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STOCHASTIC PARTIAL UPDATES OF ADAPTIVE FILTERS IN ACTIVE NOISE CONTROL[1]

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Abstract

This paper presents a Filtered-x version of the Stochastic Partial Updates (SPU) Least Mean Square (LMS) algorithm and compares its performance with other algorithms in the context of an Active Noise Control (ANC) system.

The proposed strategy, aiming at the lessening of the computational complexity, is based on partial updates (PU) of the weights of the adaptive filter. The inherent reduction in convergence rate due to the fact that only a fraction of coefficients are updated during each cycle is compensated by increasing the step size. The subset of coefficients updated at each iteration is sampled at random. In so doing, the maximum step size that can be used in the proposed algorithm is N times greater than that of the FxLMS, being N the decimating factor.

The theoretical performance of this algorithm are validated by simulation and by practical results obtained from experiments carried out in a in-vehicle implementation.

INTRODUCTION

The Filtered-x Least Mean Square (FxLMS) algorithm is the most widely used adaptive algorithm in DSP-based implementations of ANC systems [3]. As far as the length of the filter is concerned, the adaptive FIR filter may eventually require a large number of coefficients to accurately model the primary path and inversely model the secondary path. Therefore, the improvement in the performance is achieved at the expense of increasing the computational load of the control strategy. In order to lessen the complexity without shortening the length of the filter one may choose to update only a portion of the weights vector during each sample period [1], [2]. The decimating factor N is defined as the filter length divided by the number of

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coefficients updated per iteration. Nevertheless, PU algorithms suffer from one drawback: their convergence rate is reduced in proportion to *N* [1].

The Filtered-x Sequential Least Mean Square algorithm with step-size Gain (Gμ-FxSeq LMS) was proposed in previous works [4]. The Gμ-FxSeq LMS algorithm is based on Sequential PU of the coefficients as well as on the controlled increase in the step size of the adaptive filter and can be applied in active control of periodic disturbances consisting of several harmonics. The referred step-size gain determines the ratio between the maximum step size valid for the proposed algorithm and the maximum step size valid for the FxLMS. By multiplying the convergence factor by the gain it is possible to compensate the reduction in convergence rate due to PU. The analysis of the Gμ-FxSeq LMS determines that the step-size gain depends on the frequency and prevents from the use of frequencies corresponding to notches which appear in the gain of the adaptive algorithm. To sum up, the Gμ-FxSeq LMS algorithm can achieve the same performance that the FxLMS provides in terms of convergence rate and misadjustment with a minor computational complexity, as long as the undesired disturbance has no components at the frequencies where there are notches in the step-size gain.

The strategy here proposed, Filtered-x Stochastic Partial Updates Least Mean Square algorithm with Gain in step size (Gμ-FxSPU LMS), overcomes the limitations imposed by the notches in the gain of the Gμ-FxSeq LMS algorithm at the cost of a slight increase in the computational load. Not being so computationally efficient as the Sequential version, the Gμ-FxSPU LMS algorithm is, however, operationally less intensive than the conventional FxLMS.

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Gμ-Fx SPU LMS algorithm

Figure 1 shows the block diagram of a Filtered-x ANC system where the secondary path *S(z)* is placed following the digital filter *W(z)* controlled by an adaptive algorithm. Under the assumption of slow convergence, the order of the secondary path and the adaptive filter can be commuted. Besides, if the off-line estimate of the secondary path is accurately obtained, then, the resulting equivalent diagram simplifies and the output of the adaptive filter carries through directly to the error signal, Thus, standard LMS algorithm techniques can be applied to the Filtered-x version of the SPU LMS algorithm in order to determine the convergence of the mean weights and the maximum value of the step size [3].

Both conditions are assumed so as to simplify the analysis by considering the filtered reference as the regressor signal of the adaptive filter. The proposed Gμ-FxSPU LMS algorithm compensates the reduction in convergence rate by means of a gain in step size -Gμ(*N*)- applied to the maximum step size μ valid for the FxLMS. This factor μ is inversely bounded by the largest eigenvalue of the input autocorrelation matrix as follows:

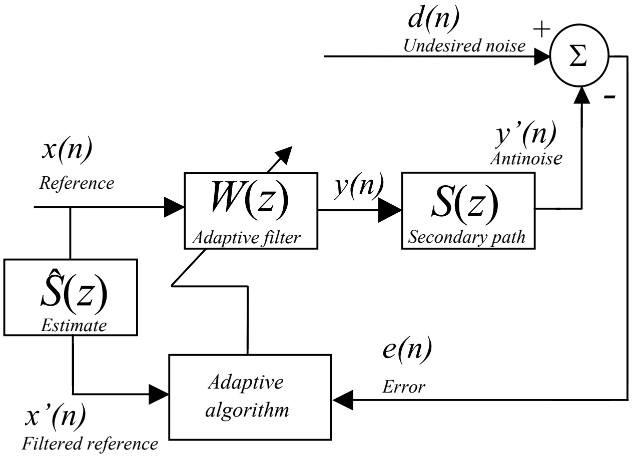

Figure 1 – Block diagram of ANC system controlled by a Filtered-x adaptive algorithm.

$$0 < \mu < \frac{2}{\lambda_{\max}} = \frac{2}{\max\left\{ eig\left(E\left[\underline{x}'(n) \cdot \underline{x}'(n)^T \right] \right) \right\}} \tag{1}$$

The question that should be answered is how larger the step size can be made if only a subset of *Lw/N* coefficients out of the *Lw* length filter are randomly updated per iteration; in other words, what is the step-size gain of the proposed strategy?.

The *l*-th coefficient of the adaptive filter is partially updated according to the SPU LMS equation expressed as follows:

$$w_l\,(n+1) = w_l\,(n) + G_\mu \cdot \mu \cdot b(l,n) \cdot e(n) \cdot x'(n-l+1) \qquad 1 \le l \le Lw \tag{2}$$

where Gμ is the gain in step size, μ is the maximum step size valid for the FxLMS and *b(l,n)* can be either 1 or 0 with the following probabilities:

$$P(b(l,n)=1) = \frac{1}{N} \quad and \quad P(b(l,n)=0) = \frac{N-1}{N} \tag{3}$$

Taking the expected value of both sides of Eq. (2) we have:

$$\begin{aligned} E[w_l(n+1)] &= E[w_l(n) + G_\mu \cdot \mu \cdot b(l,n) \cdot e(n) \cdot x'(n-l+1)] = E[w_l(n)] + G_\mu \cdot \mu \cdot E[b(l,n)] \cdot E[e(n) \cdot x'(n-l+1)] = \\ &= E[w_l(n)] + G_\mu \cdot \mu \cdot \frac{1}{N} \cdot E[e(n) \cdot x'(n-l+1)] \qquad 1 \le l \le Lw \end{aligned} \tag{4}$$

From the simplified analysis of the FxLMS outlined at the beginning of this section, the maximum step size for the Gμ-FxSPU LMS algorithm is derived as follows [3]:

$$0 < \frac{G\mu \cdot \mu}{N} < \frac{2}{\lambda_{\max}} \tag{5}$$

From the comparison of Eq. (1) and Eq. (5) it can be concluded that the increase in step size of the proposed Filtered-x SPU algorithm with respect to the maximum step size of the FxLMS is given by:

$$G_{\mu}(N) = N \tag{6}$$

and such an increase does not depend on the spectral distribution of the undesired disturbance.

Computational complexity

Table 1 shows the computational complexity of the LMS, the Sequential PU LMS and the Stochastic PU LMS algorithms in terms of the average number of operations required per cycle when used in the context of a Filtered-x implementation of a single channel ANC system. The length of the adaptive filter is *Lw*, the length of the off-line estimate of the secondary path is *Ls* and the decimating factor is *N*.

Table 1 – Computational complexity of FxLMS, Gμ-FxSeq LMS and Gμ-FxSPU LMS algorithms in terms of the average number of additions and multiplies per iteration.

	FxLMS	*Gμ-FxSeq LMS*	*Gμ-FxSPU LMS*
# Additions	$2 \cdot Lw + Ls - 1$	$\left(1+\frac{1}{N}\right)\cdot Lw + \left(\frac{Ls-1}{N}\right)$	$\left(1+\frac{1}{N}\right)\cdot Lw + Ls - 1$
# Multiplications	$2 \cdot Lw + 1 + Ls$	$\left(1+\frac{1}{N}\right)\cdot Lw + 1 + \left(\frac{Ls}{N}\right)$	$\left(1+\frac{1}{N}\right)\cdot Lw + 1 + Ls$

The Stochastic PU algorithm is slightly less efficient than the Sequential PU algorithm due to the inherent fact that is impossible to determine a priori the subset of coefficients that will be updated at the current iteration. Therefore, as there is not any clue about which samples of the regressor signal will be used during the updating process it is necessary to obtain a new sample of the filtered reference at every cycle and a *Lw*-length buffer filled with the *Lw* most recent samples of the filtered reference must be managed by the control system. Nevertheless, as it will be proved in the section devoted to the experimental results, the number of coefficients of the estimate of the secondary path can be reduced to the extreme of taking into account just the two more significant coefficients of the off-line estimate. If that is the case -*Ls*=2-, the computational complexity of the Gμ-FxSPU LMS and the Gμ-FxSeq LMS algorithms differ in a negligible amount of operations in comparison with the term depending on the length of the adaptive filter.

SIMULATION

This section describes the results achieved by the Gμ-FxSPU LMS algorithm by means of a computer model developed in MATLAB on the theoretical basis of the previous section.

Power spectral density of the undesired disturbance which has to be cancelled in this simulated example is depicted in Figure 2.a. The reference and the undesired signal were previously recorded by a two channel signal acquisition system. The secondary path is modeled -by a 4 order elliptic IIR filter- as a high pass filter whose cut-off frequency is imposed by the poor response of the loudspeakers at low frequencies. Transfer function of the secondary path *S(z)* is shown in Figure 2.b. The off-line estimate of the secondary path was accurately carried out by an adaptive FIR filter of 200 coefficients updated by the LMS algorithm solving a classical problem of system identification. The adaptive filter in all cases has 256 coefficients.

Benefits of the Gμ-FxSPU LMS algorithm over the previously outlined Gμ-FxSeq LMS are illustrated in this example my means of a simulation that compares the learning curves. The Sequential algorithm is run with two different decimating factors -N=4 and N=5-. Gain in step size in both cases are shown in Figures 2.c and 2.d, respectively [4]. According to the figures, the third harmonic (200 Hz) of the acoustic disturbance is located at the first notch of the gain in step size if N=4. As a result of that, the full strength gain Gμ=N=4 can not be applied in this case. Figure 3 compares the ensemble averages of 100 learning curves of the attenuation of the six harmonic signal by different strategies. As expected, the curves for FxLMS, Gμ-FxSeq LMS with N=Gμ=5 and Gμ-FxSPU LMS with N=Gμ=4 are almost identical. Also predicted was the instability of the Gμ-FxSeq LMS algorithm with N=Gμ=4 due to the notch at 200 Hz.

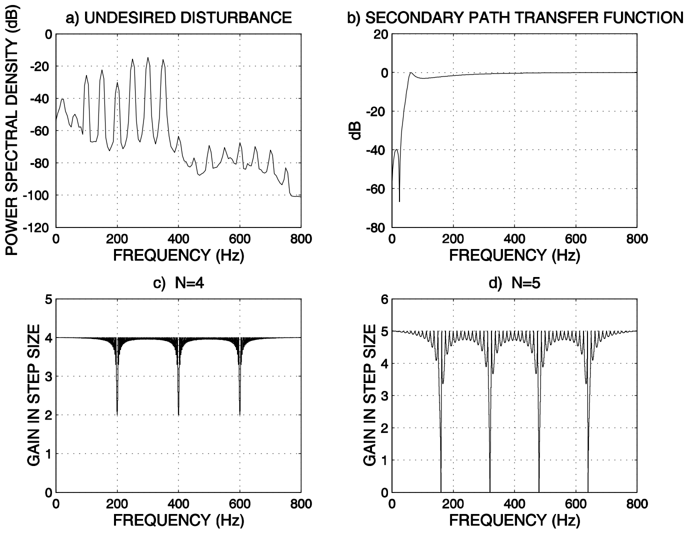

Figure 2 – a) Power Spectral Density of the undesired disturbance consisting of harmonics at 100, 150, 200, 250, 300 and 350 Hz. b) Magnitude of the secondary path. c) Gμ of the Gμ-FxSeq LMS; Lw=256 and N=4. d) Gμ of the Gμ-FxSeq LMS; Lw=256 and N=5.

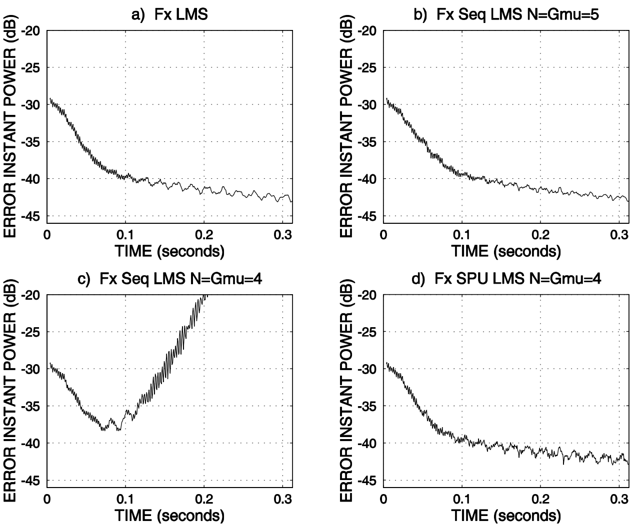

Figure 3 – Ensemble averages of 100 simulated learning curves: a) Fx LMS. b) Gμ-FxSeq LMS; N=Gμ=5. c) Gμ-FxSeq LMS; N=Gμ=4. d) Gμ-FxSPU LMS; N=Gμ =4.

EXPERIMENTAL RESULTS

The Gμ-FxSPU LMS algorithm has been put into practice in a two independent channel implementation of an ANC system placed at the front seats of a Nissan Vanette. Two error microphones are located near the head of the driver and the passenger. Tests made in the laboratory showed that cross terms can be omitted without degrading the performance of the system. Low cost microphones and loudspeakers with poor response at low frequencies were used. The distance between a microphone and its respective secondary source in this experiment of local minimization of noise is 6 cm. The main Digital Signal Processor board employed to develop the strategy was the PCI/C6600, based on the DSP TMS320C6701. The selected Input/Output board was the PMCQ20DS. This board disposes of 4 A/D and 4 D/A converters. The number of weights of the FIR off-line estimate of the secondary path was reduced to the extent of taking Ls=2 in order to minimize the term of the computational cost that does not inversely depend on the decimating factor N (see Table 1). In previous works it has been proved that an ANC system finds more difficulties in the attenuation of harmonics very close in the frequency domain [4]. This problem can be avoided by increasing the number of coefficients of the adaptive filter. So as to deal with acoustic disturbances consisting of very close harmonics the length of the adaptive filter was set to Lw=1008 coefficients. Very large decimating factors -with the subsequent gain in step size- were used to lessen the computational complexity without slowing down the convergence rate. In order to carry out a performance comparison of different control strategies it is essential to repeat the experiment in the same scenario. Due to the independence of both channels, different control algorithms can be tested simultaneously. In order to avoid fluctuations in level and frequency of the undesired disturbance, instead of starting the engine, we previously recorded a signal consisting of very close harmonics (200, 210, 220 and 230 Hz). The omnidirectional source Brüel & Kjaer Omnipower 4296 placed inside the van was fed with this signals and acted as the source of the primary noise. The attenuation of the noise was carried out by the FxLMS and the Gμ-FxSPU LMS algorithms in the same conditions and convergence rates and residual error were properly compared. Although reductions in the number of mathematical operations are an indication of the computational efficiency of an algorithm, such reductions may not directly translate to a more efficient real-time DSP-based implementation on a hardware platform. To accurately gauge such issues one must consider the freedoms and constraints that a platform imposes in the real implementation, such as parallel operations, complex addressing modes, registers available or number of arithmetic units. In our case, the control strategy and the assembler code was developed trying to take full advantage of these issues. As far as the generation of the random indexes is concerned, a table with random numbers is managed by the PC -not by the DSP- in order to provide the indexes of the coefficients to be updated at the current iteration with very little additional computational cost.

Experimental results achieved in the attenuation of the previously mentioned multi-tone acoustic disturbance are shown in Figure 4. An appropriate choice of parameters (see Figure 4.c where Ls=2 and N=Gμ=504) results in a extremely low complex ANC strategy that provides performance comparable to that of the FxLMS. It has been experimentally checked that in order to effectively attenuate harmonics as close as the components of the signal of the example (Figure 4) by means of a simple FxLMS algorithm it is necessary an adaptive filter with more that 500 coefficients. In the implementation a 1008-weight filter has been used. Our interest in the reduction in the required instructions per cycle is justified by the necessity of attenuating acoustic disturbances with spectral distribution consisting of close harmonics -the noise produced by an engine, for instance- without consuming all the resources of the DSP and enabling efficient ANC systems to be implemented on a simple processor. Test of the ANC system based on the Gμ-FxSPU LMS algorithm carried out with many other acoustic disturbances with different spectral distributions also provided satisfactory performance. Experiments have also shown that an increase in the number of coefficients of the off-line estimate of the secondary path results in a slightly faster convergence of the error. Considering the number of multiplications as a valid indicator of the overall complexity, the computational savings of the Gμ-FxSPU LMS algorithm with regard to the number of instructions required by a conventional FxLMS strategy are given by Table 2 for different values of the length of the secondary path (*Ls)* and the decimating factor (*N)*:

Table 2 – Savings in average number of multiplications per cycle. The length of the adaptive filter is Lw=1008 coefficients.

Saved Multiplications (%)	*N=42*	*N=504*
Ls=2	48.74 %	49.83 %
Ls=200	44.38 %	45.48 %

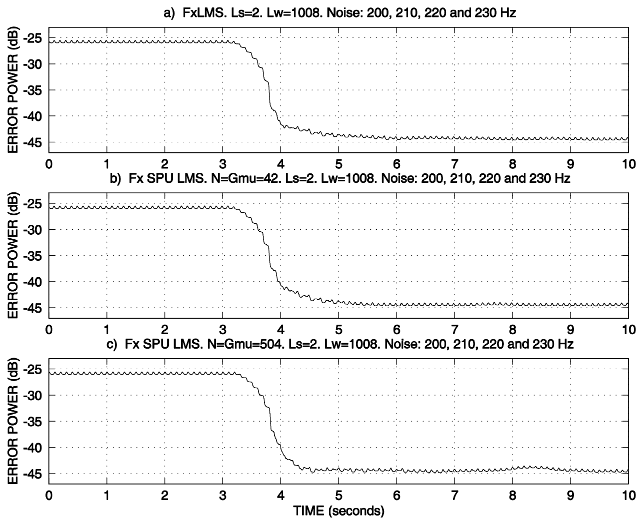

Figure 4 – Smoothed learning curves measured at the driver position. ANC system attenuating an acoustic disturbance consisting of harmonics at 200, 210, 220 and 230 Hz. a) FxLMS, b) Gμ-FxSPU LMS N=Gμ=42, c) Gμ-FxSPU LMS N=Gμ=504.

CONCLUSIONS

So as to assess the effectiveness of the Filtered-x Stochastic Partial Updates LMS algorithm with step-size Gain the proposed strategy was not only tested by simulation but was also evaluated and compared in a practical DSP-based implementation. In both cases the results confirmed the expected behavior: the bound on step size for convergence of the weight vector mean to its optimum value is N times larger for the Gμ-FxSPU LMS algorithm than for the FxLMS. Even when the number of operations per iteration is significantly reduced due to PU, the affordable increase in step size compensates the lack of adaptation of most of the coefficients. To sum up, this strategy results in an algorithm with lower computational cost and a performance very close to the conventional FxLMS. The proposed SPU algorithm overcomes the limitations that the Gμ-FxSeq LMS algorithm shows at certain frequencies due to the notches that impair the use of the full strength gain.

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