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PIEZOELECTRICAL INDUCED VIBRATIONS OF SANDWICH PANELS WITH INTERLAYER SLIP

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Abstract

Piezoelectric effects inducing flexural vibrations in viscoelastic sandwich panels are studied. Geometrically linearized conditions are considered, and the Bernoulli-Euler hypothesis is applied separately to each of the three layers. At the interfaces a linear viscoelastic slip law is assigned and a sixth-order initial-boundary value problem of the flexural vibrations is derived. In a numerical study, the frequency response function for the deflection of sandwich panels due to imposed time-harmonic piezoelectric eigenstrains is determined, which represents the important input function for computational methods in the frequency domain.

INTRODUCTION

Considering vibration control of flexible structures, the corresponding distributed actuators are frequently realized by embedding layers of piezoelectric materials, see, e.g., [2] and [5]. Such devices are often constructed in form of thin panels, plates or shells, [4]. Activation renders piezoelectrically induced strains that are imposed to generate the distributed input of the control system.

The present paper is concerned in detail with actuating piezoelectric effects in sandwich panels, where special emphasis is given to the identification of the piezoelectric actuation as a source of selfstress. It is demonstrated that piezoelectrically induced strains conveniently, within a multiple field approach, can be interpreted as eigenstrains acting in a background panel without piezoelectric actuators. The identification of piezoelectrically induced strains as eigenstrains, likewise to thermal strains, indeed may be viewed as the key for the understanding of piezoelectric actuation. It has been stated already in the literature that the analogy between the piezoelectric effect and the thermal and hygrothermal effect is important because it enables the analyst and designer to transfer all available thermoelastic and

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hygrothermal solutions to solve problems involving piezoelectric materials. This analogy is utilized in the present paper, where the layer-wise theory is applied to flexural vibrations of panels composed of three symmetrically arranged layers made of dissimilar isotropic materials. Contrary to the classical theory of perfectly bonded laminates, the following study concentrates on panels where the inter-laminar connections exhibit a viscoelastic interlayer slip between the outer faces and core. The influence of transient piezoelectric actuation is characterized by effective terms of the imposed mean strain and curvature. The corresponding sixth-order initial-boundary value problem of the flexural vibrations is formulated. A numerical example of frequency response function for the panel deflection effected by imposed time-harmonic thermo-piezoelectric sources illustrates the influence of the inter-laminar connection on the dynamic structural behavior. For an extension to thermo-piezoelectrical flexural vibrations see [3].

CONSTITUTIVE MODELING OF A UNIAXIAL PIEZOELECTRIC LAYER

In the following, a single piezoelectric beam-type layer with rectangular cross-section is studied, where the axial coordinate is denoted by x. The layer's deformation is assumed to take place in the (x,z)-plane, z denoting the transverse coordinate of the laminate. The piezoelectric layer acts as a capacitor, where the electric voltage V is constant along the metallic electrodes of the capacitor and is connected to the non-vanishing component of the quasi-static electric field density by

$$E_z = \frac{\partial V}{\partial z} \simeq \frac{V}{\bar{h}} \ , \tag{1}$$

where $\bar{h}$ denotes the (small) thickness of the piezoelectric layer. The components of the field density tangential to the layer vanish, $E_x = E_y = 0$. An isothermal, uniaxial linear constitutive equation is set up between E_z, the respective component D_z of the electric flux vector, and the axial strain ε_{xx} in the layer:

$$E_z = -a^* \varepsilon_{xx} + d^* D_z \ , \tag{2}$$

where a^* and d^* represent effective constitutive parameters. These parameters are obtained from the full set of three-dimensional constitutive equations by setting $E_x = E_y = 0$, together with $D_x = D_y = 0$, and inserting the result in order to replace the effect of the non-axial strains in Eq. (2).

Within an isothermal approximation, the mechanical constitutive equations are analogously written in the uniaxial form,

$$\sigma_{xx} = c^* \varepsilon_{xx} - a^* D_z \ , \tag{3}$$

c^* again denoting an effective elastic modulus. Inserting Eqs.(1) and (2) into Eq.(3) yields:

$$\sigma_{xx} = Y^*\left(\varepsilon_{xx} - \bar{\varepsilon}_{xx}\right) \ , \tag{4}$$

where the piezoelectric eigenstrain is given by

$$\bar{\varepsilon}_{xx} = \frac{a^*}{d^* Y^*}\frac{V}{\bar{h}} \ , \tag{5}$$

with effective modulus of elasticity,

$$Y^* = c^* - \frac{a^{*2}}{d^*} \ . \tag{6}$$

VISCOELASTIC SYMMETRIC SANDWICH PANELS

The dynamic response of slender panels composed of three symmetrically arranged piezo- viscoelastic thin layers is studied. Utilizing a layer-wise theory and taking into account also a viscoelastic interlayer slip between core and faces derives the corresponding initial-boundary value problem.

Assuming three viscoelastic layers $(i = 1,\ 2,\ 3)$ with a common retardation time ϑ, the uniaxial stress-strain relation reads

$$\sigma_{xx}(x, z_i; t) = Y^*(z_i)\left[\varepsilon_{xx}(x, z_i; t) + \vartheta\,\dot{\varepsilon}_{xx}(x, z_i; t) - \bar{\varepsilon}_{xx}(x, z_i; t)\right]. \tag{7}$$

If the usual assumptions of thin laminates are employed, both in-plane and rotatory inertia can be neglected. Considering the free-body diagram of a three-layer panel with imposed piezoelectric strains, see Fig. 1, and applying the conservation of angular momentum to all three layers gives the relations

$$M_{1,x} - Q_1 - T_1\frac{h_2}{2} - N_{1,x}\,d = 0,\ \ d = \frac{h_1 + h_2}{2} \ , \tag{8}$$

$$M_{2,x} - Q_2 + \left(T_1 + T_2\right)\frac{h_2}{2} = 0 \ , \tag{9}$$

$$M_{3,x} - Q_3 - T_2\frac{h_2}{2} + N_{3,x}\,d = 0 \ , \tag{10}$$

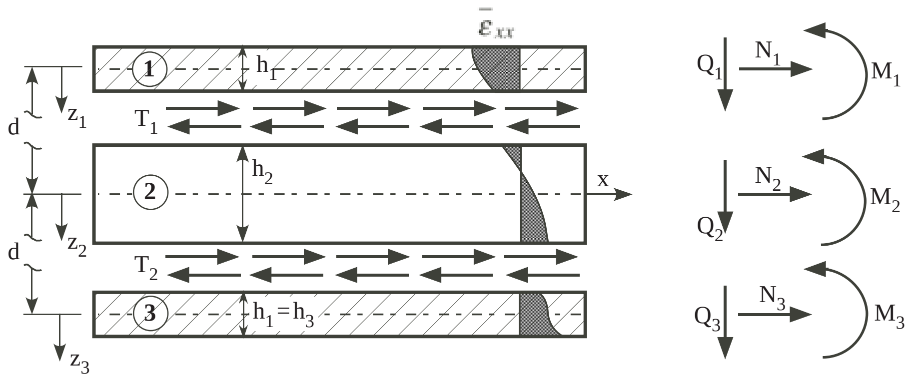

Figure 1 – Geometry and stress resultants of a symmetric sandwich panel

where T_1, T_2 denote the interlaminar shear forces per unit of length, and Q_i is the transverse shear force in the *i*-th layer. The bending moments M_i are referred to the individual layer axes. Conservation of momentum in axial and transverse directions renders

$$N_1,_x + T_1 = 0, \;\; N_2,_x - T_1 + T_2 = 0, \;\; N_3,_x - T_2 = 0 \;, \tag{11.1-3}$$

$$\sum_{i=1}^{3} Q_i,_x = \mu \ddot{w} \;. \tag{12}$$

The mass per unit panel area is denoted by μ, the abbreviation $(\,)_{,x}$ defines the spatial derivative, and $\ddot{w}(x)$ stands for the transverse panel acceleration.

Summation of Eqs. (8)-(10) yield the global conservation of momentum

$$M,_x - Q = 0 \;, \tag{13}$$

where

$$M = \sum_{i=1}^{3} M_i - (N_1 - N_3)d, \quad Q = \sum_{i=1}^{3} Q_i \;. \tag{14.1, 2}$$

In this mechanical model, all three layers are assumed to be rigid in shear. However, contrary to perfectly bonded laminates, viscoelastic interlayer slips, Δu_1 between upper face and core, and Δu_2 between lower face and core, are considered. The displacement field in the *i*-th layer is modeled according to a layer-wise Bernoulli-Euler approximation, thus

$$\begin{bmatrix} u_i \\ w_i \end{bmatrix} = \begin{bmatrix} u_i^{(0)} - z_i w,_x \\ w \end{bmatrix}, \; i = 1,2,3 \;\;, \tag{15}$$

where w represents the transverse deflection, and the axial deformations $u_i^{(0)}$ can be written as

$$u_1^{(0)} = u_2^{(0)} + \left(d\, w,_x - \Delta u_1\right), \;\; u_3^{(0)} = u_2^{(0)} - \left(d\, w,_x - \Delta u_2\right) \;. \tag{16.1, 2}$$

The constitutive relations, when expressed by means of cross-sectional resultants, become

$$\begin{bmatrix} N_i \\ M_i \end{bmatrix} = \begin{bmatrix} D_i & 0 \\ 0 & B_i \end{bmatrix} \begin{bmatrix} u_{i,x}^{(0)} + \vartheta \dot{u}_{i,x}^{(0)} - e_i \\ -\left(w,_{xx} + \vartheta \dot{w},_{xx} + \kappa_i\right) \end{bmatrix}, \; i = 1,2,3 \;\;, \tag{17}$$

where D_i and B_i are the standard extensional and bending stiffness, $D_i = Y_i^* A_i$, and $B_i = Y_i^* J_i$; the abbreviations A_i and J_i denote the layer's partial cross-sectional areas and moments of inertia, respectively. The cross-sectional means of the eigenstrains and imposed curvatures due to piezoelectrical action are defined as

$$e_i = \frac{1}{A_i} \int_{A_i} \bar{\varepsilon}_{xx}\, dA, \;\; \kappa_i = \frac{1}{J_i} \int_{A_i} \bar{\varepsilon}_{xx}\, z_i\, dA \,. \tag{18.1, 2}$$

The relative horizontal displacement between two layers causes shear tractions at the interfaces, where a linear vicoelastic relation is assigned,

$$T_1 = k\left(\Delta u_1 + \vartheta \Delta \dot{u}_1\right) = k\left[\left(d\, w,_x + u_2^{(0)} - u_1^{(0)}\right) + \vartheta\left(d\, \dot{w},_x + \dot{u}_2^{(0)} - \dot{u}_1^{(0)}\right)\right],$$

$$T_2 = k\left(\Delta u_2 + \vartheta \Delta \dot{u}_2\right) = k\left[\left(d\, w,_x - u_2^{(0)} + u_3^{(0)}\right) + \vartheta\left(d\, \dot{w},_x - \dot{u}_2^{(0)} + \dot{u}_3^{(0)}\right)\right], \tag{19.1, 2}$$

where the parameter k represents a constant slip modulus. For the sake of simplicity of the final initial-boundary value problem, the value of the retardation time, ϑ, is chosen to be common to that of the panel layers. Summation and differentiation of Eqs. (11.1) and (11.2) with respect to the axial coordinate x, and substituting Eqs. (18.1) and (18.2) together with the constitutive relations, Eq. (17), leads to

$$N_1,_{xx} - N_3,_{xx} - \frac{k}{D_1}\left(N_1 - N_3\right) + 2\, k\, d\left(w,_{xx} + \vartheta \dot{w},_{xx}\right) - k\left(e_1 - e_3\right) = 0 \,, \tag{20}$$

where, due to the symmetric layer arrangement, $D_1 = D_3$ has been considered.

Next, by means of Eq. (14.1) and Eq. (17), the force difference $\left(N_1 - N_3\right)$ can be expressed as

$$N_1 - N_3 = -\frac{1}{d}\left[B_0 \left(w,_{xx} + \vartheta\, \dot{w},_{xx} + \kappa^{(0)} \right) + M \right], \qquad (21)$$

where B_0 and $\kappa^{(0)}$ define bending stiffness and curvatures, respectively, of the non-composite actions

$$B_0 = \sum_{i=1}^{3} B_i\,, \quad \kappa^{(0)} = \frac{1}{B_0} \sum_{i=1}^{3} B_i\, \kappa_i\,. \qquad (22.1, 2)$$

Combining Eqs. (20) and (21) and considering Eqs. (13) and (14), the equation of motion, Eq. (12), can be expressed as

$$\left(w + \vartheta\, \dot{w}\right),_{xxxxxx} - \lambda^2 \left(w + \vartheta\, \dot{w}\right),_{xxxx} + \frac{\mu}{B_0} \ddot{w},_{xx} - \lambda^2 \frac{\mu}{B_\infty} \ddot{w} = \lambda^2 \kappa,_{xx} - \kappa^{(0)}_{,xxxx}\,. \qquad (23)$$

In this formulation, both bending stiffness

$$B_\infty = B_0 + 2 d^2 D_1\,, \qquad (24)$$

and imposed curvature

$$\kappa = \frac{1}{B_\infty} \int_A Y^* \varepsilon^*_{xx}\, z_2\, dA = \frac{1}{B_\infty} \left[2 D_1\, d \left(e_3 - e_1\right) + B_0\, \kappa^{(0)} \right], \qquad (25)$$

are referred to the full composite cross-section. The parameter

$$\lambda^2 = k\, B_\infty / (D_1\, B_0) \qquad (26)$$

defines an effective inter-laminar shear coefficient that allows to study also both limits $\lambda^2 \to \infty$ (perfect bond) and $\lambda^2 \to 0$ (no bond), of the panel without numerical difficulties.

The solution of Eq. (23) depends on the initial conditions, the state at time instant $t = 0$, and on the actual boundary conditions; see [3] for a compilation of classical boundary conditions.

ILLUSTRATIVE EXAMPLE

In the following example, a simply supported panel with time harmonic imposed piezoelectric eigenstains is considered, assuming that the axial distribution is given by a Fourier sine-series expansion,

$$\bar{\varepsilon}_{xx}(x,z_i;t)=\tilde{\varepsilon}^*_{xx}(x,z_i)e^{i\nu t}=\sum_{n=1}^{\infty}\varepsilon^*_n(z_i)\sin\left(\alpha_n x\right)e^{i\nu t},\ \ \alpha_n=\left(\frac{n\pi}{l}\right),\ i=\sqrt{-1}\,. \quad (27)$$

The circular forcing frequency is denoted by ν, and the coefficients of the axial eigenstrain distribution, $\varepsilon^*_n(z_i)$, are determined by the integral

$$\varepsilon^*_n(z_i)=\frac{2}{l}\int_0^l\tilde{\varepsilon}^*_{xx}(x,z_i)\sin\left(\alpha_n x\right)dx\,. \quad (28)$$

The complex frequency response function (FRF) is computed for the following piezoelectric eigenstrain distribution:

$$\tilde{\varepsilon}^*_{xx}(x,z_3)=-\tilde{\varepsilon}^*_{xx}(x,z_1)=C^{(\varepsilon)}=const.\,,\ \ \tilde{\varepsilon}^*_{xx}(x,z_2)=0\,.$$

In that specific case, the curvature terms of the individual layers vanish, compare Eq. (18.2),

$$\kappa_i=0\Rightarrow\kappa^{(0)}=0\,, \quad (29)$$

and the intensity coefficients of the eigenstain distribution according to the integral Eq. (28) result in

$$\varepsilon^*_n(z_3)=-\varepsilon^*_n(z_1)=\frac{4C^{(\varepsilon)}}{n\pi},\ \ n=1,3,5,... \quad (30)$$

The remaining driving term is of the form

$$\kappa(x,t)=\sum_{n=1}^{\infty}K_n\sin\left(\alpha_n x\right)e^{i\nu t}\,, \quad (31)$$

where the corresponding modal participation factors are given as

$$K_n=\frac{1}{B_\infty}\int_A Y^*\varepsilon^*_n z_2\,dA=2\frac{D_1 d}{B_\infty}\left[\varepsilon^*_n(z_3)-\varepsilon^*_n(z_1)\right]=\frac{16}{n\pi}\frac{D_1 d}{B_\infty}C^{(\varepsilon)}. \quad (32)$$

The FRF is derived by means of the eigenfunction expansion for the deflection of the simply supported panel, compare [1],

$$w(x,t) = \sum_{n=1}^{\infty} W_n \sin\left(\alpha_n x\right) e^{i\nu t} \,. \tag{33}$$

Inserting Eqs. (31) and (33) into Eq.(23), and comparing the coefficients of the series, finally leads to the non-dimensional complex FRF

$$\chi_w(i\nu) = \sum_{n=1,3,5,.}^{\infty} \frac{W_n}{C^{(\varepsilon)} l} = \sum_{n=1,3,5,.}^{\infty} \left(\frac{16 D_1 d}{n\pi l B_\infty \mu}\right) \frac{\lambda^2 \alpha_n^2}{\left(\frac{\alpha_n^2}{B_0} + \frac{\lambda^2}{B_\infty}\right)\left[\omega_n^2 \left(1 + i\vartheta\nu\right) - \nu^2\right]} \,, \tag{34}$$

where the natural frequencies of the elastic structure have been introduced,

$$\omega_n^2 = \alpha_n^4 \left[\alpha_n^2 + \lambda^2\right]\left[\mu\left(\frac{\alpha_n^2}{B_0} + \frac{\lambda^2}{B_\infty}\right)\right]^{-1} . \tag{35}$$

CONCLUSIONS

The sixth-order initial-boundary value problem of flexural vibrations of sandwich panels composed either completely or in part of three piezoelectric viscoelastic layers with interlayer slip is derived. Piezoelectrically induced strains are interpreted as eigenstrains acting in a background panel without actuators. Within a numerical study, a specific load case of time-harmonically imposed piezoelectric sources, applied to the outer layers of a single-span three-layer panel, is considered. The corresponding complex frequency response function for the panel deflection is determined in series representation.

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