acoustic effect of geometrical variations quickly and accurately using simulation methods. However, the current work has been limited to cases with cold no mean flow and further work needs to be done to validate the model for actual engine operating conditions with mean flow.

ACKNOWLEDGEMENTS

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