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ERRORS IN STANDARD CONFIDENCE INTERVAL ESTIMATES AND OTHER STATISTICS, IN RELATION TO SOUND EMITTED FROM ROTATING MACHINERY, WITH APPLICATION TO A TRACTOR ENGINE COOLING FAN

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Abstract

Engine cooling system is the main noise source in a tractor. In an engine cooling system, where a fan operates together with a radiator/shroud in a compact engine compartment, the radiated noise is complex containing both harmonics of the blade passing frequency (BPF) and broadband noise. Existing experimental methods used to estimate the overall sound pressure level (SPL) are insufficient to characterize the complex noise structure. The goal of this work was to characterize the stochastic structure of the noise from a tractor engine cooling system using advanced statistical signal processing tools and to decide whether a given change in the engine shape has a significant effect on the noise components by hypothesis test. A standard octave band analysis indicated the most important bands. Families of common spectral estimators were used to detect tones and estimate the tonal power at multiples of the BPF of the measured noise process. A comparison is made between the confidence interval (C.I.) of the spectral estimator from our C.I. formula and from the standard formula that is utilized in commonly used spectral analysis software, such as Matlab. Our formula considers the mixed spectral nature of the process, while the standard formula assumes purely continuous spectrum. The comparison showed that the standard formula overestimates the C.I. at the tone's frequency by a factor proportional to the window size. This overestimation may fail to identify an actual significant effect of a specific design on the noise components. The C.I. of the 1/3 octave band level (OBL) was estimated with our formula and used to decide whether the presence of engine block significantly influences the noise OBLs.

INTRODUCTION

Noises emitted from rotating machinery, such as a tractor engine cooling system, are usually mixed random processes(r.p.es) containing both periodic signals and regular wide sense stationary (WSS) processes with purely continuous spectrum. The periodic signals are caused by the rotary motion of the rotating components, for example, the fan blades. The regular random noises are generated from many sources, such as, air passing though the radiator and the engine. Spectral analysis is widely used in analyzing this type of r.p. Commonly used spectral analysis software packages, such as Matlab, assume regular r.p.es with purely continuous spectrum, when estimating the Confidence Interval (C.I.) of standard spectral estimator with the formula from [1],[2]. This formula does not hold in the case of mixed r.p.es. In this paper, we provide a new formula that can accurately estimate the C.I. of standard spectral estimators for mixed r.p.es.

To demonstrate the values of the new formula relative to the traditional one, we made a comparison of the standard spectral C.I. estimate of the sound pressure process of a tractor engine cooling system (TECS) calculated from both formulae. The comparison showed that the standard formula used in Matlab overestimates the C.I. at the tone's frequency by a factor proportional to the window size. This overestimation might lead the practitioners to fail to identify an actual significant effect of a specific design on the noise components.

Because the noise contains both continuous and line spectral components, with the latter having a masking effect and carrying the majority of the acoustic energy, existing experimental methods used to estimate the overall SPL are insufficient to characterize the noise structure. Based on the new formula, the paper demonstrates the use of new statistical analysis methods in characterizing the stochastic structure of the noise generated from the TECS and in correlating the noise components with a given design change in the engine shape.

Recently, the engine cooling system (ECS) has become the major noise source in a tractor. The noise generating mechanism of the ECS is very complicated, including air flow through the radiator, fan/shroud noise, and air flow passing the engine block. (Refer to [3] for more details.) The factors influencing the noise from the TECS include the fan type, fan speed, geometry of the shroud, the engine shape, etc. Although there exist mature theories to predict the noise generated from a rotating fan in a free field [4],[5], [6], [7] and [8], there is no theory that can predict the noise from the ECS. [9] provided a semi-empirical computer model to predict the noise spectrum from five different ECS assemblies under various working conditions. Unfortunately, the method focuses more on computation, without considering the possible mixed spectral nature of the process.

The rest of the paper is organized as follows: we first present the new formula of the standard spectral C.I. estimate. A brief description of experiment and sound pressure (SP) measurement is then provided followed by a preliminary analysis of the measured SP process. After displaying the results from standard 1/3 octave band analysis and tone detection for the SP process, a statistical analysis of the 1/3 OBL is then provided and hypothesis of whether a given change in the engine shape has a significant effect on the noise components is tested. Lastly, the key contribution of this work is summarized in the conclusions section.

SPECTRAL C.I. ESTIMATE FOR MIXED RANDOM PROCESSES

Define a mixed random process (r.p.) X(t) as:

$$X(t) = \sum_{k=0}^{K-1} A_k \cdot cos(\omega_k t + \phi_k) + \epsilon(t) \tag{1}$$

where $\{A_k, \omega_k\}$ are deterministic constants, and ϕ_k are i.i.d. Unif$[0, 2\Pi)$. To simplify the following derivation, assume $K = 1$. The noise, $\epsilon(t)$, is a regular process, and is assumed to have a continuous power spectral density (PSD) $S_\epsilon(\omega)$. The theoretical PSD of X(t) is then:

$$S_X(\omega) = \sum_k \frac{A_k^2}{4}\delta(\omega \pm \omega_k) + S_\epsilon(\omega) \tag{2}$$

where $\delta(\omega)$ is the well known Dirac-δ function. Consequently, eq(2) is only defined in the sense of a generalized function, in that only its integral, the cumulative p.s.d., is well defined with jumps at the signal frequencies.

The standard spectral estimate is an average of $N/n = H$ periodograms associated with the sampled data of size N, $\{X(i)\}_{i=1}^N$. The spectral estimate PSD(n) and its C.I. are defined in[1], [2], [10], which is used in practically all p.s.d. software packages, including Matlab. Divide $\{X(i)\}$ into $N/n = H$ non-overlapping sections of data of size n, $\{x_h(m)\}_{h=1}^H$.The standard PSD(n) at each bin frequency $\omega_p = \frac{2\pi p}{n}$, is defined as:

$$\hat{S}_X(\omega_p) = \frac{1}{H}\sum_{h=1}^{H} |X_h(\omega_p)|^2 \tag{3}$$

where $|X_h(\omega_p)|^2$ is the squared FFT of each windowed section $\{x_h(m)\}$. For a regular WSS r.p. with purely continuous spectrum (*assumption 1*) or a mixed r.p. at frequencies away from the tone, it holds that as $N/n \to \infty$, the variance of $\hat{S}(\omega_p)$ becomes approximately [11]:

$$Var\{\hat{S}_n(\omega)\} \cong nS^2(\omega)/N \qquad \text{for } |\omega - \omega_0| \gg 0 \tag{4}$$

Assuming eq(4) holds for a mixed r.p. of eq(1), at the tone's frequency ω_0, eq(4) becomes,

$$Var\left\{\hat{S}_X^{assumed}(\omega_0)\right\} = n\left[\frac{nA_0^2}{4} + S_\varepsilon(\omega_0)\right]^2 /N \tag{5}$$

However, eq(4) does not hold for a mixed r.p. in the vicinity of the tone's frequency ω_0. In fact, it can be shown that at ω_0, the variance of $\hat{S}(\omega_p)$ is [11]:

$$Var\{\hat{S}_{(n)}(\omega_0)\} = \frac{nS_\varepsilon^2(\omega_0)}{N}\left(1 + \frac{nA_0^2}{2S_\varepsilon(\omega_0)}\right) \tag{6}$$

where, $nA_0^2/(2S_\varepsilon(\omega_0))$ is the local SNR at the tone frequency. Comparing eq(5) and eq(6), they are both $O(1/N)$. However, eq(5) is $O(n^3)$, while eq(6) is $O(n^2)$. Therefore, the former overestimates the variance of standard spectral estimator by a factor proportional to the window length, n. According to the Matlab source code and its references [1],[2], etc., Matlab assumes a regular r.p. and uses eq(5) to estimate the variance or C.I. of the spectral estimate, which overestimates the variance and the C.I.

DESCRIPTION OF THE EXPERIMENT AND DATA MEASUREMENT

The experiment[1] was conducted in the anechoic acoustics lab at Iowa State University(ISU). Figure 1 shows the right view of the experimental stand with an indication of the radiator, the fan, a contoured shroud, and mock engine across the stand. The fan is 33cm behind the radiator. A plastic shield is installed surrounding the cooling system to protect the experimenter.

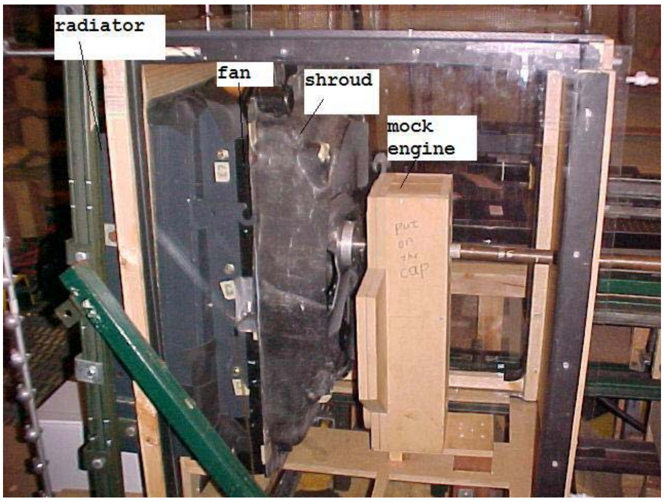


Figure 1: The right side view of the experiment stand

The sound pressure (SP) process was measured using a free field, $1/2$in microphone located 117cm away from the front surface of the radiator. The microphone is connected to the Labview DAQ system with a maximum sampling rate of 30KHz. Two data sets were obtained corresponding to the assembly with and without the rectangular mock engine block. The actual sampling rate was $f_s = 16129Hz$. The fan speed was 1930rpm. The data set nomock16000.dat with sample size $N = 16000$ corresponded to the case without mock engine. The data set mock16000.dat with sample size $N = 32000$ corresponded to the case with mock engine. The fan had 6 blades. Assuming constant fan speed, the Blade passing frequency (BPF) f_b was approximately $f_b = \frac{fan\ rpm \times 6}{60} = 192.9$Hz.

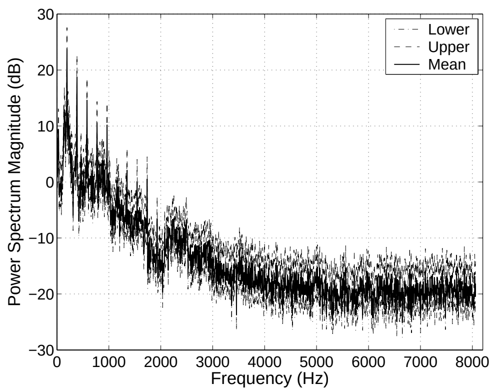
PRELIMINARY ANALYSIS OF THE SOUND PRESSURE MEASUREMENT

In this section, we analyze the sound pressure data without concern for whether it has a mixed spectral structure. This is how the data would be studied by typical practitioners and researchers who are not well versed in the theory and statistics of mixed r.p.es.

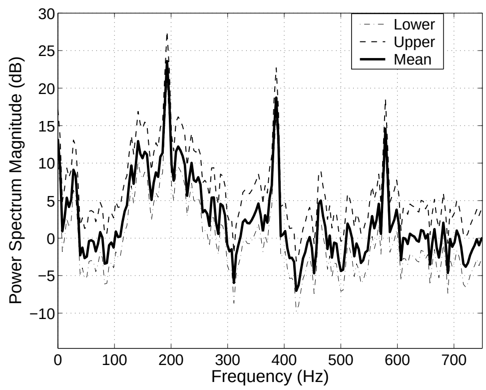
The time signal of mock16000.dat(Refer to [3] for the time signal figure.) exhibits strong periodic phenomenon and no obvious time-varying trend. Using *Matlab* with previ-

[1]Thanks to Professor Admin Mann and his graduate students at ISU for the experimental set up.

ously described standard approach, we choose to partition the SP data in mock16000.dat into 7 non-overlapping subsections of data and apply a Hanning window of size n=4096 to each subsection. Hence, the effective spectral resolution is $\Delta f = 7.88Hz$. The PSD(4096) estimate and its 95% estimated C.I. is plotted in figure 2(a). The PSD(4096) estimate in figure



(a) *The 0-8064Hz unzoomed-in PSD*



(b) *The 0-750Hz zoomed-in PSD*

Figure 2: The estimated PSD(4096) and its 95% C.I. of the measured sound pressure process $x(t)$ in mock16000.dat and the 0-750Hz zoomed-in figure.

2(a) exhibits a mixture of broadband continuous spectrum with very strong narrow peaks centered at multiples of the fundamental frequency 192.9Hz, which is the BPF. To demonstrate the spectral estimate and its 95% C.I. more clearly, the 0-750Hz zoomed-in figure is showed in figure 2(b).

STATISTICAL ANALYSIS OF 1/3 OCTAVE BAND LEVEL

Traditional 1/3 Octave band analysis includes making an estimate of the 1/3 Octave Band Level(OBL) by integrating the A-weighted PSD estimate [12] and then identifying the bands that contain the majority of the power . It was found that from a non-statistical view, the 1/3 octave bands having the majority of the power are centered on 200Hz, 400Hz, 630Hz, 800Hz and 1000Hz. These five bands contain approximately 8.9%, 11.2%, 14.6%, 14.6% and 15.2% of the total A-weighted power respectively.

Using families of AR and MV spectra [11],[3], a tone detection and tonal power estimation was developed for the process with and without mock engine. For simplicity, the results are not shown here (refer to [3] for the detailed procedure and results). It was found that the first five harmonics of BPF in ”mock16000.dat” contributed to the majority of energy (with contribution percentage of 72.4%, 78,9%, 42.1%, 23.8% and 23.7% respectively) in the corresponding 1/3 Octave bands (centered at 200, 400, 630, 800, 1000, 1250Hz). Therefore, the tones cause a masking effect.

To estimate the C.I. of each 1/3 OBL, the data is heterodyned into each band first, so that the new process has a simpler spectrum structure. The heterodyned process within each band is denoted as $x_h(t)$ (refer to [3] for more details about the heterodyning process). Usually, $x_h(t)$ contains a single tone plus broad band noise with the tone's frequency much closer to the center of the analysis Bandwidth or simply broad band noise. For example, the heterodyned process in (176, 353)Hz is a locally white noise plus a sinusoid at 193Hz. As a result, the tonal nature is easier to be detected and the frequency estimator $\hat{f}$ will be less biased. Both the mean and the variance of the autocorrelation estimator of $x_h(t)$ are estimated using eq(19) in [11]. In eq(6), the true noise spectrum and the tonal power is replaced with the estimated ones. At non-tone's frequency, the C.I. of the standard p.s.d. estimate is calculated with eq(4). At the tone's frequency, the C.I. is then calculated with eq(5)(the same as eq(4)) and eq(6) respectively. Figure 3 compares the 95% C.I. of the p.s.d. estimator using eq(5)(utilized in *Matlab*), and eq(6) respectively on the heterodyned SP process in the band of (176, 353)Hz with mock engine. From figure 3, it can be seen that at non-tone's frequencies, the standard deviation and C.I. from the two methods are the same. While at tone's frequency, there is a notable difference between results from the two equations. Both the upper and lower limit of the 1/3 C.I. from eq(6), which considers the mixed spectral nature, are more than 8dB smaller than those from eq(5). This difference will be even bigger if a larger window size is used. Therefore

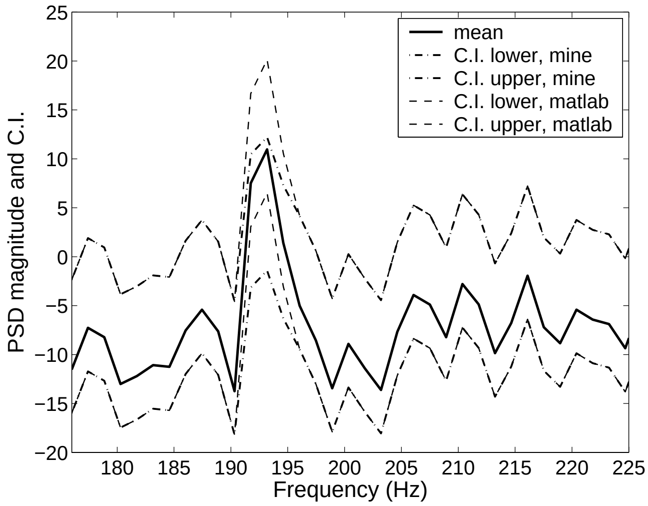


Figure 3: Comparison of the 95% C.I. of p.s.d. estimator from eq(5) (used in Matlab*) and that from eq(6) of the heterodyned process in mock16000.dat in (176, 225)Hz.*

when a mixed r.p. is involved, the C.I. of the p.s.d. estimator from eq(5) is misleading because it is based on the inappropriate assumption. Such misleading information may result in a costly mistake in military specifications for helicopter drive train or in evaluating the effect of a design change on machinery noise and vibration where the C.I. of the p.s.d. estimator plays an important role and the r.p.es involved contain both continuous and line spectrum components.

Using eq(19) in [11], the $2-\sigma$ C.I. of the 1/3 OBL(A) of the SP process in mock16000.dat and nomock16000.dat can be calculated and are shown in Figure 4. With

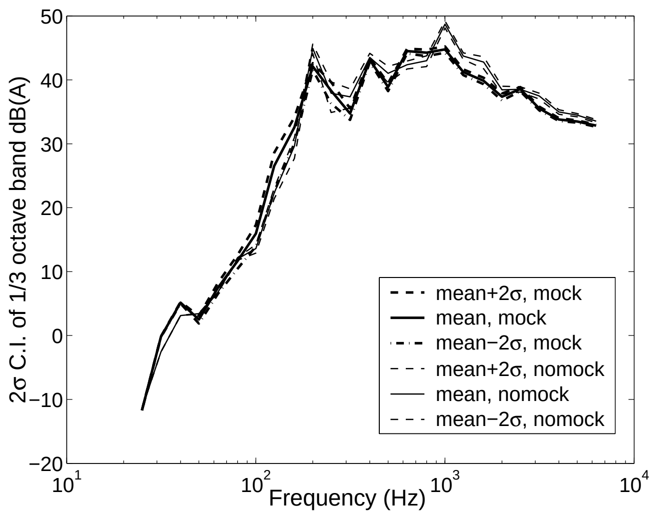


Figure 4: $2-\sigma$ *C.I. of the 1/3 OBL(A) in mock16000.dat and nomock16000.dat*

this, a more statistically sound inference could be made by comparing whether the C.I. with and without mock engine are overlapped or not. If they overlap, then the 1/3 OBL(A) of the two conditions are not significantly different at a significance level $\alpha = 0.05$. And vice versa. While a traditional analysis of 1/3 OBL(A) would only compare the mean level rather than the C.I..

With the C.I., the level of the two conditions of the 1/3 octave band centered on 125Hz that was considered different from non-statistical view, is indeed not significantly different at a 0.05 significance level. It was also found that at the 1/3 Octave bands centered on 630 and 800Hz, the 1/3 OBL(A) with mock engine is significantly higher than without mock engine at 0.05 level. While at the 1/3 Octave bands centered on 200, 315, 1000, 1250 and 1600Hz, the 1/3 OBL(A) with mock engine is significantly higher. At all other bands, the 1/3 OBL(A) with the two conditions are not significantly different. We also found that the overall C.I. of the SPL(A) with mock engine is significantly lower than without mock engine at 0.05 level, since the $2-\sigma$ C.I. of the overall SPL(A) is (52.62, 52.93)dB with mock engine and is (54.00, 54.48)dB without mock engine.

CONCLUSIONS

In summary, the key contribution of this work is that by comparing results from standard analysis and those from our proposed statistical analysis on the sound pressure process of a tractor engine cooling system, we showed that the analysis of r.p. from mechanical systems should not be done without considering the mixed spectral nature of the process. If there are tones, then the standard formula to estimate the C.I. of p.s.d. estimator at the tone's frequency is faulty and may lead to costly wrong decision. The more appropriate and reliable procedure is to first understand the physics of the phenomenon, then determine whether there are tones at specific frequencies indicated by the physics and then perform spectral analysis and associated statistical tools such as C.I. and parametric hypothesis test corresponding to the spectral nature of the process. We are not aware of any software package on the current market that is capable of performing such an advanced analysis. The practitioners have to be aware of this and apply the proposed statistical method more carefully instead of relying only on the software package to conduct the analysis.

References

[1] P. Stoica, R. L. Moses, *Introduction to Spectral Analysis*, Prentice Hall, Upper Saddle River, NJ (1997).

[2] S. Kay, *Modern Spectral Estimation*, Prentice Hall, Englewood Cliffs, NJ (1988).

[3] L. Wen, *Dissertation: Statistical signal procssing for mechanical systems*, Iowa State University, Ames, IA (2003).

[4] E. Envia, *Fan noise reduction: an overview*, Aeroacoustics, 1(1), (2002), 43–64.

[5] N. Le, S. Filleul, *An investigation of axial flow fans*, Journal of Sound and Vibration, 3, (1966), 147–165.

[6] P. E. Doak, P. G. Vaidya, *A note on the relative importance of discrete frequency and broad-band noise generating mechanisms in axial fans*, Journal of Sound and Vibration, 9, (1969), 192–196.

[7] N. Chandrashekhara, *Experimental studies of discrete tone noise from an axial flow fan*, Journal of Sound and Vibration, 13, (1970), 43–49.

[8] B. D. Mugridge, *The noise of cooling fans used in heavy automotive vehicles*, Journal of Sound and Vibration, 44, (1976), 349–367.

[9] S. F. Wu, S. Su, H. Shah, *Noise radiation from engine cooling fan*, Journal of Sound and Vibration, 216(1), (1998), 107–132.

[10] L. H. Koopmans, *The spectral analysis of time series*, Academic Press, New York and London (1974).

[11] L. Wen, C. X. Wang, P. J. Sherman, *Using variability related to families of spectral estimators for mixed random processes*, Journal of Dynamic Systems, Measurement and Control, Trans. of the ASME, 123(4), (2001), 572–584.

[12] C. M. Harris, *Handbook of Acoustical Measurements and Noise Control*, McGRAW-Hill, Inc., 3rd edn. (1991).