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APPLICATION OF THE VIDEO CAMERA MEASUREMENTS TO EXPERIMENTAL DETERMINATION OF THE STRUCTURAL ACOUSTIC VECTOR FOR VIBRATING BEAMS

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Abstract
The useful method of analysis of power flow paths for vibrating systems of surface structural elements are based on the sound structural intensity vector distribution. It can be done theoretically (numerically) or experimentally. The theoretical methods are based on the analysis of vibration of the structure by finite element method and determination of the structural intensity vector. The well-known methods of experimental determination of the considered vector are the source of the transducer mass effect. This effect can be minimized by application of the only strain signals type measurements and using relatively low mass transducers as classical strain gauges or piezoelectric elements. Application to the measurements a video signal varying in time, gives a possibility to determine the structural intensity vector with no mass effect. Analysis of the image registered by the video camera makes possible to determine the displacement of chosen points on the external surfaces of the beam and then to determine the suitable strains components. By application of the strain measurement methods the determination of power flow in beams is possible. Theoretical basis of the method and some experimental remarks are discussed in the paper.

INTRODUCTION

Determination of the structural intensity vector for vibrating technical structures are not easy to measurement. In practice, the measurements are reduced to the external surfaces, and based on these results, it is possible, assuming a suitable geometric hyphothesis, to determine the intensity vector averaged over the chosen cross-section. This attempt is called as surface structural intensity method [13].

The commonly used, one-dimensional engineering structure is beam. Experimentally determination of power flow for bending waves can be realized previously by usage of the piezoelectic transducers [2, 12, 14, 17, 16]. The main disadvantages of the method is negative effect of the transducer mass on the results of measurements. For reducing of the effect, some of piezoelectric transducers can be replaced by the strain gauged ones [4, 6, 7]. Last times the non-contact, laser Doppler velocimeter sensor head was proposed to measurement of structural surface intensity [3]. The author proposed application of the only strain measuremets to determine the power flow in beams [5]. Instead the strain gauges, the PVDF structures can be applied to measurement of strais. In the last times, it can be possible to apply the video non-contact measurement to dynamic measurements [10, 11, 15].

In the presented paper some experimental results of the determination of power flow in beams with the only video camera measurements are discussed.

THEORETICAL BACKGROUND

The formula for power flowing through the surface perpendicular to the axis of a beam is built based on the general definition of structural intensity vector for elastic structures $\underline{I}$ and power flowing through surface S, denoted as $P(S)$ (2), where: $\underline{I}$ - structural intensity vector, $\underline{\underline{\sigma}}$ - stress tensor, $\underline{v}$ - velocity, $P(S)$ - power flowing through surface S, $\underline{n}$ - normal unit vector to surface dS.

$$\underline{I} = -\underline{\underline{\sigma}} \cdot \underline{v} \tag{1}$$

$$P(S) = \iint\limits_{(S)} \underline{I} \cdot \underline{n} \; dS \tag{2}$$

For the beam case, it is assumed the Bernoulli-Euler geometric hyphothesis and the Hook constitutive law for isotropic and homogeneous material. The power flowing through a cross-section perpendicular to the axis of a beam in $+x$ direction can be formulated in terms of generalized forces and generalized velocities in the form (3), where: E - Young modulus, I - moment of inertia of cross-section, M - bending moment, Q - shear force, w - transverse displacement, φ - slope angle, $\varphi = \frac{\partial w}{\partial x}$.

$$P = M\dot{\varphi} - Q\dot{w} = EI\;[-\frac{\partial^2 w}{\partial x^2}\frac{\partial \dot{w}}{\partial x} + \frac{\partial^3 w}{\partial x^3}\dot{w}] \tag{3}$$

For the farfield the formula (3) can be simplified in two ways (4) or (5).

$$P \approx 2M\dot{\varphi} = -2EI\;\frac{\partial^2 w}{\partial x^2}\frac{\partial \dot{w}}{\partial x} \tag{4}$$

$$P \approx -2Q\dot{w} = 2EI\;\frac{\partial^3 w}{\partial x^3}\dot{w} \tag{5}$$

STRAIN METHODS

The relationships applied in the method are based on the equations of motion of the beam in the form (6). Equation is valid in the regions, where there is no external loadings. The second important relationship is connected with assumed Bernoulli-Euler hyphotesis. For such case, the relationship between strain (ε), curvature (κ) and distance of the external surface beam from the neutral axis (z) has the form (7).

$$EI\,\frac{\partial^4 w}{\partial x^4} + \rho A\,\ddot{w} = 0 \tag{6}$$

$$\varepsilon = \kappa\, z \approx \frac{\partial^2 w}{\partial x^2}\, z \tag{7}$$

After suitable manipulations, the generalized velocities can be detremined as the functions of the strains measured on the external surfaces of the beam in the forms (8), (9) and (10), where u – longitudinal displacement.

$$\dot{w} = \frac{EI}{\rho A}\frac{1}{z}\int \frac{\partial^2 \varepsilon}{\partial x^2}\, dt \tag{8}$$

$$\dot{\varphi} = \frac{\partial \dot{w}}{\partial x} = \frac{EI}{\rho A}\frac{1}{z}\int \frac{\partial^3 \varepsilon}{\partial x^3}\, dt \tag{9}$$

$$\dot{\varphi} = \frac{\dot{u}}{z_\varepsilon} \tag{10}$$

Table 1: Finite difference schemes for determination of the generalized velocities and forces based on measurement of strains for beams.

PARAMETER	FDM strain scheme
$\dot{w}_C$	$\approx \frac{EI}{\rho A}\frac{1}{z_\varepsilon}\frac{1}{\Delta^2}\int(2\varepsilon_C - \varepsilon_{-1} - \varepsilon_1)\, dt$
$\dot{w}_C$	$\approx \frac{EI}{\rho A}\frac{1}{z_\varepsilon}\frac{1}{12\Delta^2}\int(30\varepsilon_C - 16\varepsilon_{-1} - 16\varepsilon_1 + \varepsilon_{-2} + \varepsilon_2)\, dt$
$\dot{\varphi}_C$	$\approx \frac{EI}{\rho A}\frac{1}{z_\varepsilon}\frac{1}{2\Delta^3}\int(\varepsilon_{-2} - 2\varepsilon_{-1} + \varepsilon_1 - \varepsilon_2)\, dt$
$\dot{\varphi}_C$	$\frac{\dot{u}}{z_\varepsilon}$
M_C	$-\frac{EI}{z_\varepsilon}\varepsilon_C$
M_C	$\approx -\frac{EI}{z_\varepsilon}\frac{1}{2}(\varepsilon_{-1} + \varepsilon_1)$
Q_C	$\approx \frac{EI}{z_\varepsilon}\frac{1}{2\Delta}(-\varepsilon_1 + \varepsilon_{-1})$
Q_C	$\approx \frac{EI}{z_\varepsilon}\frac{1}{12\Delta}(-\varepsilon_{-2} + 8\varepsilon_{-1} - 8\varepsilon_1 + \varepsilon_2)$

Replacing differential operators by the suitable finite difference ones the measuring formulas for experimental determination of generalized velocities are obtained. The forms of formulas are connected with the applied type of finite diference operators. Some forms of operators are shown e.g. [1]. The finite difference schemes for determination of the generalized

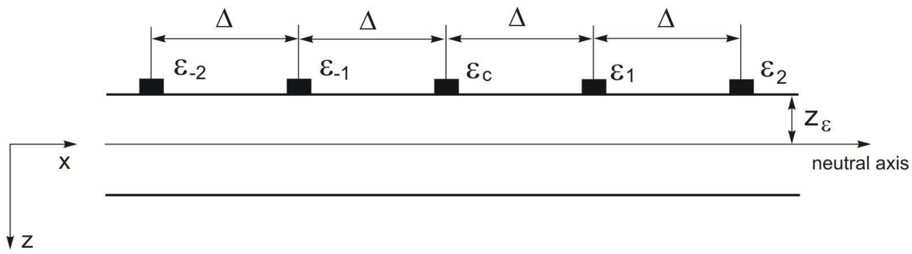

Figure 1: Position of the strain measurement points

velocities and forces based on measurement of strains for beams are given in Table 1. Position of the measuring points are shown in Fig.1. Chosing the appropriate scheems and the theoretical formula for determining the power flow (3, 4, 5), it is possible to build the measurement set-up.

APPLICATION OF THE VIDEO CAMERA IN DYNAMICAL MEASUREMENTS

For testing the possibility of application of the video camera measurements to determination of the structural intensity vector for bending vibrations of a beam, the following experiment has been done in the laboratory. The power flow was experimentally determined for a fixed–free beam, length of 0.3 m, width of 0.03 m and height of 0.002 m, made of steel. The vibrations was excited by set of four square piezoelements with dimension of 0.02x0.02 m glued in pairs to both external surfaces of beam, distanced 0.08 m and 0.2 m (central point) from the fixed end. The first mode of vibrations was considered. The frequency of excitation in experiments was equal to 17.5 Hz (the lowest theoretical eigenfrequency is equal to 18.6 Hz)

The measuring points positions were positioned on the external surface of a beam, started 0.005 m from the fixed end. The finite difference base was equal to Δ=0.005 m (see Fig.1).

In measurements there were used video camera equipment contains of the high speed video camera for signal acquisition and the computer software for signal analysis.

The measured amplitudes of vibrations (longitudinal ones) were about 1.7E-5 m, and the amplitudes of relative displacements between measuring points were about 5.0E-6 m. Longitudinal displacement and linear velocity signals for the chosen measuring point (1) are shown in Fig.2 and Fig.3. It gives the values of stain amplitudes about 1.0E-3 (0.1%). The amplitudes of longitudinal linear velocities were about 0.002 $\frac{m}{s}$. The form of spectrum for transversal displacement and linear velocities signals clearly show the peak for the analysed frequency of 17.5 Hz (see Fig.2 and Fig.3).

The form of instantaneous power flow for the cross section distanced 0.0175 m from

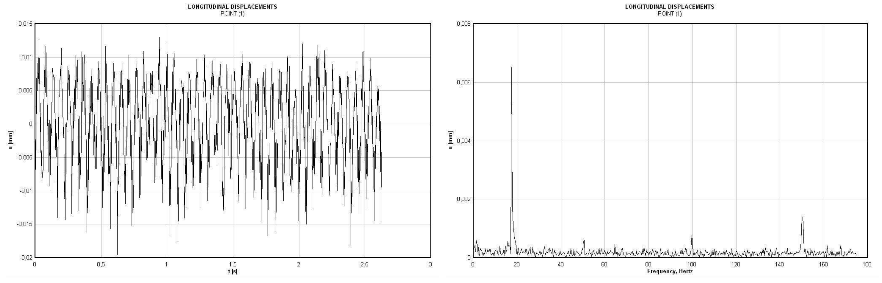

Figure 2: Longitudinal displacements in measuring point (1) and its spectrum

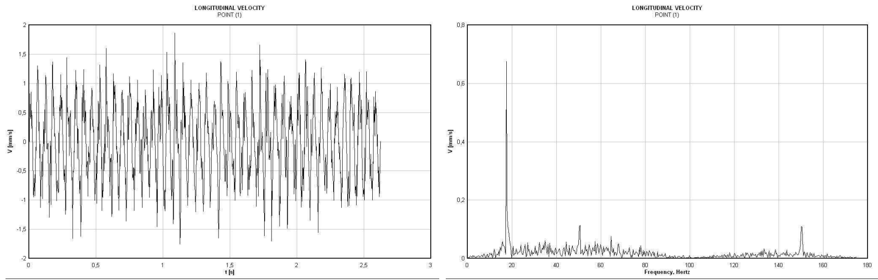

Figure 3: Longitudinal linerar velocity in measuring point (1) and its spectrum

the fixed end is shown in Fig.5. For it determination, there was applied simplified formula (4). Angular velocity and bending moment in central point (C), were obtained based on fourth and fifth formulas given in Table 1, and were shown in Fig.4.

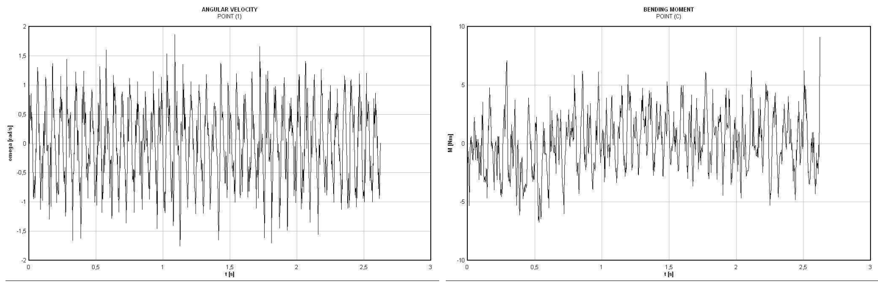

Figure 4: Angular velocity (left) and bending moment (right) in measuring point (C)

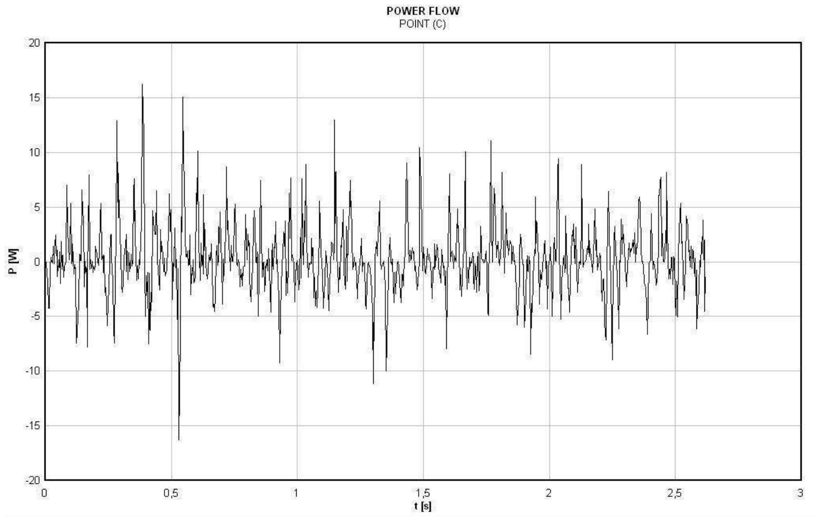

Figure 5: Power flow in measuring point (C)

CONCLUSIONS

The measurement show possibility to apply the video camera measurements to experimental determination of the structural intensity vector or power flow of bending waves in beams. The sensitivity of the method is enough good, for relatively small amplitudes of vibrations. The method are not source of the mass of transducers. The limitation of the method is the upper frequency, which can be analysed by the camera. Therefore the method can be applied in low frequencies.

The application of the video camera measurements are the natural way to determine of the structural intensity vector for longitudinal waves in beam.

It is probably true, that the method can be applied do determination of the structural intensity vector for bending vibrations of plates according to the relationships given in [7, 8], and patented as measuring matrix [9].

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