



DETERMINING ACOUSTIC PROPERTIES OF OPEN-CELL METAL FOAMS USING THE MULTI-MICROPHONE TRANSFER MATRIX METHOD

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Abstract

Due to their special combination of material and structure, novel porous materials like open-cell metal foams are increasingly often used in technical applications. In many cases, they are inserted in systems with mean air flow. For this reason, the influence of the flow on the damping characteristics of the metal foam is of particular interest. One method to describe the acoustic properties of an acoustic system is the network model, which combines the transfer matrices of acoustical elements. A transfer matrix relates the sound pressure and acoustic velocity on each side of each element. The transfer matrix of sound-absorbing materials is usually expressed as a function of its bulk properties (complex characteristic impedance and complex wave number). In the literature, several analytical models exist to predict the bulk properties of common porous media, which mainly differ in the values of their model parameters. In this work the multi-microphone transfer matrix method is used to measure the bulk properties and the flow resistance of open-cell metal foams with varying pore structure and pore volume density for different element lengths. The experiments are carried out over a wide frequency range up to 1000 Hz at different mean flows with Mach numbers below 0.05. The effects of flow velocity on the damping characteristics and on the flow resistance are investigated. The frequency dependent complex characteristic impedances and complex wave numbers for the measured metal foams are compared to those calculated by analytical models of related porous materials. Specific model parameters are determined by using a regression analysis based on their measured bulk properties. A relation for the influence of the mean flow on the model parameters is provided. The results indicate that the multi-microphone transfer matrix method combined with a regression analysis is a simple and reliable way to describe the acoustic properties of open-cell metal foams.

INTRODUCTION

A 1D-acoustical system can be represented as a network of multi-ports, which connect the related components of the system to each other. Every component is considered as a “black box” and their acoustical properties are fully expressed by its transfer matrices. A transfer matrix describes the transformation of the acoustic field variables - i.e. fluctuations of velocity u' and pressure p' - by an acoustic element as a function of the angular frequency ω .

$$\begin{pmatrix} \frac{p'}{\bar{\rho} \cdot \bar{c}} \\ u' \end{pmatrix}_u = T(\omega) \cdot \begin{pmatrix} \frac{p'}{\bar{\rho} \cdot \bar{c}} \\ u' \end{pmatrix}_d \quad (1)$$

Herein u and d denote the up- and downstream locations of the acoustical element.

Compact porous sound-absorbing materials are often applied in acoustical systems with mean air flow mainly for affecting their sound absorbing properties. For the computation of an acoustical network model of the whole system it is necessary to determine the transfer matrices of each element. A porous sound-absorbing component can be treated as a lossy homogeneous medium [1]. Its acoustic transfer matrix is mainly a function of its bulk properties (complex characteristic impedance and complex wave number) [2]. The goal to characterize the acoustical behavior of porous media is achieved by predicting its bulk properties.

In the literature, several discussions about the prediction of the bulk properties of acoustic absorbers exist. Delany and Bazley measured the values of the characteristic impedance and propagation constant for a range of fibrous absorbent materials [3]. Mechel and Ver made an empirical prediction on the bases of regression analyses of measured data [1]. They showed that the bulk properties of fibrous materials can be determined as a function of frequency, flow resistivity and their specific regression coefficients. They found that all measured fibrous materials can be classified into two categories - namely mineral/basalt wool and glass fibers – and presented their regression parameters. Braccesi and Bracciali provided a least squares estimation of the bulk properties of fibrous materials by an iterative procedure on the basis of a simple mathematical model [4]. Recently, Chen et al. determined the regression parameters of black open-cell plastic foam by applying the two-microphone transfer function method [5]. Tao et al. measured the bulk properties of polyester materials by using the two source transfer matrix method [2]. They also applied the transfer matrix method for layered materials and combined the transfer matrices of the individual layers in a network model.

In this work the bulk properties, flow resistivity and regression parameters for open-cell metal foams with different pore size and length are investigated. The experiments are carried out over a range of mean air flow for Mach numbers below 0.05. The influence of the mean air flow on the flow resistivity and on the regression parameters is shown. Therefore, the multi-microphone transfer matrix method is applied. The

bulk properties are determined by a regression analysis of the measured data and the model of Delany and Bazley [3].

THEORY

The four-pole transfer matrix of a given compact porous element with length l is fully described by its complex wavenumber k' and its complex characteristic impedance Z_c .

$$\begin{pmatrix} \frac{p'}{\bar{\rho} \cdot \bar{c}} \\ u' \end{pmatrix}_u = \begin{pmatrix} \cos(k'l) & j \cdot Z_c \cdot \sin(k'l) \\ j \cdot \frac{1}{Z_c} \cdot \sin(k'l) & \cos(k'l) \end{pmatrix} \cdot \begin{pmatrix} \frac{p'}{\bar{\rho} \cdot \bar{c}} \\ u' \end{pmatrix}_d \quad (2)$$

Since the experimental transfer matrix consist of the four complex elements T_{11} , T_{12} , T_{21} and T_{22} the frequency dependent experimental bulk properties can be calculated out of the measured data by

$$k' = \frac{1}{l} \cdot \cos^{-1} T_{11}, \quad (3)$$

and

$$Z_c = \sqrt{\frac{T_{12}}{T_{21}}}, \quad (4)$$

respectively. Assuming that open-cell metal foams are homogeneous, isotropic and porous, the empirical relationship for the bulk properties given by Delany and Bazley can be applied [3]. The relationships are

$$Z_c = 1 + c_1 \cdot \left(\frac{f \cdot \rho_a}{\sigma} \right)^{-c_2} - j \cdot \left(c_3 \cdot \left(\frac{f \cdot \rho_a}{\sigma} \right)^{-c_4} \right) \quad (5)$$

and

$$k' = \frac{\omega}{\bar{c}} \cdot \left(1 + c_5 \cdot \left(\frac{f \cdot \rho_a}{\sigma} \right)^{-c_6} - j \cdot \left(c_7 \cdot \left(\frac{f \cdot \rho_a}{\sigma} \right)^{-c_8} \right) \right) \quad (6)$$

where f is the frequency, ρ_a the air density, c_1, c_2, \dots, c_8 the material specific regression parameters and \bar{c} the velocity of sound. The flow resistivity σ is a function of the mean air velocity u_m on the element face and the pressure loss Δp of the porous component [6].

$$\sigma = \frac{\Delta p}{l \cdot u_m} \quad (7)$$

MATERIALS AND METHODS

Various samples of open-cell metal foams with different pore size and thickness were investigated. The available materials all have a porosity of 0.91. The experimental set-up for the multi-microphone method is shown in Figure 1.

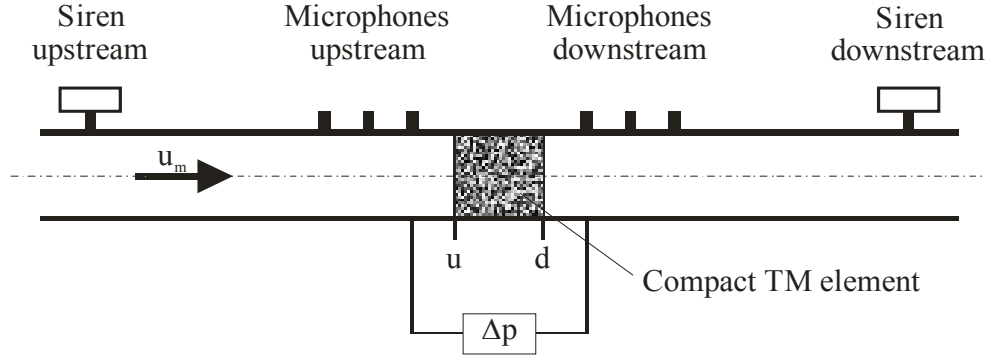


Figure 1: Experimental setup for the multi-microphone transfer matrix measurements

The test element is inserted in a tube and charged with mean air flow over a range up to a Mach number of 0.05. A siren is used for the excitation in a frequency range from 20 to 950 Hz. The dynamic pressure data is measured at three locations each up- and downstream of the test element using microphones. To obtain all four transfer matrix elements it is necessary to carry out the measurements for two different excitation cases – i.e. the siren can be employed at the up- and downstream end of the test-rig, to provide excitation with or versus the mean air flow direction. For the determination of the pressure loss Δp a simple differential pressure transducer is used.

Once the experimental pressure drop data is available for a given material and mean air flow, the flow resistivity σ can be determined with (7). The complex characteristic impedance Z_c and complex wavenumber k' are calculated via (5) and (6) over the entire frequency range. For the specific regression parameters the recommended values of Mechel and Ver [1] or Chen et al. [5] are applied. Furthermore the eight regression parameters of relationship (5) and (6) are optimized to fit the measured bulk properties calculated via (3) and (4). A relation of the flow resistivity σ and the regression parameters to the mean air flow is given.

RESULTS AND DISCUSSION

Figure 2 shows the experimental values of the flow resistivities σ for a 20 ppi (pores per inch) and a 30 ppi open-cell aluminium foam with different lengths. The flow resistivity of the investigated samples all show a linear increase over the mean velocity u_m . Due to the fact, that σ is defined as the steady pressure difference normalized over the sample length [6], its values for the thinner samples (30mm) are

larger compared to those with higher thickness (60mm). For comparison the flow resistivity of black foam, which is composed of plastic with porous and open-cell structure, is about 10000 N s/m^4 , according to Chen et al. [5]. Due to the fact, that the pressure drop of porous materials follows Darcy's Law, the flow resistivities of typical porous materials are independent for different lengths and different mean air flows for low Mach numbers. According to Figure 2, this assumption is not valid for the investigated open-cell metal foams.

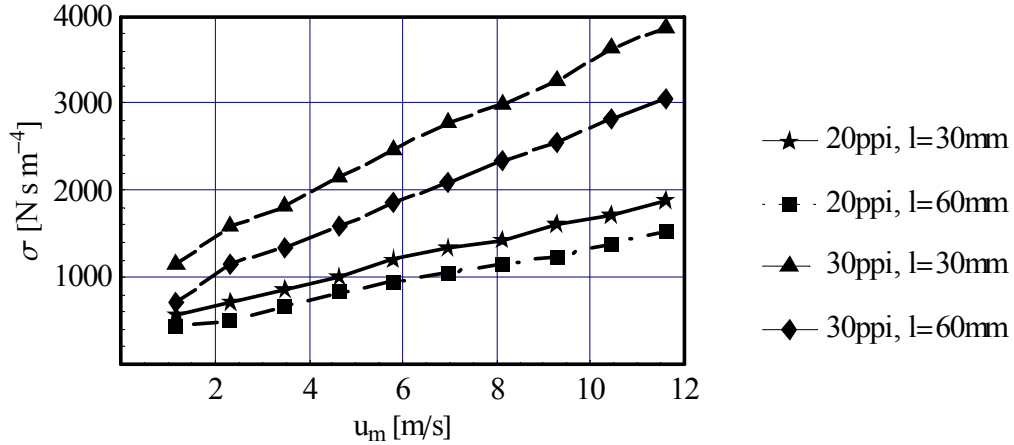


Figure 2: Flow resistivity σ over mean velocity u_m for four different samples with varying pore size and length l

Assuming that the material specific regression coefficients c_1, c_2, \dots, c_8 in equations (5) and (6) are independent at frequency and constant over the given Mach number range, their values can be optimized by fitting the calculated bulk properties to the experimental data. Therefore, a regression analysis of the eight parameters is used combining all material related measurements. Table 1 shows the optimized regression coefficients for a 20ppi and a 30ppi open-cell metal foam compared to values for sound-absorbing materials found in the literature. The method gives negative values for c_1 and c_2 , which are both corresponding together to calculate the additive part of the real part of the complex characteristic impedance Z_c via Equation (5). The experimental data shows that the real part of Z_c is close to 1 over the frequency range for all investigated configurations, which indicates a negligible small value for the additive real part in (5).

Table 1: Material specific regression coefficients according to equations (5) and (6)

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
Open-cell metal foam 20ppi	-0,35	-0,55	0,083	0,662	0,023	2,002	0,513	0,755
Open-cell metal foam 30ppi	-0,23	-1,08	0,106	0,914	0,049	1,293	0,244	0,842
Black foam [5]	0,271	0,381	0,089	0,802	0,271	0,454	0,163	0,681
Fibrous materials [1]	0,056	0,725	0,127	0,665	0,103	0,716	0,179	0,663

Since the flow resistivity σ and the regression parameters of the investigated foams are known, the complex wave number k' and the complex characteristic impedance Z_c can be described as a function of the frequency. Figure 3 shows the measured real and imaginary parts of k' and Z_c of an open-cell metal foam with 30ppi, $l = 60\text{mm}$, $\text{Ma} = 0,024$. It is demonstrated, that the calculated values of k' and Z_c are in better agreement with the experimental data when using the optimized regression coefficients instead of applying appropriate values from literature.

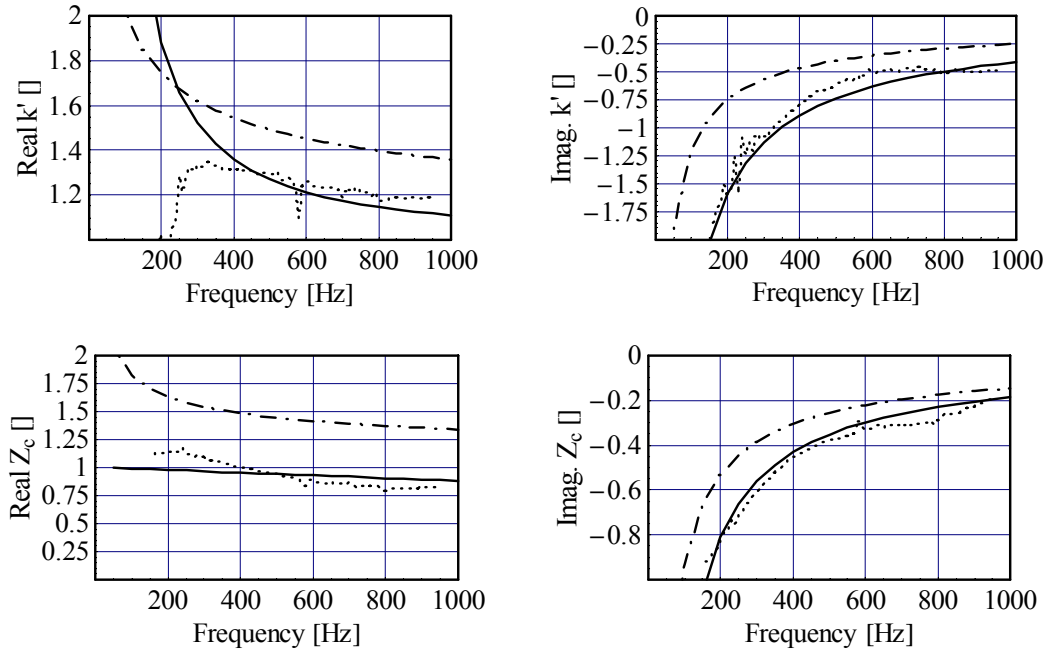


Figure 3: Real and imaginary part of the complex wave number k' and the complex characteristic impedance Z_c over frequency of an open-cell metal foam with $u_m = 8,16\text{m/s}$, 30ppi, $l = 60\text{mm}$, $\text{Ma} = 0,024$. Comparison between the experimental values (points) and the calculated values via equations (5) and (6) with a) the optimized regression coefficients constant over u_m and length (full line) and b) the regression coefficients for black foam given by Chen et al. (dot-dashed) [5].

To verify the assumption of constant regression coefficients over the appropriate Mach number range, their values are further optimized and fitted separately for every investigated range of mean velocities for an open-cell metal foam with 30ppi, $l = 60\text{mm}$. Their relation is plotted in Figure 4. It can be seen that the parameters c_3 , c_4 , c_5 , and c_7 are constant over the velocity range u_m . The parameters c_1 , c_2 , c_6 , and c_8 have varying values over the given Mach number range and it seems, that the assumption mentioned above is not strictly.

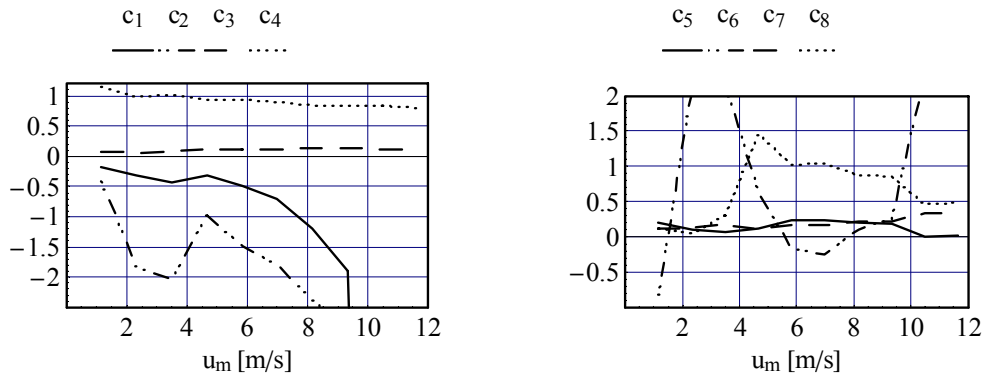


Figure 4: Optimized regression coefficients $c_1, c_2 \dots$ and c_8 as a function of the mean velocity for an open-cell metal foam with 30ppi, $l = 60$ mm.

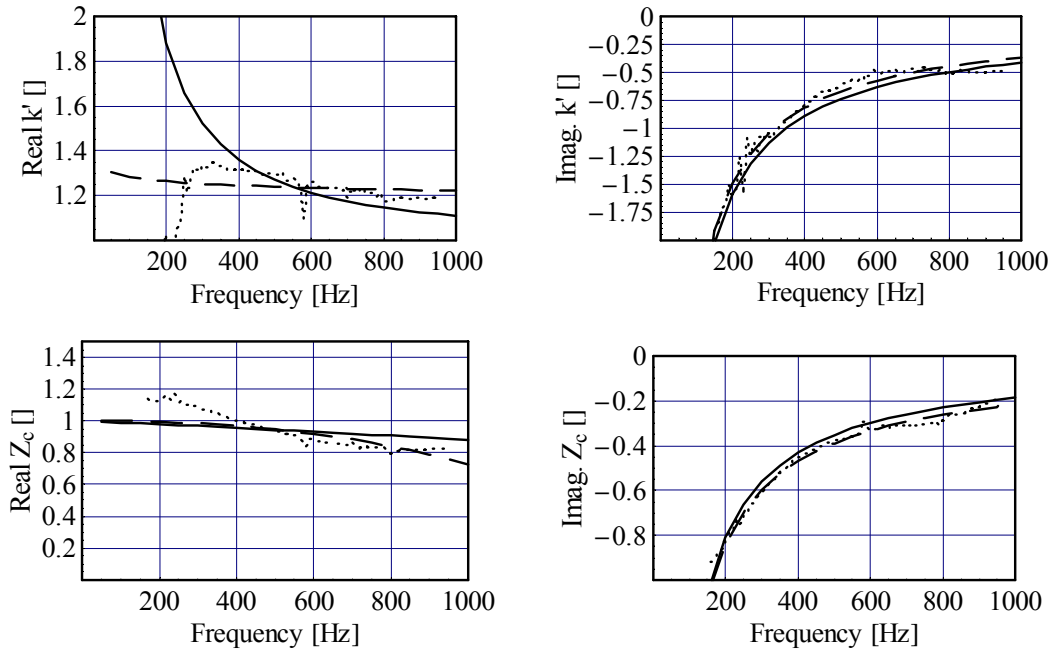


Figure 5: Real and imaginary part of the complex wave number k' and the complex characteristic impedance Z_c over frequency of an open-cell metal foam with $u_m = 8,16$ m/s, 30ppi, $l = 60$ mm, $Ma = 0,024$. Comparison between the experimental values (points) and the calculated values via equations (5) and (6) with a) the optimized regression coefficients constant over u_m and length (full line) and b) the optimized regression coefficients for the given test case (dashed).

Figure 5 shows a comparison between the calculated bulk properties with two different sets of regression coefficients. One set is constant over the investigated Mach number range, and the other set is optimized for the specific test case with $Ma = 0,024$. By using the second set of regression coefficients, a significant better fit to

the experimental data can be seen only for the real parts of k' and Z_c and for frequencies smaller than 300 Hz. Since changing the flow resistivity has much more influence on the calculated bulk properties than changing the regression coefficients, their effect on the agreement to the experimental values is negligible comparing regression coefficients varying with mean air flow. Using Equations (5) and (6) has substantial advantages from the application point of view. For this reason it is proposed to use the regression parameters in Table 1, which are constant over the investigated Mach number range and which represent the reality satisfactorily.

SUMMARY AND CONCLUSIONS

This work uses an experimental method for the determination of the transfer matrix combined with a regression analysis to describe the bulk properties of different samples of open-cell metal foams for frequencies up to 1000 Hz. The related parameters – the flow resistivity σ , the complex wave number k' and the complex characteristic impedance Z_c – are investigated over a range of charged mean air flow. Based on a mathematical relation of Delany and Bazley [3], the specific coefficients are determined and compared to those given in the literature. New values for the specific parameter coefficient of the measured open-cell metal foams are proposed, and their dependency on the mean air flow is investigated. With the simplified assumption of constant values for the coefficients, the values of the modelled bulk properties showed good agreement to the experimental data.

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