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SOUND RADIATION BY THE COLOURED NOISE EXCITED VISCOELASTIC SHALLOW SHELLS

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Abstract
Shallow shells are commonly used as constructional elements in cages of heavy duty machines. These elements, are sources of structural noise radiated during the machine operation. In the previous authors' papers the analysis of influence of curvature of the shallow shell elements on the level of their structural vibrations and sound radiation for different kinds of excitation (deterministic: harmonic, polyharmonic; random of white noise type) were discussed. In the presented paper the method based on the acoustic power radiated by the shell is applied to estimate the sound radiated by randomly excited shallow shells with different radii of curvatures. The shell is loaded by the continuous force, distributed over the middle surface, and randomly varying in time. The random, coloured noise type of excitation is assumed. The estimations are valid for the acoustic far-field.

INTRODUCTION

Shallow shells are commonly used as constructional elements in cages of heavy duty machines. These elements, are sources of structural noise radiated during the machine operation. In the previous authors' papers the analysis of influence of curvature of the shallow shell elements on the level of their structural vibrations for different kinds of excitation (deterministic: harmonic, polyharmonic; random: white and coloured noise) were discussed [5, 6, 7, 8, 9]. In the presented paper the method based on the acoustic power radiated by the shell was applied to estimate the sound radiated by randomly excited shallow shells with different radii of curvatures. The estimations are valid for the acoustic far-field.

MATHEMATICAL DESCRIPTION OF VIBRATION

Geometry, equations of motion, boundary conditions, solution

Let us consider the vibrating shallow shell with internal viscoelastic damping. As shallow shell it is understood a shell for whom for every point connected to its middle surface, the angle between tangent plane and its perpendicular projection on the plane is enough small [10] i.e. is smaller than about 18 deg [15]. The other definition says that for shallow shell the ratio between strzalka and the characteristic dimension $l = min(a, b)$ is smaller than 0.2 [3] (see Fig.1).

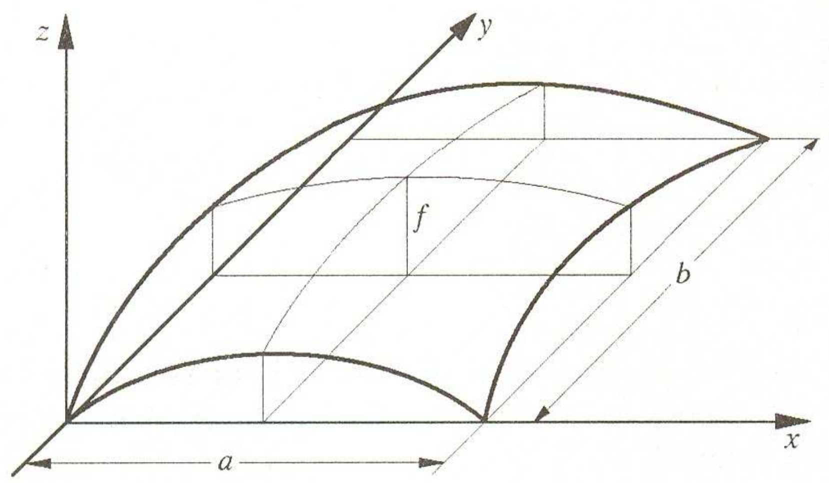

Figure 1: Geometry of shallow shell

The Voigt-Kelvin model of damping was assumed. The detailed description of the model are given in the paper [9]. Some theoretical background of the applied shell models can be found in [2, 10, 13, 14]. The geometry of shallow shell is shown in Fig.1. The inertial components which represents the motion in tangential directions of the middle surface are neglected in the model. Moreover it is assumed that the velocity components in the in-plane equations of motion are neglected, and the only excitation in the Oz direction are taken into account. The excitation has the form of surface distributed function, which is multiplication of the two components: constant value z_0 and the randomly variabled time function $f(t)$. The obtained set of equations of motion have the forms (1), where the symbols denote: u, v, w – displacements in directions Ox, Oy and Oz respectively, R_x and R_y – radii of main curvatures, h – thickness, E – Young modulus, ν - Poisson ratio, D – bending stiffness, B – in-plane stiffness, ε - viscous damping coefficient, ρ - material density.

$$\left.\begin{array}{l}\frac{\partial^2 u}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2}+\frac{1+\nu}{2}\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial}{\partial x}(\frac{w}{R_x}+\nu\frac{w}{R_y})\\ +\varepsilon\frac{\partial^2 \dot{u}}{\partial x^2}+\varepsilon\frac{1-\nu}{2}\frac{\partial^2 \dot{u}}{\partial y^2}+\varepsilon\frac{1+\nu}{2}\frac{\partial^2 \dot{v}}{\partial x\partial y}+\varepsilon\frac{\partial}{\partial x}(\frac{\dot{w}}{R_x}+\nu\frac{\dot{w}}{R_y})-\frac{\rho h}{B}\ddot{u}=0\\ \\ \frac{\partial^2 v}{\partial y^2}+\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial}{\partial y}(\frac{w}{R_y}+\nu\frac{w}{R_x})\\ +\varepsilon\frac{\partial^2 \dot{v}}{\partial y^2}+\varepsilon\frac{1-\nu}{2}\frac{\partial^2 \dot{v}}{\partial x^2}+\varepsilon\frac{1+\nu}{2}\frac{\partial^2 \dot{u}}{\partial x\partial y}+\varepsilon\frac{\partial}{\partial y}(\frac{\dot{w}}{R_y}+\nu\frac{\dot{w}}{R_x})-\frac{\rho h}{B}\ddot{v}=0\\ \\ \frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}+\frac{12}{h^2}(\frac{1}{R_x}+\frac{\nu}{R_y})\frac{\partial u}{\partial x}+\frac{12}{h^2}(\frac{1}{R_y}+\frac{\nu}{R_x})\frac{\partial v}{\partial y}\\ +\frac{12}{h^2}(\frac{1}{R_x^2}+\frac{1}{R_y^2}+\frac{2\nu}{R_xR_y})w+\varepsilon\frac{\partial^4 \dot{w}}{\partial x^4}+2\varepsilon\frac{\partial^4 \dot{w}}{\partial x^2\partial y^2}+\varepsilon\frac{\partial^4 \dot{w}}{\partial y^4}+\frac{12}{h^2}\varepsilon(\frac{1}{R_x}+\frac{\nu}{R_y})\frac{\partial \dot{u}}{\partial x}\\ +\frac{12}{h^2}\varepsilon(\frac{1}{R_y}+\frac{\nu}{R_x})\frac{\partial \dot{v}}{\partial y}+\frac{12}{h^2}\varepsilon(\frac{1}{R_x^2}+\frac{1}{R_y^2}+\frac{2\nu}{R_xR_y})\dot{w}+\frac{\rho h}{D}\ddot{w}=\frac{1}{D}z_0 f(t)\end{array}\right\} \tag{1}$$

The boundary condition of the shell have the following form: the simply supported one for bending vibrations (2) and the free-fixed one for the in-plane vibrations (3).

$$\left.\begin{array}{ll} w(x,y,t)_{|x=0}=0 & M_x(x,y,t)_{|x=0}=-D\frac{\partial^2 w(x,y,t)}{\partial x^2}_{|x=0}=0\\ w(x,y,t)_{|x=a}=0 & M_x(x,y,t)_{|x=a}=-D\frac{\partial^2 w(x,y,t)}{\partial x^2}_{|x=a}=0\\ w(x,y,t)_{|y=0}=0 & M_y(x,y,t)_{|y=0}=-D\frac{\partial^2 w(x,y,t)}{\partial y^2}_{|y=0}=0\\ w(x,y,t)_{|y=b}=0 & M_y(x,y,t)_{|y=b}=-D\frac{\partial^2 w(x,y,t)}{\partial y^2}_{|y=b}=0\end{array}\right\} \tag{2}$$

$$\left.\begin{array}{ll} v(x,y,t)_{|x=0}=0 & N_x(x,y,t)_{|x=0}=B[\frac{\partial u(x,y,t)}{\partial x}+\nu\frac{\partial v(x,y,t)}{\partial y}]_{|x=0}=0\\ v(x,y,t)_{|x=a}=0 & N_x(x,y,t)_{|x=a}=B[\frac{\partial u(x,y,t)}{\partial x}+\nu\frac{\partial v(x,y,t)}{\partial y}]_{|x=a}=0\\ u(x,y,t)_{|y=0}=0 & N_y(x,y,t)_{|y=0}=B[\frac{\partial v(x,y,t)}{\partial y}+\nu\frac{\partial u(x,y,t)}{\partial x}]_{|y=0}=0\\ u(x,y,t)_{|y=b}=0 & N_y(x,y,t)_{|y=b}=B[\frac{\partial v(x,y,t)}{\partial y}+\nu\frac{\partial u(x,y,t)}{\partial x}]_{|y=b}=0\end{array}\right\} \tag{3}$$

Randomly excited vibrations

When analysing the free vibrations case, it can be shown that the eigenfunctions for the $(m,n)-th$ mode of flexural vibration can be written in the form (4) [4, 9].

$$w_{mn}(x,y)=\sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y) \tag{4}$$

Let us assume that the time function $f(t)$ is a random stationary process with zeroes average value $< f(t) >= 0$, and well-known correlation function $K_{ff}(t_1,t_2)=K_{ff}(\tau)$; $\tau=t_2-t_1$. The solution of the problem for function of transversal vibrations $w(x,y,t)$

can be written in the series form (5), with unknown functions of time $S_{mn}(t)$. Solving the problem for determining functions $S_{mn}(t)$ in the way described in [9], the formula for unknown autocorrelation function K_{ww} takes the form (6). This formula is written as a function of unknown transfer functions $h_{mn}(t)$.

$$w(x,y,t) = \sum_{m,n=1,3,5,\ldots}^{+\infty} w_{mn}(x,y)\, S_{mn}(t) \tag{5}$$

$$\begin{aligned} K_{ww}(t_1,t_2) &= \frac{256}{\pi^4}\, \frac{z_0}{\rho^2 h^2} \sum_{m,n=1,3,5,\ldots}^{+\infty} \frac{1}{mn} \sum_{j,k=1,3,5,\ldots}^{+\infty} \frac{1}{jk} w_{mn}(x,y)\, w_{jk}(x,y) \cdot \\ & \int_0^{t_1} h_{mn}(t_1-\tau_1) \int_0^{t_2} h_{jk}(t_2-\tau_2)\, K_{ff}(\tau_1,\tau_2)\, d\tau_1\, d\tau_2 \end{aligned} \tag{6}$$

Dispersion of transversal displacements $\sigma_w^2(t)$ can be obtained directly based on the correlation function of excitation function by assuming contition $t_1 = t_2 = t$. For such a case, the correlation fuction of the excitation process has a form (7). The integral of transfer functions h_{mn}, takes the form (9), where α and β are the coefficients applied to define the form of coloured noise. When $\alpha \to +\infty$, it is obtained the limit case of the noise – the white nose. In the other case, for $\alpha = 0$ it is obtained the harmonic, deterministic function.

$$K_{ff}(\tau_1,\tau_2) = e^{-\alpha|\tau_1-\tau_2)|}\ \cos(\beta(\tau_1-\tau_2)) \tag{7}$$

$$\left.\begin{aligned} \sigma_w^2(x,y,t) &= 2\, \frac{256}{\pi^4}\, \frac{z_0^2}{\rho^2 h^2} \sum_{m,n=1,3,\ldots}^{+\infty} \frac{1}{mn} w_{mn}(x,y) \sum_{j,k=1,3,\ldots}^{+\infty} \frac{1}{jk} w_{jk}(x,y) \cdot \\ & \int_0^{t} h_{mn}(t-\tau_1) \int_0^{\tau_1} h_{jk}(t-\tau_2)\, K_{ff}(\tau_1,\tau_2)\, d\tau_1\, d\tau_2 \end{aligned}\right\} \tag{8}$$

$$\begin{aligned} & \int_0^{t_1} h_{mn}(t_1-\tau_1) \int_0^{t_2} h_{jk}(t_2-\tau_2)\, K_{ff}(\tau_1,\tau_2)\, d\tau_1\, d\tau_2 = \\ & \frac{\sin(\omega_{mn}\sqrt{1-\zeta_{mn}^2})\ \sin(\omega_{jk}\sqrt{1-\zeta_{jk}^2})}{\beta^2+(\alpha+\omega_{jk}\zeta_{jk})^2} \{ e^{-t(\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})} \\ & (\frac{(-1+e^{t(\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})})(\alpha+\omega_{jk}\zeta_{jk})}{\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk}} + \frac{\beta^2-(\alpha-\omega_{mn}\zeta_{mn})(\alpha+\omega_{jk}\zeta_{jk})}{\beta^2+(\alpha-\omega_{mn}\zeta_{mn})}) - \\ & e^{-t(\alpha+\omega_{jk}\zeta_{jk})} \frac{(\beta^2-(\alpha-\omega_{mn}\zeta_{mn})(\alpha+\omega_{jk}\zeta_{jk}))\cos(t\beta)+\beta(2\alpha-\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})\sin(t\beta)}{\beta^2+(\alpha-\omega_{mn}\zeta_{mn})^2} \} + \\ & \frac{\sin(\omega_{mn}\sqrt{1-\zeta_{mn}^2})\ \sin(\omega_{jk}\sqrt{1-\zeta_{jk}^2})\ e^{-t(\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})}}{(\beta^2+(\alpha+\omega_{mn}\zeta_{mn})^2)\ (\beta^2+(\alpha-\omega_{jk}\zeta_{jk})^2)\ (\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})} \\ & \{ e^{t(\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk})}\ (\beta^2+(\alpha+\omega_{mn}\zeta_{mn})^2)\ (-\alpha+\omega_{jk}\zeta_{jk}) + \\ & (\alpha+\omega_{mn}\zeta_{mn})\ (\beta^2+(\alpha-\omega_{jk}\zeta_{jk})^2) + e^{t(\alpha+\omega_{mn}\zeta_{mn})}(\omega_{mn}\zeta_{mn}+\omega_{jk}\zeta_{jk}) \\ & ((-\beta^2+(\alpha+\omega_{mn}\zeta_{mn})(\alpha-\omega_{jk}\zeta_{jk}))\cos(t\beta) + \\ & \beta(2\alpha+\omega_{mn}\zeta_{mn}-\omega_{jk}\zeta_{jk})\sin(t\beta)) \} \end{aligned} \tag{9}$$

In the considered case, the dispersion of the normal displacements, can be written in the form (8). The complete funcion is defined after calcullation of integrals for suitable corelation funcion of random coloured noise type funtion of excitation.

The detailed form of the function can be obtained after assuming the suitable form of the correlation function of external excitation K_{ff}. For the analysed case of the coloured noise (7), it has the form (9).

Based on the given relationships, the calcullation of the dispersion of transversal displacements or velocty is possible.

Estimation of acoustic radiation

The knowledge of the dispersion of displacements in Oz direction gives possibility to estimate the dispersion of the sound pressure level in chosen control point in acoustic medium. The method of estimation is based on the analysis of sound power radiated by vibrating panel. The acoustic sound power can be estimated based on the simplified formula (10) [1]. On the other hand, the pressure in the far-field, can be estimated based on the formula (11) [11]. After suitable manipulations, based on the formulas (10) and (11), it is possible to estimate the averaged pressure level L_p. The applied symbols denote: $< V_n^2 >$ - squared normal velocity averaged over the shell surface, ρ_0 – density of the acoustic medium, c – speed of sound in acoustic medium, S - area of the shell, S^* - area of the hemisphere with radius equal to approximate distance of the control point from the radiating panel, W_{rad} – sound power radiated by the shell, σ_{rad} – radiation efficiency coefficient, p_{av} – averaged sound pressure.

$$W_{rad} = \sigma_{rad}\, \rho_0\, c\, S < \overline{V_n^2} > \tag{10}$$

$$\rho\; c\; W_{rad} = p_{av}^2\; S^* \tag{11}$$

NUMERICAL EXAMPLE

Let us consider the shallow shell in the form of elliptical paraboloid made of steel, whose equation of middle surface has the form (12). Their projection on the xy plane has the form of a square a=b=1 m. The thickness is h=0.002 m, and the damping coefficient is equal $\varepsilon = 0.001$. The detailed values of geometrical and material parameters are the same like in [9]. The amplitude of excitation is equal to $z_0 = 1N/m^2$.

$$z(x, y) = f\; [1 - \frac{(2x - a)^2}{2a^2} - \frac{(2y - b)^2}{2b^2}] \tag{12}$$

The β parameter definig the colured noise was assumed as equal to $60\frac{rad}{s}$, which corresponds with the lowest natural frequencies for plate ($f = 0, \omega_{(1,1)} = 61.6\frac{rad}{s}$). The two group of estimation have been done. For the first one, the variable parameter was value of deflection f, and the second parametrer defining the coloured noise α was constant value equal to 0.1. The estimated values of the pressure dispersion levels in the control points distanced 1 m, 3 m and 5 m over the centre of the middle surface, for different values of deflection f are given in Table 1. In the Table 2, values of the same parameters estimated in the same control points, but analysed for plate (deflection $f = 0$) and different values of parameter α. The effect of increasing of the sound pressure level in accordance with increasing of the parameter α is connected with different energy of the considered signals.

Table 1: Powered dispersion of transversal displacements and velocity, averaged over the surface of element, and dispersion of sound pressure in control points, for different values of deflection f, and constant value of parameter $\alpha = 0.1$.

f	$<\sigma_w^2>$	$<\sigma_{\dot{w}}^2>$	$L_{\sigma_p^2}$ $[dB]$		
$[m]$	$[m^2]$	$[\frac{m^2}{s^2}]$	R=1 m	R=3 m	R=5 m
0	$5.635 \cdot 10^{-6}$	$8.589 \cdot 10^{-3}$	117.9	108.4	103.9
0.01	$4.948 \cdot 10^{-6}$	$9.291 \cdot 10^{-3}$	118.3	108.7	104.3
0.02	$3.991 \cdot 10^{-6}$	$8.445 \cdot 10^{-3}$	117.8	108.3	103.9
0.05	$6.796 \cdot 10^{-8}$	$2.934 \cdot 10^{-4}$	103.2	93.7	89.3
0.1	$1.884 \cdot 10^{-8}$	$9.585 \cdot 10^{-5}$	98.4	88.8	84.4
0.2	$1.231 \cdot 10^{-9}$	$6.354 \cdot 10^{-6}$	86.6	77.1	72.6

Table 2: Powered dispersion of transversal displacements and velocity, averaged over the surface of plate, and dispersion of sound pressure in control points, for variable value of parameter α.

α	$<\sigma_w^2>$	$<\sigma_{\dot{w}}^2>$	$L_{\sigma_p^2}$ $[dB]$		
[–]	$[m^2]$	$[\frac{m^2}{s^2}]$	R=1 m	R=3 m	R=5 m
0	$5.322 \cdot 10^{-6}$	$7.876 \cdot 10^{-3}$	117.5	108.0	103.6
0.05	$5.478 \cdot 10^{-6}$	$8.225 \cdot 10^{-3}$	117.7	108.2	103.7
0.1	$5.635 \cdot 10^{-6}$	$8.589 \cdot 10^{-3}$	117.9	108.4	103.9
1.0	$9.502 \cdot 10^{-6}$	$1.188 \cdot 10^{-2}$	121.3	111.8	107.3

CONCLUSIONS

The results of analysis show an important influence of the shallow shell radii of middle surface curvatures on dispersion of the pressure in chosen control point in the acoustic surrounding medium.

With increasing of the shell radii of curvatures, which is connected with increasing the shell deflection f, the values of basic natural frequencies rapidly growths. The effect of increasing is stronly higher for the parameter of modal damping coefficient.

References

[1] L.Beranek (Ed.), *Noise and Vibration Control*. (Institute of Noise Control Engineering, Washington DC, 1988)

[2] M.Epstein (Ed.), *A World of Shells.Festschrift In Honour of Peter G.Glockner on the Occasion on his Retitement from Teaching*. Proceedings of the Conference Held at the Bannf Centre, Alberta. (The University of Calgary, Calgary, 1995)

[3] N.V.Kolkunov, *Foundations of the Analysis of Elastic Shells*. (Vishaya Shkola, Moscow, 1972 (in Russian), Cracow University of Technology, Cracow, 1973 (in Polish)).

[4] M.Kozień, "Influence of the Inertial Components in Equations of Motion on Eigenfrequencies for the Vibrating Shallow Shells", Zeitschrift fur Angewandte Mathematik und Mechanik (ZAMM), Vol.76, Suppl.5, 271-272 (1996)

[5] M.Kozień, J.Nizioł, "Reduction of the values of normal velocity for vibrations of the glass of heavy duty machine TD20 E - the theoretical analysis", Proceedings of the Eight Symposium of Dynamic of Constructions. Technical Papers of Rzeszów University of Technology Nr.117 – Mechanics Vol.38, 71-78 (1993) (in Polish)

[6] M.S.Kozień, J.Nizioł, "Randomly Excited Vibrations of Viscoelastic Shallow Shells", Proceedings of the Fifth Workshop: Active Methods of Vibration and Noise Control, Krakow-Krynica, 225-232 (2001) (in Polish)

[7] M.Kozień, J.Nizioł, "Analysis of the value of normal surface velocity for the vibrating shallow shell" Technical Bulletin of the Cracow University of Technology – Mechanics, No. 4-M, 1-18 (1993) (in Polish)

[8] M.S.Kozień, J.Nizioł, "Sound Radiation by the White Noise Excited Viscoelastic Shallow Shells", *Mechanics of the 21st Century – Proceedings of the 21st International Congress of Theoretical and Applied Mechanics ICTAM04*, (Eds.: W.Gutkowski, T.A.Kowalewski), (Springer, e-book SM25_12511, 2005)

[9] J,Nizioł, M.S.Kozień, "White Noise Excited Vibrations of Viscoelastic Shallow Shells", J.Theoret.Appl.Mech., 2, 38: 351-366, (2000)

[10] Z.E.Mazurkiewicz, R.T.Nagórski, *Shells of Revolution*. (PWN Warsaw – Elsevier Amsterdam-Oxford-New York-Tokyo, 1991)

[11] R.Panuszka, "Research on the Sound Radiated by Circular Plate", Bulletin of the Academy of Mining and Metallurgy – Mechanics, 1, 125-145 (1982) (in Polish)

[12] K.Piszczek, J.Nizioł, *Random Vibration of Mechanical Systems*. (PWN Warszawa – Ellis Horwood Ltd. Chichester, 1986)

[13] Z.Q.Xia, S.Łukasiewicz, "On Internal Resonance of Sandwich Cylinndrical Panels Under Periodic Loadings", in: [2], 201-225 (1995).

[14] Z.Waszczyszyn (Ed.), *Mechanics of Buildings. Computer Formulation Vol.3*. (Arkady, Warsaw, 1995 (in Polish)).

[15] Cz.Woźniak (Ed.), *Technical Mechanics, Vol.VIII: Mechanics of Elastic Plates and Shells*. (PWN, Warsaw, 2001 (in Polish)).