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OVER-DETERMINATION IN ACOUSTIC TWO-PORT DATA MEASUREMENT

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Abstract

Measurement of plane wave acoustic transmission properties, so called two-port data, of flow duct components is important in many applications. It is an important tool for instance in the development of mufflers for IC-engines. Measurement of two-port data is difficult when the flow velocity in the measurement duct is high because of the flow noise contamination of the measured pressure signals. The plane wave acoustic two-port is a 2x2 matrix containing 4 complex quantities at each frequency. To experimentally determine these unknowns the acoustic state variables on the inlet and outlet side must be measured for two independent test cases. The two independent test cases can be created by: changing the acoustic load on the outlet side leading to the so-called two-load technique or by using one acoustic source on the inlet side and one acoustic source on the outlet side leading to the so-called two-source technique. In the latter case the independent test cases are created by first using the source on the inlet side and then the source on the outlet side. As pointed out by Åbom it is also possible to run both sources simultaneously to create more than two independent test cases. This over-determination could be used to improve the measurement results for instance if the data is contaminated by flow-noise. In this paper over-determination is tested by applying up to 5 different test cases. This procedure has been applied to a single orifice test object.

1. INTRODUCTION

There are several parameters that describe the acoustic performance of a muffler and/or its associated piping. These include noise reduction (NR), insertion loss (IL) and transmission loss (TL). The NR is the sound pressure level difference across the muffler. Though the NR can be easily measured, it is not particularly helpful for muffler design. The IL is the sound pressure level difference at a point, usually outside the system, without and with the muffler present. Though the IL is very useful to industry, it is not so easy to calculate since it depends not only on the muffler geometry itself but also on the source impedance and the radiation impedance. The TL is the difference in sound power level between the incident and the transmitted sound wave when the muffler termination is anechoic. It is a property of the duct element under test only so it is helpful for instance in muffler design. In many cases the acoustic properties such as transmission and insertion losses can not be determined analytically, owing for instance to the complex geometry or the presence of mean flow. Therefore experimental techniques must be used. The standard technique today for measuring acoustic plane wave properties in ducts, such as absorption coefficient, reflection coefficient and impedance is the two-microphone method (TMM) [1], [2]. The sound pressure is decomposed into its incident and reflected waves and the input sound power may then be calculated. Transmission loss can in principle be determined from measurement of the incident and transmitted power using the TMM on the upstream and downstream side of the test object provided that a fully anechoic termination can be implemented on the outlet side. It is however very difficult, to design an anechoic termination that is effective at low frequencies. An acoustical element, like a muffler, can also be modelled via its two-port data relating the acoustic state variables on the inlet and outlet sides [3], [4]. Using the two-port parameters, the transmission loss of a muffler can be readily calculated. Furthermore, if the source impedance is known, the two-port data parameters of the muffler can be used to predict the insertion loss of the muffler system [4]. The experimental determination of the two-port data has been investigated by many researchers. The most usfull method is that propsed by Doige and Munjal [5], the two different states variables required to calculate an acoustic 2-port are obtained by changing the source location, with the rest of the system kept unchanged. As demonstrated in reference [5] the two-source method typically gives better results compared to the two-load method and it does not affect on the mean flow field, since the geometry of the system is kept unchanged [5]. Åbom [6], [7] presented and tested a method for measuring the two-port data in the form of a scattering-matrix, describing the relationship between the traveling wave amplitudes of the pressure on either side of the test object. This technique can easily be extended to the case of an arbitrary number of ports. He also suggested the idea of over-determination by running both sources simultaneously and thereby creating more independent acoustic test cases. A method to suppress disturbing flow noise was also described in reference [7], using a reference signal correlated with the acoustic field. The aim of this work is to test this over-determination technique for improving the measurement results for instance if the data is contaminated by flow-noise.

2. THEORITICAL BACKGROUND

A two-port is a linear system with an input and output. The properties of acoustical two-ports can be determined either by theoretical models or by measurements. The relation between the input and the output states of a time-invariant, linear and passive two-port can, in the frequency domain, be written:

$$\mathbf{X} = \mathbf{T}\ \mathbf{Y} \qquad (1)$$

where, ***X/Y*** are the state vectors at the input/output and **T** is a [2×2]-matrix, which is independent of ***Y***. To determine **T**, from measurements four unknown must be determined. To make a complete experimental determination of the properties of an acoustical two-port, two independent tests must be carried out.

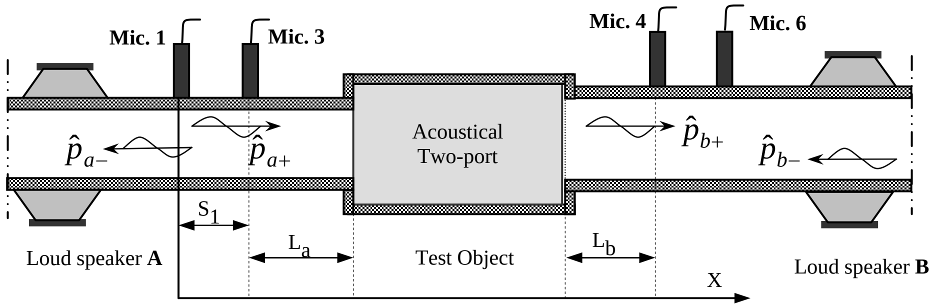

Figure *1*. *Black box relating two pairs of state variables x and y*

Any pair of state variables, i.e. a state vector, belonging to a two-port defines a linear 2D state-space. This means that from a given state vector an infinite set of alternative state vectors can be generated by linear transformations. Acoustic 2-port model is then an appropriate formalism and a common choice of state variables is the plane wave acoustic pressure ***p*** and volume velocity ***q***.
The transfer-matrix form uses the acoustic pressure p and the volume velocity q, i.e. $X = [p_a, q_a]$ and $Y = [p_b, q_b]$, here "***a***" and "***b***" represent two different ducts cross-section. If there are no internal sources inside the two-port element the transfer-matrix could be written in the following form [7]:

$$\begin{pmatrix} p_a \\ q_a \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b \\ q_b \end{pmatrix} \tag{2}$$

where,

$$p_a = p_+ \exp\left(-i\, k_+^a L_a\right) + p_- \exp\left(i\, k_-^a L_a\right) \tag{3}$$
$$q_a = \frac{A_a}{\rho c} \left\{\left(p_+ \exp\left(-i\, k_+^a L_a\right) - p_- \exp\left(i\, k_-^a L_a\right) \right)\right\}$$

and

$$p_b = p_+ \exp\left(-i\, k_+^b L_b\right) + p_- \exp\left(i\, k_-^b L_b\right) \tag{4}$$
$$q_b = \frac{A_b}{\rho c} \left\{\left(p_+ \exp\left(-i\, k_+^b L_b\right) - p_- \exp\left(i\, k_-^b L_b\right) \right)\right\}$$

In order to calculate the transfer matrix, the transfer function between a reference signal and traveling wave amplitudes in positive and negative direction see Figure *1* is required, the electric signal driving the external source, ***e***, is chosen. By using the assumptions in references [6], [7], the pressure amplitudes in positive and negative directions in ***a*** side can be written as:

$$\hat{p}_{+} = \frac{H_{e3}\ \exp(i\,k_{-}\ s_1) - H_{e1}}{\exp(i\,k_{-}\ s_1) - \exp(-\,i\,k_{+}\ s_1)}$$
$$\hat{p}_{-} = \frac{-\,H_{e3}\exp(-\,i\,k_{+}\ s_1) + H_{e1}}{\exp(i\,k_{-}\ s_1) - \exp(-\,i\,k_{+}\ s_1)} \qquad (5)$$

Due to the deviation from the ideal case, which introduces amplitude and phase shifts, relative calibration of the microphone measurement chain is therefore needed. It sufficient to measure the transfer function between the used microphones and a reference microphone say microphone *1*, then the calibrated transfer function can be presented as:
$H_{rm}^{cal} = H_{rm} / H_{1m}$, where *m* is the microphone number and *r* refers to the reference side. By using the calibrated transfer function, equation (5) gives:

$$\hat{p}_{+} = \frac{H_{13}\ \exp(i\,k_{-}\ s_1) - H_{11}}{\exp(i\,k_{-}\ s_1) - \exp(-\,i\,k_{+}\ s_1)}$$
$$\hat{p}_{-} = \frac{-\,H_{13}\exp(-\,i\,k_{+}\ s_1) + H_{11}}{\exp(i\,k_{-}\ s_1) - \exp(-\,i\,k_{+}\ s_1)} \qquad (6)$$

At both testes we determine the pressure and volume velocity spectra using two-microphone method, and the unknown two port matrix **T** is determined from the matrix equation

$$\begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix} \qquad (7)$$

where, 1 and 2 refers to the measured data when the signal comes from the upstream and downstream side respectively.
The transfer matrix can be solved for if equation (7) is satisfied, i.e.

$$\det\begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix} \neq 0 \qquad (8)$$

Then the transfer matrix can be calculated from:

$$\begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} = \begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix}^{-1} \qquad (9)$$

If more than two independent acoustic test cases are created the unknown two port matrix **T** is determined from the following matrix equation:

$$\begin{pmatrix} \hat{p}_a^1 & \hat{p}_a^2 & \hat{p}_a^3 & \cdots & \hat{p}_a^N \\ \hat{q}_a^1 & \hat{q}_a^2 & \hat{q}_a^3 & \cdots & \hat{q}_a^N \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} \hat{p}_b^1 & \hat{p}_b^2 & \hat{p}_b^3 & \cdots & \hat{p}_b^N \\ \hat{q}_b^1 & \hat{q}_b^2 & \hat{q}_b^3 & \cdots & \hat{q}_b^N \end{pmatrix} \tag{10}$$

The transfer matrix can be calculated from:

$$\begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} = \begin{pmatrix} \hat{p}_a^1 & \hat{p}_a^2 & \hat{p}_a^3 & \cdots & \hat{p}_a^N \\ \hat{q}_a^1 & \hat{q}_a^2 & \hat{q}_a^3 & \cdots & \hat{q}_a^N \end{pmatrix} \begin{pmatrix} \hat{p}_b^1 & \hat{p}_b^2 & \hat{p}_b^3 & \cdots & \hat{p}_b^N \\ \hat{q}_b^1 & \hat{q}_b^2 & \hat{q}_b^3 & \cdots & \hat{q}_b^N \end{pmatrix}^{-1} \tag{11}$$

3. TEST PROCEDURE

Experiments were carried out at room temperature using the flow acoustic test facility at The Marcus Wallenberg Laboratory (MWL) for Sound and Vibration research at KTH. The test duct used during the experiments consisted of a standard steel-pipe with a wall thickness of 3 mm, duct inner diameter 57 mm and overall length of around 7 meters. The test object was a single diaphragm orifice with concentric holes and a diameter of 30 mm. Four loudspeakers were used as external acoustic sources. The loudspeakers were divided equally between the upstream and downstream side as shown in Figure 2. The distances between the loudspeakers were chosen to avoid any pressure minima at the source position. Six flush mounted condenser microphones (B&K 4938) were used, three upstream and three downstream of the test object giving two microphone separations 33 mm and 283 mm approximately covering frequency ranges 530-4200 Hz and 60-490 Hz.. The cut-on frequency of the first higher order mode in a circular duct is: $f_{cut-on} = 1.84\,c\left(1 - M^2\right)/\pi d$, where d is the duct diameter, or around 3400 Hz in this case. The flow speed was measured upstream of the test section using a small pitot-tube connected to an electronic manometer at a distance of 1000 mm from the upstream loudspeakers section.

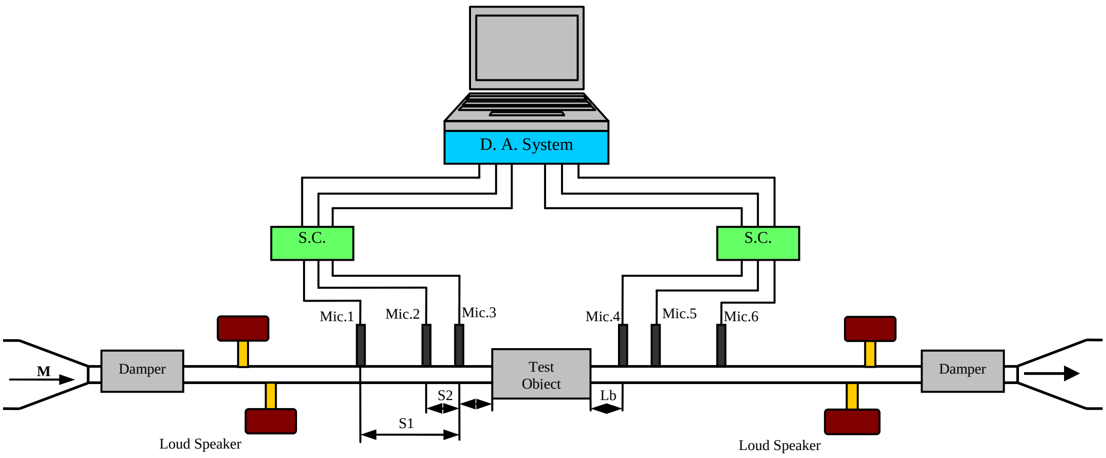

Figure 2. *Measurement configuration for plane wave decomposition at MWL.*

The flow speed was measured in the middle of the duct and before and after each acoustic measurement and the average was used. To generate over-determination by independent acoustic test cases, the two sources were first used one at a time, then both sources were used simultaneously, the phase was then changed 180 degrees on the upstream side and then on the downstream side. This produced five independent test cases. To be able to test the efficiency of the technique by having a reasonably low signal-to-noise-ratio random noise excitation was used and 100 averages were made.

4. RESULTS AND DISCUSSIONS

The acoustic two port, of a single diaphragm orifice, has been determined using over-determination as described in equation (11) with up to five independent acoustic test cases. The signal-to-nose ratio was reasonably low as shown in Figure 3. The computed results have been compared with results from the original two source location method. The new procedure gives a better result compared to the theoretical results as shown in Figure 4. The theoretical result has been calculated using 3D FEM software FEMLAB [10] and the impedance of the orifice has been modelled using the tested Bauer formula [11]. An improvement of the results can be obtained by using the same procedure with higher input source level.

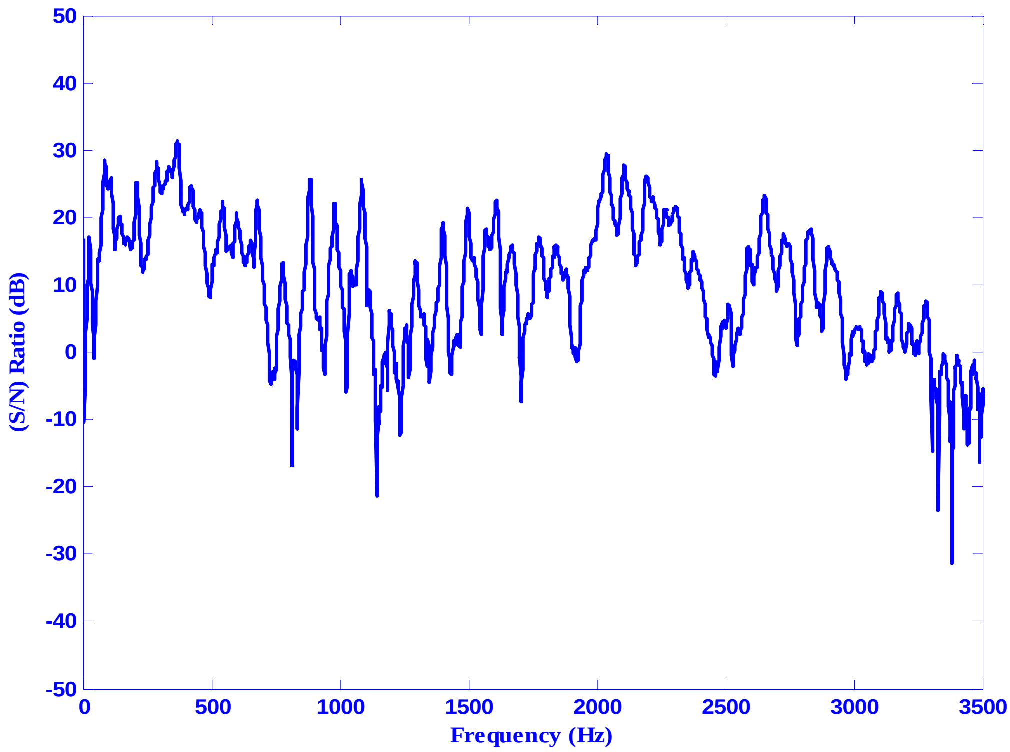

Figure 3. *Signal to noise ratio at reference microphone when the signal comes from upstream side.*

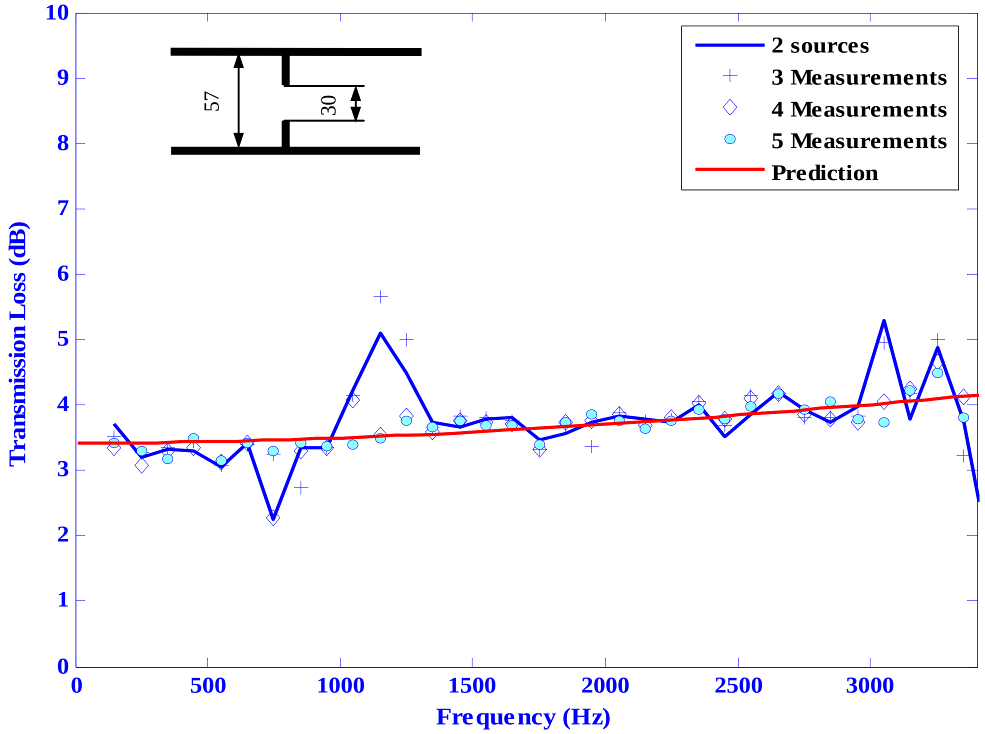

Figure 4. *Effect of number of over-determination using additional acoustic test cases on the measured transmission loss compared to theoretical results for single orifice.*

5. SUMMARY AND CONCLUSIONS

To experimentally determine acoustic two-port matrices for flow duct components the acoustic state variables on the inlet and outlet side must be measured for two independent test cases. In the so-called two-source location technique the independent test cases are created by first using a source on the inlet side and then a source on the outlet side. It was pointed out by Åbom [7] that it is also possible to run both sources simultaneously to create more than two independent test cases. This over-determination could be used to improve the measurement results for instance if the data is contaminated by flow-noise. A technique for creating a number of independent test cases has been suggested in the present paper and tested, on a single orifice test object, with up to 5 different test cases. The results show that significant improvement of the experimental results can be obtained using this technique.

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