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ON ACOUSTICAL PROPERTIES OF ANCIENT CHINESE MUSICAL BELLS

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Abstract
The history of Chinese music bells can be traced back to the Shang dynasty (1600–1100 B.C.). In addition to their significances in history and metallurgy, the unique acoustical properties of Chinese music bells not only indicate level of applications of acoustical know-how at that time, but also present interesting questions to modern acoustics. They differ from carillon/church bells and oriental temple bells by their almond-shaped cross sections, which acoustically result in two distinct tones, and short decay time. The interval between the two strike notes is often found to be minor third. An interval of major third is also found between the normal-strike tones of the adjacent bells. In this paper we discuss the tonal properties of those music bells, and their relevance to the design of 12 semi-tones music scale in an octave frequency band and to the balance of the bell sound as a set.

INTRODUCTION TO THE TWO TONE FEATURE

Figure 1 shows the Marquis Zeng Yi's musical bells, which were discovered in the Chinese province of Hubei in 1978, dated from more than 2400 years ago. The set consists of 65 bells hanging in three rows on the elaborately carved L-shaped frame. In this paper, it is numbered in three groups by I, J, K, and each group has three rows. The sound and vibration of music bells at Beijing Bell Museum were also measured to provide more experimental evidences. Figure 2 shows the sound pressure spectrum of the normal and side-strike tones of the bell #J1.1 at the Museum. When the bell is excited near the normal-strike position (upper Figure 2), a fundamental frequency of 392.6 Hz (G_4) is observed in the spectrum. A lower peak at 468.8Hz ($^bB_4$) is due to the contribution of the second bell mode, which can be best excited at the side-strike position. As high level of G_4 is heard right at the beginning of the sound decay, and the higher frequency partials decay at the faster rates, a clear note of G_4 can be heard. When the bell is excited near the side-strike position (lower Figure 2), the sound pressure level at the second peak is more than 20 dB higher than that at the first. Thus a clear note of $^{b}B_4$ results. The two tone feature of the bells results from the degeneration of the fundamental modes (the circumferential modes $(2,0)$) of the music bells due to the breakdown of the symmetry of the cross section. Figure 3 shows the FEA vibration mode shapes of the symmetrical $(2,0)_S$ (with respect to the major axis of the cross section) and anti-symmetrical mode $(2,0)_A$ of a music bell.

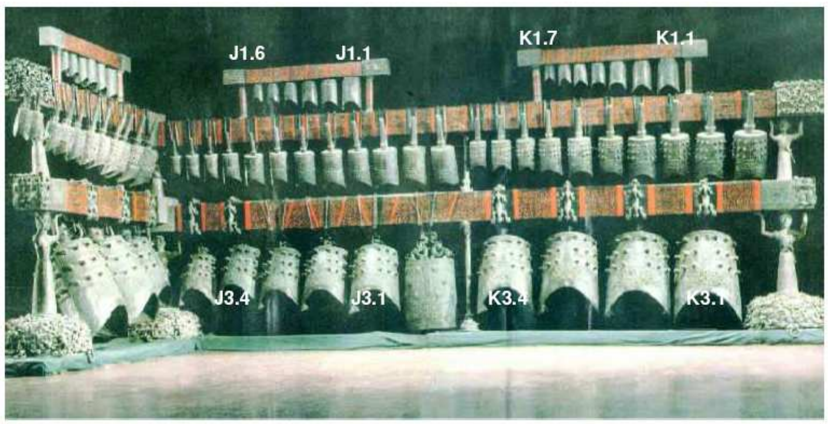


Figure 1 Marquis Zeng Yi's musical bells.

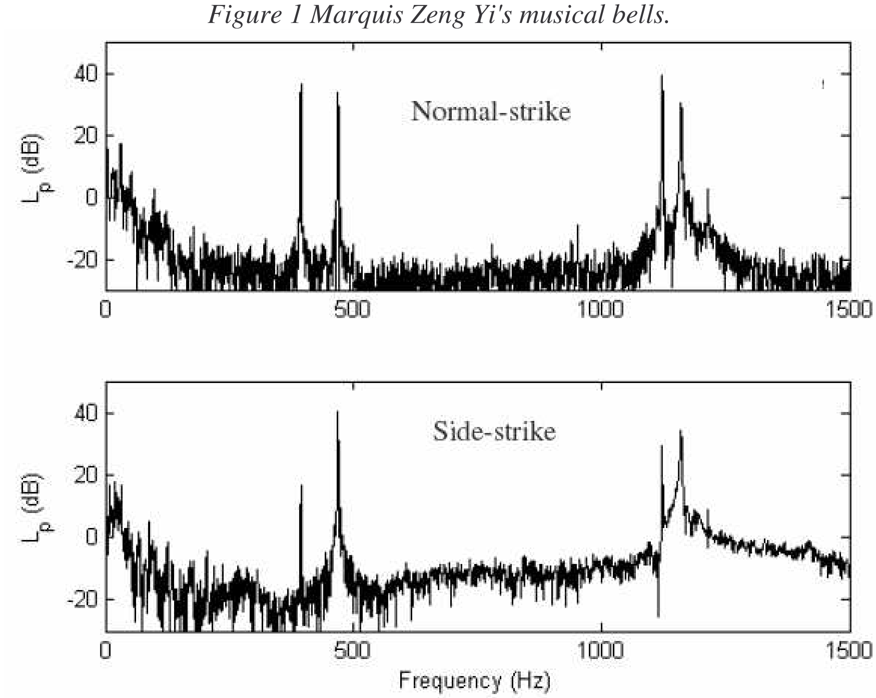


Figure 2 Power spectrum densities of the normal and side-strike tones of the bell #J1.1 at Beijing Bell Museum.

The design of minor third interval between the normal and side-strike tones has an obvious advantage of consonant intervals. In addition, the arrangement of major third interval between the normal-strike tones of the adjacent bells makes the production of 12 semi-tones in an octave band possible. To illustrate this point, Figure 4 shows a combination of the Marquis Zeng Yi's musical bells from the two top rows of group J and K. In the vertical direction of this "bell matrix" in Figure 4, the frequency interval is minor third and determined by the normal and side-strike tones of each bell. The diagonal direction of the matrix has major third interval due to the frequency interval between the adjacent bells. A semi-tone interval is thus established in the horizontal direction of the bell matrix. Clearly these frequency properties of ancient Chinese music bells indicates that the 12 semi-tones music scale is supported by the design of frequency intervals of the normal and side-strike tones of each bell and of the normal-strike tones between adjacent bells.

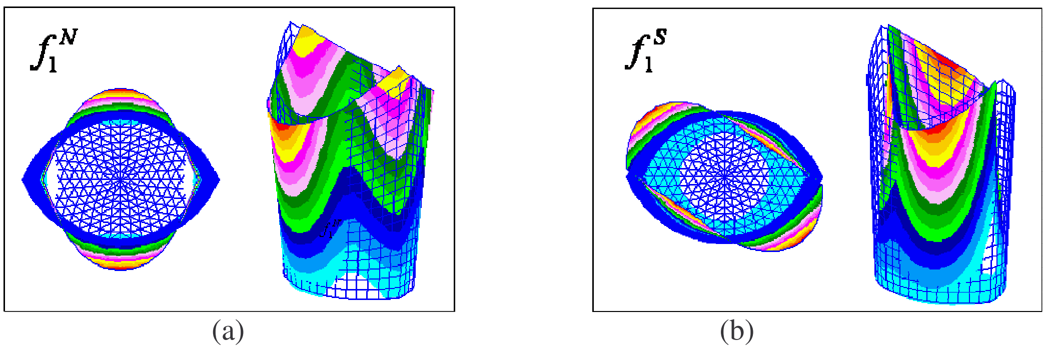


Figure 3 (a) Symmetrical mode $(2,0)_S$ *and (b) Anti-symmetrical mode* $(2,0)_A$.

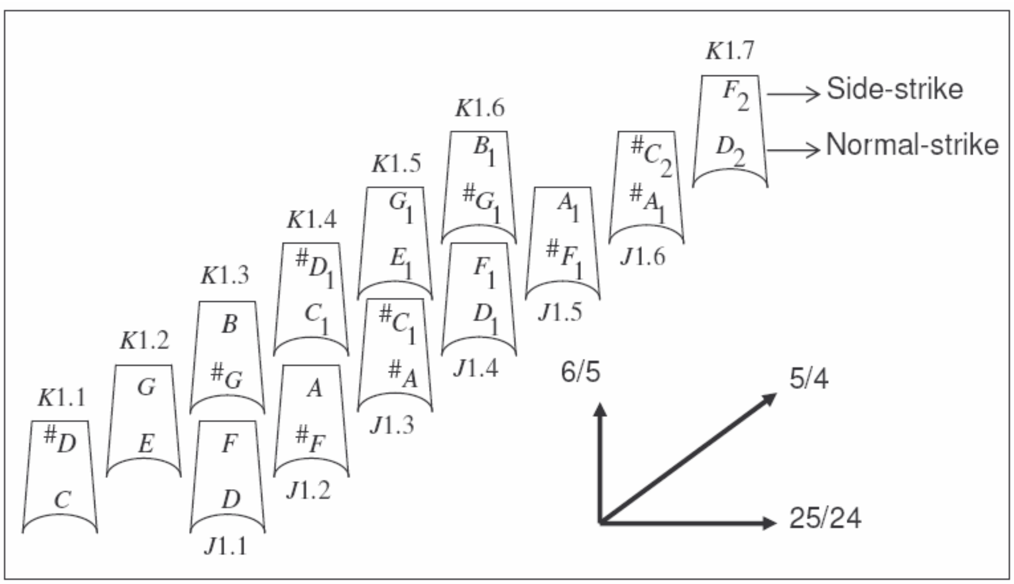


Figure 4 Tonal intervals of two top rows (J1.1 – J1.6 and K1.1 – K1.7) in Marquis Zeng Yi's musical bells (See Figure 2). For convenience of the notation the lowest tone at 365.1 Hz is defined as C.

BELL SHAPE AND ITS MINOR THIRD INTERVAL

Although the almond-shaped cross section of the music bell allowed the degeneration of the fundamental modes into two different natural frequencies with a close minor third interval, there are clear evidences that Marquis Zeng Yi's musical bells were fine tuned by shaping the inner wall of the bells and in both major and minor axis directions. Modern bell makers are quite aware of the importance of minimizing the effort in the bell tuning by designing the right bell dimensions. The relative dimensions of Chinese music bells are illustrated in Figure 5. In general, all the bell dimensions are scaled in terms of the bell height H. Among all the bell dimensions, the distribution of the wall thickness of the bell as a function of the angular position appeared to be the most sensitive factor to the successful design and tunning of the required minor third interval. Such distribution seems to be already determined at the design and casting stages of the bells. The thickness distribution is illustrated in Figure 6(a). In the circumferential direction of the bell lip, the normal thickness regions are found in the areas close to $\theta = [0, \pi/2, \pi, 3\pi/2]$, where evidences of shaping and smoothing were found. These areas are called sound ditches. The bell wall increases up to 1.5 times of the normal thickness in areas close to $\theta = [\pi/6, 5\pi/6, 7\pi/6, 11\pi/6]$, where some evidences of shaping and smoothing were also found. These areas are called sound spines. Along the vertical direction of the bell (see Figure 5), the wall thickness and angular width of the sound spines gradually decrease to the normal thickness and zero respectively when the position changes from the bell lip to half of the bell height. Finite element analysis has been used to investigate the effect of the location and angular width of sound spines upon the natural frequencies of the bells and the tuning required for the minor third interval. Figure 6(b) shows a FEA model of a music bell with dimensions similar to bell #K3.1 in Figure 1. The normal wall thickness is 12.5mm and the thickness of the sound spines is 25mm. The Young's modulus, mass density and Poisson's ratio are respectively $E = 1.22 \times 10^{11} N/m^2$, $\rho = 8900 kg/m^3$ and $\nu = 0.35$. Three cases are considered in this analysis. They respectively correspond to 60°, 45° and 30° of the angular width $\Delta\phi$ of the sound ditches at $\theta = \pi/2$ and $\theta = 3\pi/2$. The angular width of the sound spines, $\Delta\theta$, is used as a variable for each case. The natural frequencies of the first two modes of the bell and their ratio for given $\Delta\phi$ and $\Delta\theta$ are shown in Table 1. The analysis indicates that the distribution of the bell wall thickness significantly affects the frequency ratio between the side and normal-strike tones. As shown in Table 1 ($\Delta\phi = 60^\circ$), the frequency ratio is critically dependent upon $\Delta\theta$, and the minor third interval of the two tones can be fine tuned by varying $\Delta\theta$ in the range of $(35^\circ, 37.5^\circ)$. The tuning of the minor third interval is not unique. For example, if $\Delta\phi = 45^\circ$ the minor third should be tuned by letting $\Delta\theta = 37.5^\circ$, and if $\Delta\phi = 30^\circ$, $\Delta\theta$ should be within $(40^\circ, 45^\circ)$. We also observed that each case gives different frequency of normal-strike tone and the natural frequency of higher order modes. Therefore which case was actually used for the minor third tuning must also be constrained by the tuning of the major third between the adjacent bells and bell's tonal quality (frequency relationship with partials).

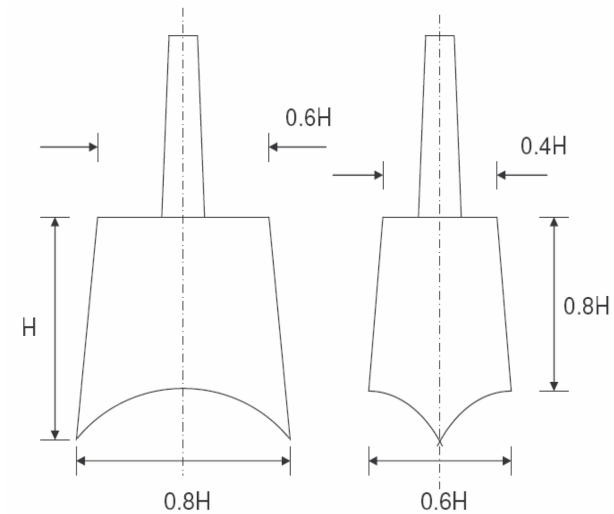
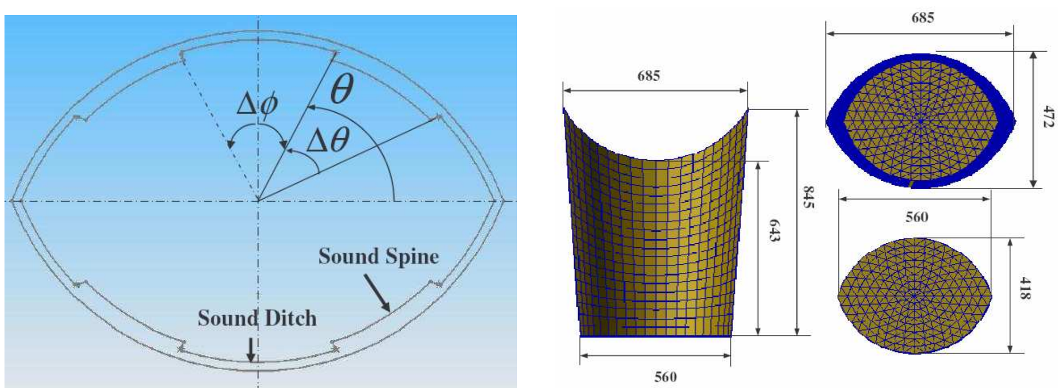


Figure 5 Relative dimensions of Chinese music bells in terms of the bell height H .



(a) (b)

Figure 6 (a) Illustration of bell wall thickness distribution. (b) Dimensions of the model bells for FEA study of the effect of wall thickness distribution on the minor third tonal intervals o the music bells.

Table 1. The natural frequencies of the first two vibration modes of the music bell described in Figure 6(b) for $\Delta\phi = 60^\circ$.

$\Delta\theta$	f_1^N (Hz)	f_1^S (Hz)	f_1^S / f_1^N
30°	96.0	110.0	1.15
<u>35°</u>	97.4	116.1	1.19
<u>37.5°</u>	98.1	119.4	1.22
40°	98.8	123.0	1.25

BELL SHAPES AND MINOR THIRD INTERVAL BETWEEN BELLS

For circular bells, the dependence of the natural frequency of the n^{th} mode upon the characteristic dimensions and material properties is described by the bell formula:

$$f_n = C\frac{t}{R_o^2}\sqrt{\frac{E}{\rho(1-\nu^2)}} \quad (1)$$

where C is a proportional constant, t and R_o are respectively the thickness and average radius of the shell. E, ρ and ν are respectively the Young's modulus, density and Poisson's ratio. Equation (1) suggests that if the same material is used, the natural frequency of the same mode can be designed by scaling t/R_o^2. When applied to music bells, R_o is replaced by the bell height H. In Figure 8(a), the frequencies of the normal-strike note of the 64 Marquis Zeng Yi's musical bells are plotted as a function of the corresponding H. The experimental data is fitted by two curves. The dash curve obeys

$$f_1^N \propto 1/H\,, \quad (2)$$

where $\propto$ represents the proportional relationship between the two sides of the equation. We observed that Eq. (2) describes the bell's normal-strike tone well when $H > 400mm$. For the large bells the thickness is related to the height by $t \propto H$ and thus Eq. (2) is consistent with Eq. (1). For smaller bells where $H < 400mm$, the solid curve in Figure 8(a) suggests

$$f_1^N \propto 1/H^3. \quad (3)$$

This dependence of the normal-strike tone of small bells upon the bell height is due to increased bell's thickness with deceased bell height. In Figure 8(b) the bell thickness for small bells is scaled as

$$t \propto 1/H \quad . \quad (4)$$

Substituting Eq. (4) into Eq. (1), we found that Eq (3) for smaller bells is also consistent with the bell formula. In summary, we found that two scaling rules were used to achieve the major third frequency interval between the bells. For large music bells, the frequency interval between adjacent $(i+1)^{th}$ and i^{th} bells is $f_{1,i+1}^N / f_{1,i}^N = 5/4$, which was realized by a single scaling rule for the bell height and thickness:

$$H_{1,i+1}/H_{1,i} = 4/5, \quad t_{1,i+1}/t_{1,i} = 4/5. \quad (5)$$

For small bells, the major third interval was achieved by scaling the bell height and thickness separately:

$$H_{1,i+1} / H_{1,i} = \left(4/5\right)^{1/3},\ t_{1,i+1} / t_{1,i} = \left(5/4\right)^{1/3}. \tag{6}$$

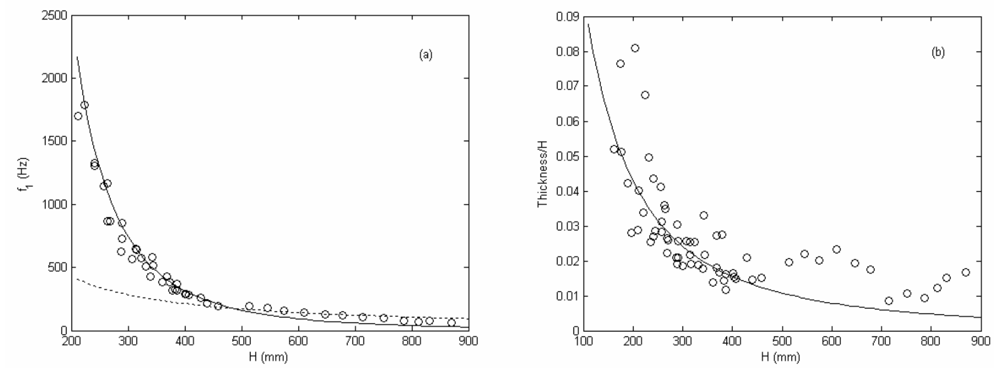


Figure 8 (a) frequencies of the normal-strike notes of Marquis Zeng Yi's musical bells as a function of the corresponding bell height. The dash curve is described by E. (2) and the solid curve by Eq (4). (b) t/H (at the sound spine locations) of the bells as a function of bell height. The solid curve is described by $t \propto 1/H$.

TONAL BALANCE OF MUSIC BELLS

The tonal balance of music bells, as a music instrument, is important, as the ear must judge the sound of the bells on the basis of similar loudness. The loudness of a bell sound is dependent upon the sound power radiated Π_i and also on the sensitivity of human ear to the same level of sound intensity at different frequencies. If the sound power the normal-striking note (the same analysis applies to the side-striking note) is of concern, the radiated sound power is

$$\Pi_i = \rho_o \sigma_i < \overline{V}_i^{\,2} > S_i \tag{7}$$

where σ_i, $< \overline{V}_i >$ and S_i are respectively the modal radiation efficiency, spatially averaged modal velocity and surface area of bell sound radiation. When sound radiation of the bell's normal-striking mode is considered, the following proportion relationships can be established

$$\sigma_i \propto f_{1,i}^{N} H_i,\ \ S_i \propto H_i^2 . \tag{8}$$

Thus the sound power ratio between adjacent bells for large bells is

$$\Pi_{i+1} / \Pi_i = (4/5)^2 (< \overline{V}_{i+1}^{\,2} > / < \overline{V}_i^{\,2} >) . \tag{9}$$

For small bells, the ratio becomes

$$\Pi_{i+1} / \Pi_i = (< \overline{V}_{i+1}^2 > / < \overline{V}_i^2 >) . \tag{10}$$

Assuming an equal spatially averaged modal velocity of all the bells, Eqs. (10) and (11) illustrate that the sound power level of the large bells decreases by 1.94dB when frequency of sound increases by a major third or by a factor of 5/4, while the ratios of sound powers for small bells remain as a constant. In Figure 9, the relative sound power of the normal-strike tone of the music bells and the A-weighting correction curve are presented as a function of frequency ratio. Clearly, the increased sound power for large bell height compensates the reduced sensitivity of human ears at lower frequencies, while constant sound power for small bells corresponds well with the relative flat sensitivity in the higher frequency rage. The fact that the scaling rules do not provide the exact compensation to the ear's sensitivity suggests that the modal velocities of the bells are not the same. Indeed, they vary with the size of the hammer and speed of impact, which normally results in larger velocity for larger bells.

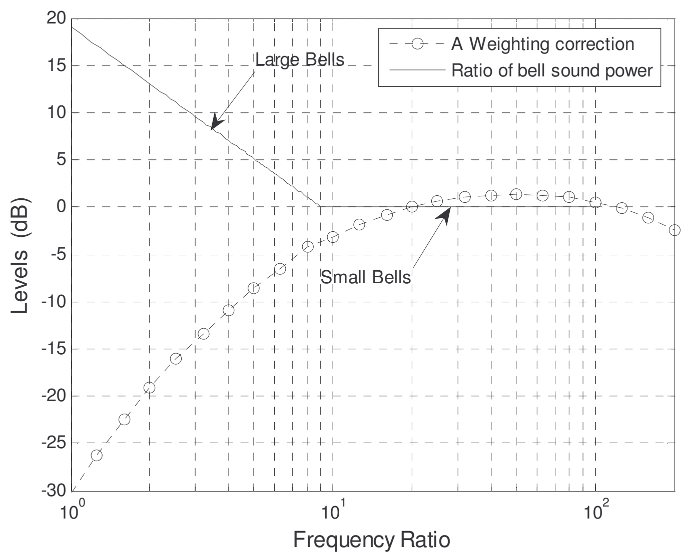


Figure 9 A-weighting level correction and ratio of the radiated sound powers by bells of different sizes. The reference value for the frequency ratio is 50Hz.

CONCLUSIONS

The minor third between side and normal strike notes of the bells and major third interval between adjacent bells allows the successful establishment of 12 semi-tones in an octave when the music bells are used for performance. The minor third frequency interval of a music bell is determined by the specific shape and wall thickness distribution at the sound ditches and spines. The major third frequency interval between adjacent bells is realized by the principle of similarity. However the different scaling rules were used for bells with different sizes. Such scaling rules compensate the reduced sensitivity of human ears at lower frequencies and provide balanced tones among bells of different sizes.