MECHANIZED MATHEMATICS MODELLING AND ALGORITHM ANALYSIS FOR 200MW TURBO-GENERATOR SYSTEM

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Abstract:

For 200MW turbo-generator system, considering higher order and higher dimension of nonlinear differential equations governing the motion of its rotor-bearing system, the Mechanized Mathematics Method can be used as modeling and analyzing means of rotor-bearing system in order to obtain the analyzed solutions of the 200MW turbo-generator system. When the order of the differential model has been reduced by fixed interface mode synthesis method, it may be transformed into an algebraic expression set by Wu Elimination Method （WEM）. Then lower dimension algebraic equations included nonlinear oil force expressions are obtained, because of the nonlinear parts of oil force expressions can be remained after eliminating the linear coupling variable of node displacements of rotor-bearing system of 200MW turbo-generator system. Differential Control Method (DCM) that is different to analyzed method and the classical numerical method was presented in this paper. According to proposed method the model of 200MW turbo-generator system has been described and its node displacement response of system was analyzed and predicted. The analysis results based on proposed method stand with very good agreement with previous numerical calculate results.

Key Words: Mechanized Mathematics Method; Wu Elimination Method; Differential Control Method; Modeling; Nonlinear Oil force

1 INTRODUCTION

A rotor bearing system of a large turbo-generator system can be described by a higher order or higher dimension differential equations only due to some complex nonlinear dynamic characteristics that it possesses. For such dynamic system, exact solutions are generally too difficult and approximate solutions can be obtained numerically. A commonly used method is to simulate the dynamic behavior by classical numerical method[1]. It must be extremely time-consuming, especially for large order dynamic system. The expressions of influence of strongly nonlinear components upon the dynamic behavior of rotor bearing system can't be obtained by numerical method. It leads to the inherent in engineering dynamic system is difficult to mastery. So it must be explore all possible method for solving the large rotational machinery, other than numerical method in order to obtain analytical expression of system[2-5].

Mechanized Mathematics-Wu Elimination Method（WEM）presented by Chinese scholar Wu Wentsun based on the good idea of ancient mathematics in China. The WEM offers several potential advantages over numerical method for solving the algebraic equation system. This superior performance of WEM has raised a wide application in the field of mathematical science and computational science. According to characteristic set of WEM, the expression of the solutions of nonlinear algebraic equations can be obtained rapidly. The essence of motion behavior of a type of dynamic system can be given clearly and directly by WEM. Combine with classical numerical method, Mechanized Mathematics is shown the excellent performance that it is able to greatly improve solving efficiency and accuracy of nonlinear algebraic equations. Owing to above advantage, as an efficient method, WEM will be played an important part in analyzing nonlinear dynamic problem[6-7].

The focus of this research is on studying how combining the WEM with classical numerical method. By this work the rules of momentarily controlling steady state of a type of higher dimension nonlinear dynamic system can be given. When a nonlinear dynamic system is analyzed, it must be transform the differential equation that described the model of system into algebraic equation first, due to WEM is only suitable for solving the algebraic expression. The theory of Mechanized Mathematics, dynamic modeling method and idea of transforming nonlinear differential equation into algebraic expression are researched for solving higher dimension nonlinear dynamic problem[8-9].

Based on above studies, Differential Control Method（DCM）is taken into account especially in this paper. The feasibility and advantages of proposed method are illustrated with an example of 200MW turbo-generator system with nonlinear supports of China. According to this system, a nonlinear vibration differential equation with 112 freedoms is given and the number of freedom can be reduced to 16 by the fixed interface mode synthesis method[10-12]. After transforming the differential equation with 16 freedoms into algebraic equation, reduced approach algebraic expressions with 8 freedoms can be obtained by WEM. The approach algebraic expressions are condensed to a small order system that only related to the system physical coordinates associated with the nonlinear components. The approach algebraic expressions of displacement response that included nonlinear oil force acting at the journals can be only analyzed by DCM.

2 CHARACTERISTIC SET OF WU ELIMINATION METHOD

The set of complex nonlinear algebraic equations that has been obtained from nonlinear differential equations governing the rotor dynamic system can be reduced or solved by WEM. The idea of WEM is illustrated as following section.

For any two polynomials F and G including the leading variable x_l, they are arranged as following form:

$$F = c_0 x_l^{\,m} + c_1 x_l^{\,m-1} + \cdots + c_m \tag{1}$$

$$G = d_0 x_l^{\,M} + d_1 x_l^{\,M-1} + \cdots + d_M \tag{2}$$

where the constant coefficients $c_0, c_1, \cdots, c_m$ and $d_0, d_1, \cdots, d_M$ are real numbers , the parameters M, m, l are integer, moreover, $d_0 \neq 0$， $M \geq m$. We call the polynomial r the Remainder of polynomial G with respect to polynomial F, if they satisfy following terms.

$$[I(F)]^s \cdot G = qF + r \quad (3)$$

Where s is the minimum nonnegative integer, $I(F)$ is called Initial of the polynomial F. The polynomial r is unique and can be noted as

$$\mathrm{r} = \mathrm{Rem}(G, F, x_l) \quad (4)$$

Signs $\deg_{x_l} r$, $\deg_{x_l} F$, $\deg_{x_l} G$ denote the order of polynomial r, polynomial F and polynomial G respectively. They satisfy $\deg_{x_l} r < \deg_{x_l} F$ and $\deg_{x_l} r < \deg_{x_l} G$.

The polynomial r (the Remainder of polynomial G for polynomial F) is the reduced algebraic polynomial from polynomial G and F for the variable x_l. The expression of the variable x_l can be solved by Eq.(4) easier based on Maple software, if the solutions of Eq.(4) exist.

For any series of polynomial $FS = \{F_1, F_2, \cdots, F_r\}$, where $F_i (i = 1,2,\cdots r)$ are the polynomial in the increasing order for the leading variable x_l. The polynomial $F_i (i = 1,2,\cdots r)$ is similar to the above polynomial F in Eq.(2). The Remainder of any polynomial G with respect to the series FS is considered here.

In particular, we denote by notation R_{r-1} the Remainder of G for F_r. Denote by R_{r-2} the Remainder of polynomial R_{r-1} for polynomial F_{r-1}. Continue the above procedure, denote by notation R_{i-1} the Remainder of polynomial R_i for polynomial F_i ($i = 0,\cdots,r$). We call polynomial R_0 the Remainder of G for increasing series FS， if it satisfy the following equation.

$$I_1^{s_1} \cdot I_2^{s_2} \cdots \cdot I_r^{s_r} \cdot G = Q_r \cdot F_r + Q_{r-1} \cdot F_{r-1} + \cdots + Q_1 \cdot F_1 + R_0 \quad (5)$$

where I_i ($i = 0,\cdots,r$) are called the Initial polynomial of F_i, s_i ($i = 0,\cdots,r$) are the nonnegative integer, Q_i $(i = 1,2,\cdots,r)$ is the polynomial equation. We denote by notation $\mathrm{Rem}(G/FS)$ the Remainder of G for increasing series FS, i.e. $R_0 = \mathrm{Rem}(G/FS)$.

In general, an increasing series of polynomial $PS = \{P_1, P_2, \cdots, P_m\}$ are called characteristic set of polynomial equation set $QS = \{Q_1, Q_2, \cdots, Q_r\}$, if they satisfy

$$\mathrm{Rem}(P_i / QS) = 0 \,, \quad i = 1,2,\cdots,m \quad (6a)$$

$$\mathrm{Zero}(PS) \subset \mathrm{Zero}(QS) \quad (6b)$$

We denote by $\mathrm{Zero}(PS)$ and $\mathrm{Zero}(FS)$ the assemblage of solutions of PS and assemblage of solutions of FS, respectively. Let $p_i = 0$ $(i = 1,2,\cdots,m)$, the expression of leading variable of x_i $(i = 1,2,\cdots,m)$ can be obtained, if they exist.

3 MODELLING OF 200MW TURBO-GENERATOR SYSTEM

A physical model of rotor-bearing system of low-pressure cylinder of 200MW turbo-generator is shown in Fig.1. Where, k_p denotes oil film stiffness coefficient, k_b denotes equivalent static stiffness coefficient, M_b denotes base and its equivalent mass of bearing, ww(nn) denote mass of disk, ns denotes the degree of freedom of reduced system, nb indicate the number of bearing. Assuming that, the number of structure node *nn*=28; degree of freedom of every node displacement *nf*= 4; the total number of structure displacement $ms = nf \cdot nn = 112$. As shown in Fig.1, 4.th and 24.th nodes are supported by two bearings. It indicates that, the nonlinear node displacements (x_4, y_4) and (x_{24}, y_{24}) will lead to the nonlinear response of rotor-bearing system of low-pressure cylinder of 200MW turbo-generator system occurs. The displacement response of these two nodes will be especially analyzed in this paper.

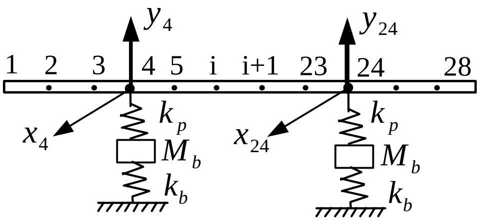


Fig.1 physical model of 200MW turbo-generator

Among the 28 structure nodes, 4.th and 24.th nodes are the nonlinear, the other are linear. When any classical method of modal reduction is applied to analyze the rotor-bearing system, assuming that, the matrix of modal denote $\Phi(112\times16)$; the mass matrix of reduced system denote $M(16\times16)$; the damping matrix denote $C(16\times16)$; the stiffness matrix denote $K(16\times16)$. Before reducing dimension the mass matrix denote $M_0(112\times112)$; the damping matrix denote $C_0(112\times112)$; the stiffness matrix denote $K_0(112\times112)$. The following relations are satisfied: $M=\Phi^T M_0 \Phi$; $C=\Phi^T C_0 \Phi$; $K=\Phi^T K_0 \Phi$. It indicated that, the number of freedom is reduced from 112 to 16 by above procedure. In the end the algebraic characteristic expressions, including displacements of nodes 4.th (x_4, y_4), 5.th (x_5, y_5), 23 (x_{23}, y_{23}), 24.th (x_{24}, y_{24}) and etc. are obtained by WEM. They are shown as following:

$$f_1 := 307.8760749\ fx24 - .5961896351\ 10^7\ y_{24} + .6070718385\ 10^7\ x_{24} + .01976795045 + .5583013843\ 10^7\ x_5 - .5484079962\ 10^7\ y_5 \tag{7a}$$

$$f_2 := -314.1592599\ fy24 + .5302131298\ 10^7\ y_{24} - .5442045340\ 10^7\ x_{24} - .01957457382 - .5006317215\ 10^7\ x_5 + .4879116609\ 10^7\ y_5 \tag{7b}$$

$$f_3 := 307.8760747\ fx4 + 53993.8581\ y_{23} - 55033.53771\ x_{23} - .124735179\ 10^7\ x_4 + .122378721\ 10^7\ y_4 - 22592.1741 - .3457673059\ 10^{10} / \omega^2 \tag{7c}$$

$$f_4 := -314.1592601\ fy4 - 47929.3874\ y_{23} + 49266.12579\ x_{23} - .124735303\ 10^7\ x_4 + .122378530\ 10^7\ y_4 + 22592.1761 + .3528238128\ 10^{10} / \omega^2 \tag{7d}$$

$$f_5 := 307.8760747 fx24 + .503429336610^7\ y_{24} - .512734233910^7\ x_{24} - .01440529147 - .449912294110^7\ x_5 + .441838106710^7\ y_5 \tag{7e}$$

$$f_6 := -314.1592600 fy24 - .447527000410^7\ y_{24} + .459490434110^7\ x_{24} + .01450746603 + .403308473110^7\ x_5 - .392927380810^7\ y_5 \tag{7f}$$

$$f_7 := 307.8760747\ fx4 + .1120131761\ 10^7\ y_{23} - .1141700564\ 10^7\ x_{23} - 55032.94\ x_4 + 53994.25\ y_4 + 607.4301 - .3184001323\ 10^{10}/\omega^2 \tag{7g}$$

$$f_8 := -314.1592603\ fy4 - 994321.0069\ y_{23} + .1022052308\ 10^7\ x_{23} - 55034.89\ x_4 + 53995.04\ y_4 - 607.4265 + .3248981236\ 10^{10}/\omega^2 \tag{7h}$$

Eq.(7) can't be solved for any given classical numerical method due to exist of complex differential expressions of nonlinear oil force fx_4, fy_4, fx_{24}, fy_{24}. It is necessary to exploit the other method in order to solve similar equations. Above nonlinear oil force fx_4, fy_4, fx_{24}, fy_{24} has been derived in the following dimensionless form:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \frac{-[(x-2y')^2+(y+2x')^2]^{1/2}}{1-x^2-y^2}\begin{bmatrix} 3x\cdot V(x,y,\alpha)-sin\alpha\cdot G(x,y,\alpha)-2cos\alpha\cdot S(x,y,\alpha) \\ 3y\cdot V(x,y,\alpha)-cos\alpha\cdot G(x,y,\alpha)-2sin\alpha\cdot S(x,y,\alpha) \end{bmatrix} \quad (8a)$$

There,

$$G(x,y,\alpha)=\int \frac{d\vartheta}{1-x\cos\vartheta-y\sin\vartheta}=\frac{2}{(1-x^2-y^2)}[\frac{\pi}{2}+\text{arctg}\frac{y\cos\alpha-x\sin\alpha}{(1-x^2-y^2)^{1/2}}] \quad (8b)$$

$$V(x,y,\alpha)=\frac{2+(y\cos\alpha-x\sin\alpha)G(x,y,\alpha)}{1-x^2-y^2} \quad (8c)$$

$$S(x,y,\alpha)=\frac{x\cos\alpha+y\sin\alpha}{1-(x\cos\alpha+y\sin\alpha)^2} \quad (8d)$$

In Eq. (8) variables α and θ denote the parameter of nonlinear oil force.

4 IDEA OF DIFFERENTIAL CONTROL METHOD

Based on WEM the idea of Differential Control Method is given in order to solve the nonlinear algebraic-differential equations that described model of rotor-bearing system of low-pressure cylinder of 200MW turbo-generator system. The idea of DCM will be introduced as following.

To define displacement x is a function of time variable *t*. For any minimum length of time (t_1, t_2), its corresponding displacements can be denoted (t_1, x_1) and (t_2, x_2). According to mathematics the difference x' of displacement x may be defined as followings

$$x'=\frac{\Delta x}{\Delta t}=\frac{x_2-x_1}{t_2-t_1} \quad (9)$$

Similar to above procedure the difference y' of displacement y can be also expressed in its algebraic expression form

$$y'=\frac{\Delta y}{\Delta t}=\frac{y_2-y_1}{t_2-t_1} \quad (10)$$

Considering the engineering condition of Eq. (7) , it can’t be solved by analytical method or classical numerical method, therefore the idea of DCM is necessary to study. By Visual PowerStation software, the calculation procedure is given here. When $x=x_1$, in order to obtain moment time t_1, corresponding programmable statements can be written as follows

$$\text{CALL GETTIM (ihr, imin, isec, i100th)} \tag{10a}$$

$$\text{t1=ihr*3600.0+imin*60.0+isec*1.0+i100th/100.0} \tag{10b}$$

Above characters mean that, the ihr denote the hour; the imin denote the minute; the isec denote the second; i100th denote the precision of calculate.

Similar to above discuss, when $x = x_2$, the moment time t_2 can be obtained by following statements

$$\text{CALL GETTIM (ihr, imin, isec, i100th)} \tag{11a}$$

$$\text{t2=ihr*3600.0+imin*60.0+isec*1.0+i100th/100.0} \tag{11b}$$

The length of time *t* can be calculated from following expression

$$\Delta t = t_2 - t_1 \tag{12}$$

Corresponding the length of displacement x can be obtained in the following form

$$\Delta x = x_2 - x_1 \tag{13}$$

Substitution of variables $\Delta t, \Delta x$ into Eq.(9) yields the x'. Repeat similar procedure y' can be written in the form of a algebraic expression that like Eq.(10). Above algebraic difference equations Eq.(9) and Eq.(10) can be solved by Newton iterative algorithm. The relations between the linear displacements and nonlinear oil force are shown in Eq.(7). Flow chart of analyzing similar rotor-bearing system by DCM are shown as follows

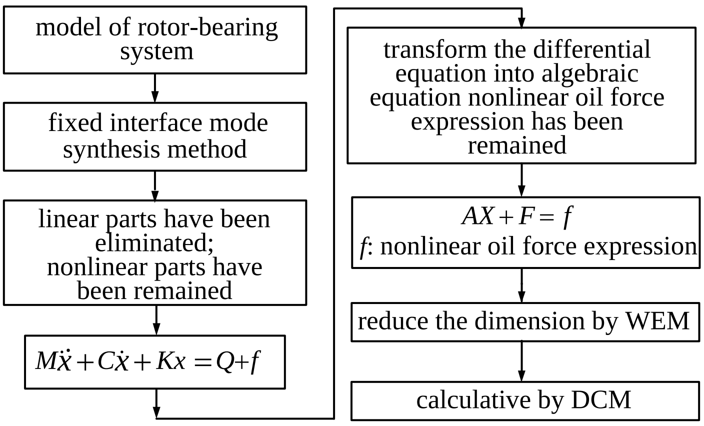
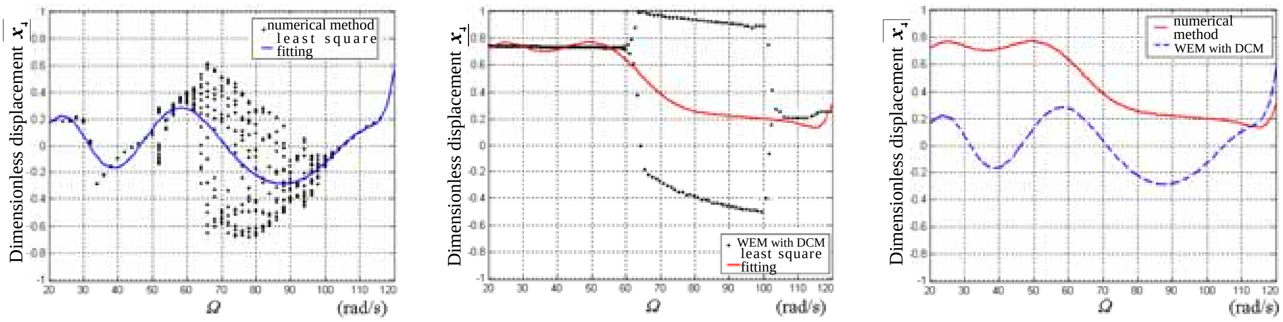


Fig.2 the flow chart of DCM

4 ANALYSIS OF 200MW TURBO-GENERATOR BASED ON DCM

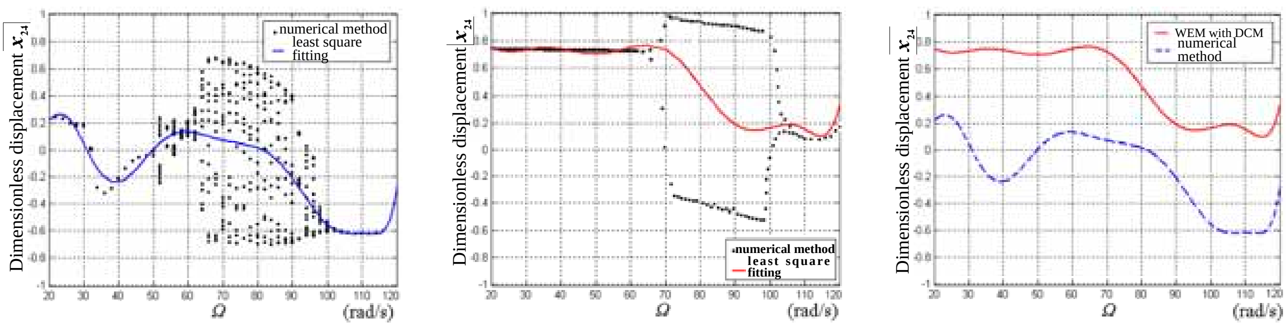
As shown in Fig.1, parameters of low-pressure cylinder of 200MW turbo-generator are given as following. There, the oil films stiffness coefficient $k_p = 2.45 \times 10^6$ kN/m , equivalent static stiffness coefficient $k_b = 3.92 \times 10^6$ kN/m , base and its equivalent mass of bearing $M_b = 17.64$t . Considering the mass matrix M , damping matrix C , stiffness matrix K and eccentricity matrix are more complex, they aren't given detailed. In order to analyze the response of node displacement the proposed method is used to this example. It's only taken 1 minute to solve this problem by computer of CPU of 1.5G. It's at least taken 26 hours or more time to solve unreduced differential equations in same conditions. The effectiveness of WEM combined with DCM is attested by following results.

Fig.3 show the response curves of node displacement x_4 of rotor-bearing system of 200MW turbo-generator that obtained by DCM. There the bifurcation curves of displacement are represented by dotted line. The other smooth curves described by solid line are the fitting curves of the bifurcation response dots witch are obtained by least square theory. The results shown in Fig.3(a) and Fig.3(b) are well compared to those of using Newton method and proposed method. It's noted that at initial point $\omega \approx 62.5\text{rad/s}$ and confluence point $\omega \approx 108\text{rad/s}$ results obtained by classical method are well consistent with that one and by DCM. As shown in Fig.3(c), the response curve obtained by numerical method is compared to that one by DCM in order to analyze simulation effect of chaos phenomena by this two methods between $62.5\text{rad/s} \le \omega \le 108\text{rad/s}$. The comparison shows that the trend curves obtained by these two methods are in very good agreement excepting their amplitudes.



(a) (b) (c)

Fig.3 Displacement response of node coordinate x_4



(a) (b) (c)

Fig.4 Displacement response of node coordinate x_{24}

Fig.4 show the response curves of node displacement x_{24} of rotor-bearing system of 200MW turbo-generator. Similar above analysis procedure, it show that the trend curves obtained by numerical method and DCM are consistent each other very well between $62.5\text{rad/s} \le \omega \le 108\text{rad/s}$. According to Fig.3 and Fig.4 following conclusions can be given

① When the rotational speed ω reaches 62.5rad/s, the bifurcation phenomena of dimensionless displacement x_4 and x_{24} occurs in present case of parameters.

② When $62.5\text{rad/s} \le \omega \le 108\text{rad/s}$, the chaos occurs. When the rotational speed ω reaches 108rad/s, the response of displacement became the period motion.

There is some difference of amplitude of response between these two methods. It is the disadvantage of proposed method and we will work hard to perfect it in the feature.

5 CONCLUSIONS

The characteristics of exact analysis and high efficiency are the merits of proposed method for classical numerical method. Based on WEM the DCM has been studied in this paper in order to analyze the nonlinear oil force model of 200MW turbo-generator. The nonlinear algebraic equations of nodes displacement that combined with expression of nonlinear oil force can be solved by proposed method. The DCM and WEM are supplemented each other, they are one of the ideas to solve the nonlinear problems of large rotational machinery such as 200MW turbo-generator system.

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