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NUMERICAL MODEL FOR THE INTERACTION OF A GAS BUBBLE WITH A STRONG ACOUSTIC FIELD

Alexander V. Teterev*[1], Natalya I. Misychenko[1], Larisa V. Rudak[1], and Alexander A. Doinikov[2]

[1]Department of Applied Mathematics and Computer Science, Belarus State University
4 Prospect Nezavisimosti, Minsk 220030, Belarus
[2]Institute of Nuclear Problems, Belarus State University
11 Bobruiskaya Street, Minsk 220050, Belarus
teterev@bsu.by

Abstract

A numerical model has been developed for simulating the oscillatory dynamics of a gas bubble in a strong acoustic field. The radial pulsation of the bubble is calculated by combining a Rayleigh-Plesset-like equation with a one-dimensional gas-dynamic problem that describes the gas motion inside the bubble. Solving the internal gas-dynamic problem together with the Rayleigh-Plesset-like equation is provided by appropriate boundary conditions at the gas-liquid interface. Numerical results obtained by using the above model are compared with those obtained on the basis of the conventional approach in which only the Rayleigh-Plesset-like equation is used and the gas pressure within the bubble is assumed to be spatially homogeneous and specified by the adiabatic law. It is shown that the difference between the two approaches increases with time of computation. The new model predicts the effect of energy pumping into the bubble which results in increasing mean bubble radius.

INTRODUCTION

Simulation of the interaction of a gas bubble suspended in a liquid with an acoustic field is commonly carried out by using a Rayleigh-Plesset-like equation. If the imposed acoustic field is moderate so that the compressibility of the ambient liquid and resulting radiation losses are negligible, the original Rayleigh-Plesset equation is applied [6,7]. For stronger forcing, when the velocity of the bubble surface is comparable to the sound speed in the ambient liquid and the acoustic radiation losses due to the finite liquid compressibility are no longer negligible, more complicated models are used, such as the Herring-Flynn equation [2,4,12], the Keller-Miksis equation [5], or the Gilmore model [3,10]. All the above models assume that the gas

pressure is spatially homogeneous within the bubble and obeys the adiabatic law. However, this is not always the case. For example, bubble pulsations can be too fast for the gas pressure to have time to become even throughout the bubble volume. In such cases, the description of the internal gas dynamics of the bubble by means of general hydrodynamic equations is more justified. It is the purpose of the present paper to develop such a model and compare its predictions with results provided by Rayleigh-Plesset-like equations.

NUMERICAL MODEL OF BUBBLE OSCILLATIONS IN A LIQUID

In this section, a model is developed that describes the radial motion of a gas bubble in an acoustic field, the gas pressure within the bubble being calculated by gas-dynamic equations. The following computational procedure is applied. The velocity of the bubble surface is calculated from a Rayleigh-Plesset-like equation and then used as the boundary condition at the gas-liquid interface in the gas-dynamic problem which describes the gas motion within the bubble for a given time layer. The gas pressure obtained by solving this problem is then used to determine the velocity of the bubble surface for the next time layer.

Mathematical model

As the problem under consideration is one-dimensional, it is reasonable to solve it using the Lagrangian method. The continuity equation, written in Lagrangian mass coordinates, for the case of spherical symmetry, and with respect to the density per unit spatial angle, takes the form [13]:

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial\left(ur^2\right)}{\partial m}, \tag{1}$$

where ρ and u are the density and the velocity of the gas, respectively, and the relation between the Euler and the mass coordinates is given by $dm = \rho r^2 dr$.

The equation of motion in the Lagrangian coordinates is written as

$$\frac{\partial u}{\partial t} = -r^2 \frac{\partial p}{\partial m}, \tag{2}$$

where p is the gas pressure.

The small size of the bubble and the high value of the speed of sound lead to very small time steps. As a result, the solution of the problem is reached over a very large number of time steps. In addition, in some cases it is necessary to trace the medium parameters for a large number of oscillations. These circumstances make high demands to the accuracy of energy computation. Therefore it is reasonable to take the equation of energy in the divergent form:

$$\frac{\partial}{\partial t}\left(\varepsilon+\frac{u^2}{2}\right)=-r\frac{\partial}{\partial m}\left(r^2 pu\right), \tag{3}$$

where ε is the internal energy per unit mass.

The surface tension pressure on the bubble surface is given by

$$p_\sigma=\frac{2\sigma}{R}, \tag{4}$$

where σ is the surface tension coefficient and R is the radius of the bubble.

To close the set of (1) – (4), it should be supplemented with a state equation, $p = p(\rho,\varepsilon)$. If the temperature distribution within the gas is required, an equation for temperature, $T = T(\rho,\varepsilon)$, should be added as well. For a perfect gas, these equations can be written as $p=(\gamma-1)\rho\varepsilon$ and $T=\varepsilon/c_v$, where γ is the ratio of specific heats and c_v is the specific heat at constant volume.

The boundary condition $u_R = v$ at the spherical bubble surface is sort of a piston, whose velocity v is calculated from a Rayleigh-Plesset-like equation which takes in general the form:

$$\frac{dv}{dt}=F\left(R,v,P_0,P_g,P_{ac},\sigma,\eta,\rho_0,c,\gamma\right),\quad v=\frac{dR}{dt}, \tag{5}$$

where P_0 is the initial pressure in the ambient liquid, P_g is the gas pressure, P_{ac} is the imposed acoustic pressure, η is the liquid viscosity, ρ_0 is the equilibrium liquid density, and c is the speed of sound in the liquid. A particular form of (5) is chosen depending on the problem parameters.

Numerical model

Solution of the set of gas-dynamic equations (1) – (4) can be written on a mass non-uniform grid: $m = \{m_i, m_{i+1/2}, m_{i+1} = m_i + h_i, m_{i+1/2} = m_i + 0.5h_i, i = 0,1,..., N-1, m_0 = 0, m_N = M\}$, where M is the mass of the gas. The grid functions of the radius $r_i = r$ and the velocities $u_i = u$ are related to the integer points of the grid, while the pressures $p_{i+1/2} = p$, the densities $\rho_{i+1/2} = \rho$, the internal energies $\varepsilon_{i+1/2} = \varepsilon$, and temperatures to the half-integer points $m_{i+1/2}$. The set of difference equations can then be written as [11]

$$\frac{u_i^{n+1}-u_i^n}{\Delta t^n}=R_i^{n+1/2}\frac{p_{i+1/2}^{n+1/2}-p_{i-1/2}^{n+1/2}}{0.5\left(h_{i-1}-h_i\right)},$$

$$R_i^{n+1/2}=\frac{1}{3}\left[\left(r_i^{n+1}\right)^2+r_i^n\cdot r_i^{n+1}+\left(r_i^n\right)^2\right],$$

$$\frac{r_i^{n+1} - r_i^n}{\Delta t^n} = u_i^{n+1/2},$$

$$\frac{(1/\rho)^{n+1} - (1/\rho)^n}{\Delta t^n} = \frac{R_{i+1}^{n+1/2} u_{i+1}^{n+1/2} - R_i^{n+1/2} u_i^{n+1/2}}{h_i}, \tag{6}$$

$$\frac{E_{i+1/2}^{n+1} - E_{i+1/2}^n}{\Delta t^n} = -\frac{\hat{p}_{i+1}^{n+1/2} R_{i+1}^{n+1/2} u_{i+1}^{n+1/2} - \hat{p}_i^{n+1/2} R_i^{n+1/2} u_i^{n+1/2}}{h_i},$$

$$E_{i+1/2}^n = \varepsilon_{i+1/2}^n + \frac{1}{4}\left[\left(u_i^n\right)^2 + \left(u_{i+1}^n\right)^2 \right],$$

$$\hat{p}_i^{n+1/2} = \frac{h_i p_{i-1/2}^{n+1/2} + h_{i-1} p_{i+1/2}^{n+1/2}}{h_i + h_{i-1}}.$$

The difference scheme (6) approximates the set of the gas-dynamic equations with accuracy up to the second order in both the spatial and time coordinates. The implicit scheme (6) was solved by an iteration method. Convergence of the specific internal energy ε over every time layer for every computational cell was used as the convergence condition. The gas pressure at the bubble surface obtained by (6) was used as an input parameter for (5), which was solved by the explicit scheme accurate to first or second order in time.

The 1D program for modeling the above-stated problem was built up on the basis of the well-known programming system OLYMPUS [1,9]. This latter is based on the principle of computation with splitting into different physical processes. The bundled software includes the specially developed internal system of graphic output SIGO, which makes it possible to monitor computational results in interactive mode. In addition, the stand-alone system of animation and graphic output, SAGO, allows computational results to be represented both in graphic form and as animation films created on the basis of the problem parameters.

NUMERICAL SIMULATION

In this section, the potential of the developed model is demonstrated by numerically simulating radial oscillations of an air bubble in water. Physical conditions characteristic of experiments on sonoluminescence were set. The radial motion of the bubble is described by the Keller-Miksis equation [5], which is written in terms of the velocity of the bubble surface v as

$$R\frac{dv}{dt}\left(1-\frac{v}{c}\right)+\frac{3}{2}v^2\left(1-\frac{v}{3c}\right)=\left(1+\frac{v}{c}\right)\frac{P_R - P_0}{g_0}+\frac{R}{c\rho_0}\frac{dP_R}{dt}, \tag{7}$$

where P_R is given by

$$P_R = P_g(t) - \frac{2v}{R} - \frac{2\sigma}{R} - \frac{4\eta v}{R} - P_{ac}(t). \tag{8}$$

The time-varying gas pressure $P_g(t)$ is calculated either from the adiabatic law,

$$P_g(t) = \left(P_0 + \frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\gamma}, \tag{9}$$

in the case that the calculation is carried out by using the Keller-Miksis equation alone, or from the gas-dynamic problem, as the value of the gas pressure at the inner bubble surface.

The imposed acoustic pressure $P_{ac}(t)$ is specified by

$$P_{ac}(t) = P_A \sin(2\pi ft), \tag{10}$$

where P_A is the pressure amplitude and f is the driving frequency. The values of the physical parameters required for these calculations are $\gamma = 1.4$, $\sigma = 0.0725$ N/m, $\eta = 0.001$ Pa s, $P_0 = 1.014 \cdot 10^5$ Pa, $\rho_0 = 1000$ kg/m^3, and $c = 1500$ m/s. The other parameters were as follows: the initial bubble radius $R_0 = 5 \cdot 10^{-6}$ m, the driving frequency $f = 2 \cdot 10^4$ Hz, and the acoustic pressure amplitude $P_A = 1.26 P_0$. The initial bubble radius was covered with a computational grid, which consisted of 100 cells, in the following way. The first eight computational cells counting from the center of symmetry were set equal in mass. The rest of the computational grid consisted of cells with an equal spatial step. This partition made possible correcting the strong mass-irregular character of the computational grid in the proximity of the coordinate origin for the spatially uniform grid. Simulations by the developed model and by (7) alone were carried out simultaneously with the same time step Δt, which made possible avoiding additional errors related to the different precision of approximating the equations at different time steps. The value of the time step in the gas-dynamic problem was chosen by Courant's condition [8]. The small size of the time step, especially at the moments of the maximum bubble contraction, resulted in the fact that the accuracy of the solution of (7) in the first and the second orders coincided with the graphical accuracy.

Figure 1 shows the variation of the bubble radius over approximately 6.5 cycles of the acoustic wave. One can see that the main discrepancy between the solution obtained from (7) alone (black line) and that obtained by combining (7) with (1) – (4) (red line) lies in different amplitudes of the high-frequency rebound oscillations of the bubble, which are noticeably smaller in the latter case. In addition, if the plot scale is increased, one can see an increase in the bubble radius at the moments of maximum expansion, which is growing cycle by cycle. To understand this effect, let us consider

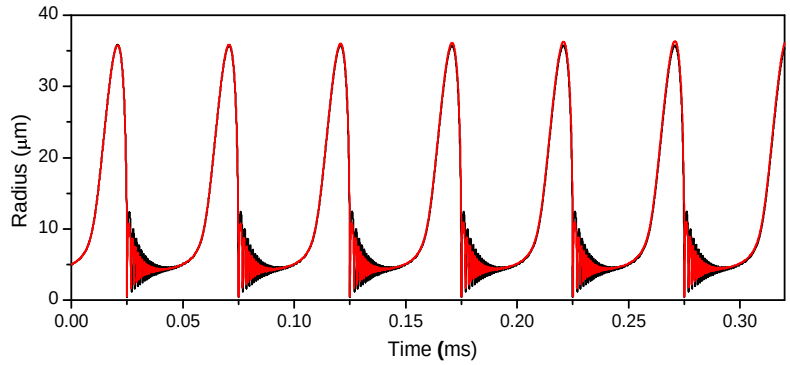

Figure 1. Bubble radius as a function of time. The black line corresponds to the solution of the Keller-Miksis equation alone. The red line shows the result given by the developed model.

work that is done by the ambient liquid on the bubble, see Figure 2. As one would expect, the total work done on the bubble over each oscillatory period for the case of the Keller-Miksis equation is zero, which corresponds to the adiabatic law used in this equation. A different pattern occurs if the gas-dynamic problem for the gas motion within the bubble is solved. In this case, energy pumping takes place, which results in

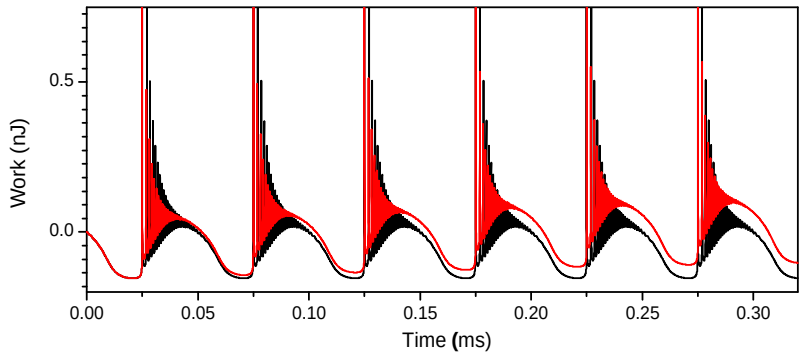

Figure 2. Work done by the ambient liquid on the bubble as a function of time.

increasing mean bubble radius. The effect of the energy pumping into the bubble is explained by Figure 3, which shows that work at the gas compression occurs when the pressure inside the bubble monotonically increases from the center of the bubble towards the gas-liquid interface. Such a moment is just displayed in Figure 3. The expansion of the bubble occurs at reduced pressure at the gas-liquid interface relative to the pressure in the inner bubble area. This behavior of the pressure profile results in the energy pumping into the bubble during every acoustic cycle and renders the adiabatic process irreversible. Numerous high-frequency rebound oscillations following the main oscillation lead to a tangible energy pumping into the bubble.

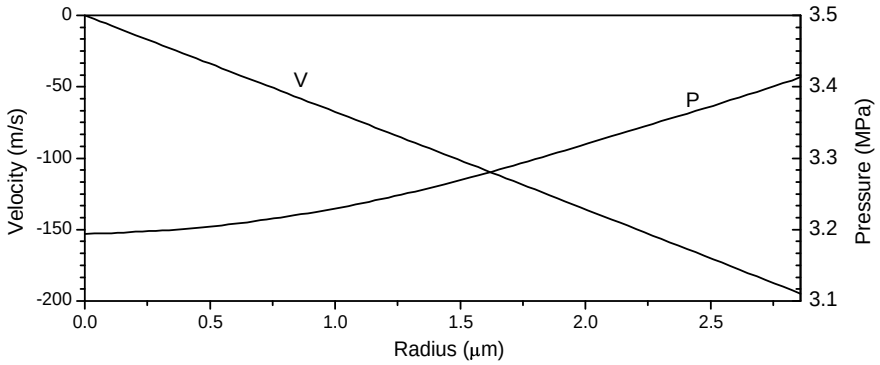

Figure 3. Velocity (v) and pressure (p) along the bubble radius for t = 124 μs.

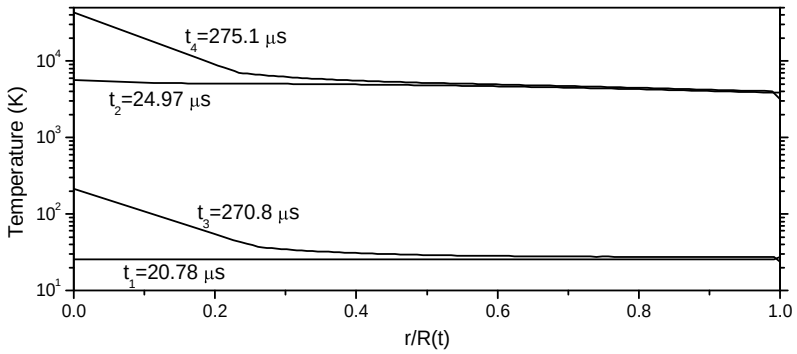

Figure 4. Gas temperature at different instants of time. The instants t_1 and t_3 correspond to the maximum expansion of the bubble, while t_2 and t_4 to its maximum compression.

Finally, Figure 4 demonstrates the temperature profiles inside the bubble which approximately correspond to some points in time when the expansion or compression of the bubble is a maximum. The time points in Figure 4 are for the first and the sixth cycles of the acoustic wave. The temperature, to which the area nearby the bubble center is warmed up, is so high that the effects of thermal conductivity and radiation are no longer negligible. The temperature growth at the bubble center over every period of the acoustic wave (Figure 4) is due to the above-stated process of energy pumping. Clearly such a system can arrive at a steady state in the case that losses due to, for example, re-radiation are equal to the acoustic-wave energy pumping.

CONCLUSIONS

The developed numerical model, based on combining a Rayleigh-Plesset-like equation with the gas-dynamic problem for the interior of a gas bubble, can be applied in many numerical investigations. Simulations carried out by using the above

model predict the effect of energy pumping into the bubble which results in increasing mean radius of the bubble. The data on the distribution of the gas parameters within the bubble volume suggest that the model can be improved by incorporating the effects of thermal conductivity and radiation transfer of energy as well as by replacing the perfect gas law with equations of state for real gases, such as table relationships. The set of Rayleigh-Plesset-like equations available in the bundled software makes possible comparison between numerical simulations performed under various starting assumptions. The capabilities of the model can be extended by setting a virtual oscillating surface at a distance inside the ambient liquid rather than at the very gas-liquid interface. In this case, surface effects at the gas-liquid interface will fall within the domain of the gas-dynamic problem.

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