CONTRIBUTIONS TO THE ANALYSIS OF THE DYNAMIC RESPONSE OF A HEXAPOD TYPE MOBILE ROBOT

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Abstract:
The paper presents an intricate mathematical formalism useful in the kinematic and dynamic study of the kinematic chains in the structure of the industrial and biological robots.
The presented analysis method completes a very proficient computational system of the mechanical systems from the structure of the robots, especially of the biological robots. This system contains the mathematical methods [1], [5], useful in the kinematical and dynamical study, in the precision control. This formalism is based on the Newton Euler method, completed by the Lagrange multipliers and adapted to the complex study of the mobile spatial systems.
The hexapod robot that constitutes the object of this paper was obtained through the structural, kinematic and dynamic study of a hexapod insect, from the Blatodea zoological order. The processing of the dynamic models on a computer and the three dimensional simulation of its behavior offers the necessary information to proceed with the dynamic study on the experimental model

1. INTRODUCTION

The methods used for the kinematic and dynamic analysis of the spatial mechanism, in particular, are many and varied.
Nevertheless, we consider that the possibility of processing the obtained mathematical models and especially the interface that they can offer for modeling and simulating the mobile system functioning in real, practical conditions are important.
In the papers [3], [7], we have studied some aspects regarding the structure and the kinematics of a hexapod walking robot. The structural and kinematical models had as a source of inspiration the movement from the living world namely the movement of the hexapod insects from the Blatodea order.

2. Kinematic Modeling [3]

It is considered a kinematic linkage made by "n" rigid solids, connected through "n-1" kinematics pairs (fig.1).

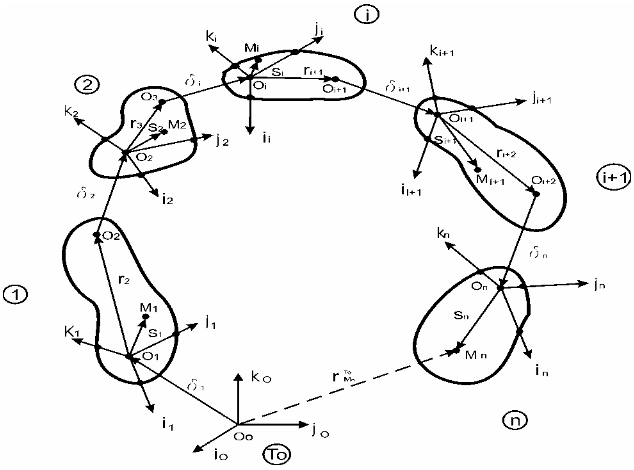

Fig. 1. The kinematic linkage

We make the following notations:

- $T_i\left(\bar{i}_i, \bar{j}_i, \bar{k}_i\right)$ - the reference frame attached to the element i, with unit vectors base $\overline{W}_i\left(\bar{i}_i, \bar{j}_i, \bar{k}_i\right)$; $i = \overline{1,n}$.
- $T_0\left(\bar{i}_i, \bar{j}_i, \bar{k}_i\right)$ - the global reference frame with unit vectors base $\overline{W}_0\left(\bar{i}_0, \bar{j}_0, \bar{k}_0\right)$;
- $\vec{\delta}_i$ - the relative translation vector between the elements i-1 and i, with respect to existed trihedral, if there is a prismatic pair between elements i-1 and i; ($i = \overline{1,n}$).
- $\bar{r}_i$ - the position vector with respect to the reference frame T_i, of O'_I point, from which begins the relative translation, ($i = \overline{1,n}$);
- $\overrightarrow{S_i}$ - the position vector of M_i, in proportion to T_i , attached to element i.

The position vector of M_n point with respect to the global reference frame is given by the relationship:

$$\vec{r}_{M_n}^{\,T_o} = \overrightarrow{O_o M_n} = \sum_{i=1}^{n} (\vec{r}_i + \vec{\delta}_i) + \vec{S}_n \tag{1}$$

We introduce the coordinate transformation matrix from a reference frame to another.

$$\left\{\overleftrightarrow{W}_{i\text{-}1}\right\} = \left[A_{oi\text{-}1}\right]\left\{\overleftrightarrow{W}_o\right\} \tag{2}$$

$$\vec{r_i} = \{r_i\}^T \{\overrightarrow{W}_{i\text{-}1}\} = \{r_i\}^T [A_{oi\text{-}1}]\{\overrightarrow{W}_o\} \tag{3}$$

$$\overrightarrow{\delta_i} = \{\delta_i\}^T \{\overrightarrow{W}_{i\text{-}1}\} = \{\delta_i\}^T [A_{oi\text{-}1}]\{\overrightarrow{W}_o\} \tag{4}$$

$$\overrightarrow{S_n} = \{S_n\}^T \{\overrightarrow{W}_{i\text{-}1}\} = \{S_n\}^T [A_{oi\text{-}1}]\{\overrightarrow{W}_{i\text{-}1}\} \tag{5}$$

Introducing the relations (3), (4) and (5) in relation (1) we obtain:

$$\vec{r}^{\,T_o}_{M_n} = \overline{O_o M_n} = \left(\sum_{i=1}^{n} \begin{pmatrix} \left(\{r_i\}^T + \{\delta_i\}^T\right)[A_{oi\text{-}1}] + \\ + \{S_n\}^T [A_{on}] \end{pmatrix} \right) \{\overrightarrow{W}_0\} \tag{6}$$

3. THE MOVEMENT EQUATIONS IN NEWTON-EULER FORMALISM

The movement equations in Newton-Euler formalism are:

$$\delta q^T \cdot \left[M \cdot q - Q^a\right] = 0, \tag{7}$$

where:

M is the mass matrix;

q- the generalized coordinates vector;

Q^a- the active generalized forces matrix.

The mechanism configuration leads to an equation system:

$$\phi(q,t) = 0. \tag{8}$$

By differentiating the equations (8) we get:

$$J_q \cdot \delta_q = 0, \tag{9}$$

where: j_q- the evaluated Jacobean for the q coordinates which satisfy the equation (8);

δ_q - the virtual displacements.

According to the theorem of Lagrange multipliers the movement equation becomes:

$$\delta_q^T \cdot \left[M \cdot \ddot{q} - Q^a\right] + \lambda^T \cdot J_q \cdot \delta_q = 0, \tag{10}$$

or:

$$\left[M \cdot \ddot{q} - Q^a\right]^T \cdot \delta_q + \lambda^T \cdot J_q \cdot \delta_q = 0; \tag{11}$$

$$M \cdot \ddot{q} + J_q^T \cdot \lambda = Q^a . \tag{12}$$

By differentiating the equations (13) with respect to time we get:

$$J_q \cdot \dot{q} = -\frac{\partial \phi}{\partial t} \overset{(not)}{=} v. \tag{13}$$

By differentiating the equations (8) with respect to time we obtained:

$$\left[J_q\right] \cdot \ddot{q} = -\left[\dot{J}_q\right] \cdot \dot{q} - \left[\frac{\partial^2 \phi}{\partial t^2}\right] \overset{(not)}{=} [a]. \tag{14}$$

We obtain the equations:

$$\ddot{q} = \left[J_q\right]^{-1} \cdot [a]. \tag{15}$$

By combining the equations (12) and (15) we get:

$$\begin{bmatrix} M & J_q^T \\ J_q & 0 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q^a \\ a \end{bmatrix} \tag{16}$$

where: M = diagonal ($m_1, m_2, ..., m_n$) is the mass matrix;
λ - Lagrange multiplier

4. THE COMPUTATIONAL CALCULATION OF THE LIAISON FORCES

4.1 The mathematical modeling [1]

We consider the reference systems: Ri' and Rj' solidary to the elements i and j, Ri" and Rj" centered in the joint K and solidary to the elements i and j (fig.2.2).
The torque of the liaison forces has the components F"k and T"k, expressed through the orthogonal systems Ri" and Rj" made up of three axis.
The components of the liaison forces torque in the kinematic couple k are:

$$\begin{cases} \left\{F_i''^k\right\} = -\left[R_{ii''}\right]^T \cdot \left[A_{oi}\right]^T \cdot \left\{J_{r_i}^k\right\}^T \cdot \left\{\lambda^k\right\} \\ \left\{T_i''^k\right\} = \left(\left\{S_i'^M\right\}^T \cdot \left[P_{oi}\right]^T \cdot \left\{J_{r_i}^k\right\}^T - \left\{J_{\varphi_i}^k\right\}^T\right) \cdot \left\{\lambda^k\right\} \end{cases} \text{[1]} \quad \frac{d}{d\varphi}\left[A_{oi}\right] = \left[P_{oi}\right] \text{, where,} \tag{17}$$

$\left[R_{ii}''\right]$ - The transformation matrix of the coordinates when converted from system Ri" to system Ri'.

4.2 The fifth class rotational joint [1]

For the rotation coupling, the torque of the forces reported to the Ri" system is:

$$\begin{cases} F_i''^{r(ij)} = -[R_{ii''}]^T \cdot [A_{oi}]^T \cdot [\lambda]^{r(i,j)} \\ T_i''^{r(ij)} = \left[\{S_i'^M\}^T \cdot [P_{oi}]^T \cdot [I] - \{S_i'^M\}^T \cdot [Poi]^T \cdot [\lambda^{r(i,j)}] \right] \cdot \end{cases} \quad [1] \tag{18}$$

4.3 The fifth class translation joint [1]

We will mathematically model the existence condition of the translation joint in order to evaluate the corresponding Jacobean.
The torque of the liaison forces from the translation joint, with respect to the axis system, will have the following components:

$$F_i''^{tr(i,j)} = \left[R_{ii''} \right]^T \cdot \left[A_{oi} \right]^T \cdot [R] \cdot \left[A_{oi} \right] \cdot \left[w'_i \right] \cdot \lambda^{tr(i,j)}; \qquad [1] \tag{19}$$

$$\begin{aligned} T_i''^{tr(i,j)} = &\left[(r_j - r_i)^T \cdot [A_{oi}] + [S_j'^P]^T \cdot [A_j]^T, [w_j]^T \cdot [A_j]^T \right] \cdot \\ &\cdot [w_i] \cdot \lambda^{tr(i,j)}. \end{aligned} \qquad [1] \tag{20}$$

The equations (19) and (20) give the components of the liaison forces torque from the translation couple.

5. APPLICATION

The paper presents the dynamic movement of the left anterior leg, from the experimental model (fig.2). This model strictly observes the structure of the equivalent mechanism of the hexapod insect, scale 20:1.
For the dynamic analysis, we assume that the geometric parameters and the components of the inertia torques are already known, following the determination of the variation laws of the generalized coordinates and of the liaison forces torques in the kinematic couplings.
For the inverse dynamic analysis we used as input data the variation laws of the generalized coordinates, which define the motor joints movements and the variation laws of the generalized forces.
We studied the requirements in a dynamic regime of the liaison forces from the kinematic joint of the hexapod mechanism.
The transformation matrices of the coordinates are:

$$\left[A_{i'i} \right] = \begin{bmatrix} \cos q_i & \sin q_i & 0 \\ -\sin q_i & \cos q_i & 0 \\ 0 & 0 & 1 \end{bmatrix}; i = 1 \ldots 6 \tag{21}$$

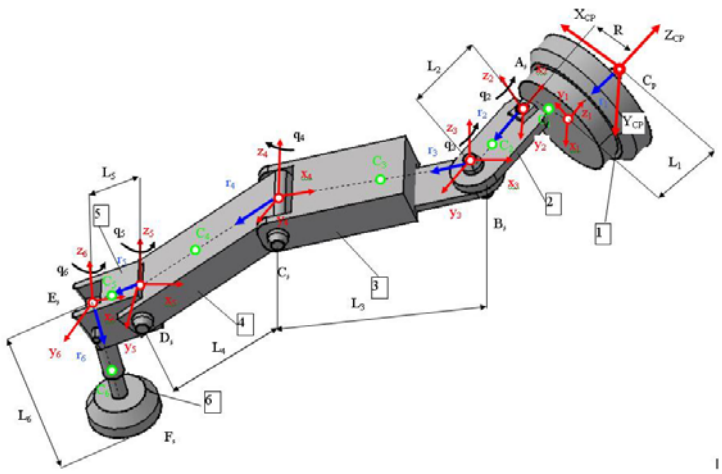

Fig.2 The model of the left fore-leg

The base changes are defined as follows:

$$\left\{\overrightarrow{W_i}\right\} = \left[A_{i'i}\right]\bullet\left\{\overrightarrow{W_{i'}}\right\}; i = 1 \ldots 6 \text{ or } \left\{\overrightarrow{W_i}\right\} = \left[A_{Cpi}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\}; i = 1..6 \tag{22}$$

$$\vec{r_i} = \left\{r_i\right\}^T\bullet\left[A_{Cpi}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\}; i = 1..6 \quad \vec{S_6} = \left\{S_6\right\}^T\bullet\left[A_{Cp6}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} \tag{23}$$

where:

$C_{ii+1'}$ - conversion matrix for passing from the system R_i to the system $R_{i'}$

The position of the point F_S in the reference system solidary to the pronot is:

$$\begin{aligned}\overrightarrow{r_{C_p}^{F_S}} = \overrightarrow{C_pF_S} = &\left\{r_{C_1}\right\}^T\bullet\left[C_{p1'}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \left\{r_2\right\}^T\bullet\left[A_{Cp1}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \\ &+ \left\{r_3\right\}^T\bullet\left[A_{Cp2}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \left\{r_4\right\}^T\bullet\left[A_{Cp3}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \\ &+ \left\{r_5\right\}^T\bullet\left[A_{Cp4}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \left\{r_6\right\}^T\bullet\left[A_{Cp5}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\} + \left\{S_6\right\}^T\bullet\left[A_{Cp6}\right]\bullet\left\{\overrightarrow{W_{C_p}}\right\}\end{aligned} \tag{24}$$

We determine the coordinates of the mass centers of the kinematic elements, reported to the reference system linked to C_p.

$$\begin{aligned}&\vec{r}_{C1}^{Cp} = \vec{r}_1 + \left[\vec{r}_{c1}\right]^T\cdot\left[A_{Cp1}\right]\cdot\vec{W}_{Cp}; \vec{r}_{C2}^{Cp} = \vec{r}_1 + \vec{r}_2 + \left[\vec{r}_{c2}\right]^T\cdot\left[A_{Cp2}\right]\cdot\vec{W}_{Cp} \\ &\vec{r}_{C3}^{Cp} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \left[\vec{r}_{c3}\right]^T\cdot\left[A_{Cp3}\right]\cdot\vec{W}_{Cp}; \vec{r}_{C4}^{Cp} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \left[\vec{r}_{c4}\right]^T\cdot\left[A_{Cp4}\right]\cdot\vec{W}_{Cp} \\ &\vec{r}_{C5}^{Cp} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \left[\vec{r}_{c5}\right]^T\cdot\left[A_{Cp5}\right]\cdot\vec{W}_{Cp}; \vec{r}_{C6}^{Cp} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6 + \left[\vec{r}_{c6}\right]^T\cdot\left[A_{Cp6}\right]\cdot\vec{W}_{Cp}\end{aligned} \tag{25}$$

We identify the equation system which describes the kinematic configuration of the mechanism.

$$\phi(q,t)=\left\{x_1-x_{C_1}=0;x_2-x_{C_2}=0\ldots x_6-x_{C_6}=0\right\} \tag{26}$$

The system is to be differentiated (26), to identify the Jacobean $J_q[18\times 24]$, reported to the variables x_1, y_1, z_1,q_1, .. x_6, y_6, z_6,q_6. The mechanic masses and inertia moments are introduced to define the matrix of the masses M.

The system of differential equations are formed and solved numerically (16), using the programs Maple and MATLAB.

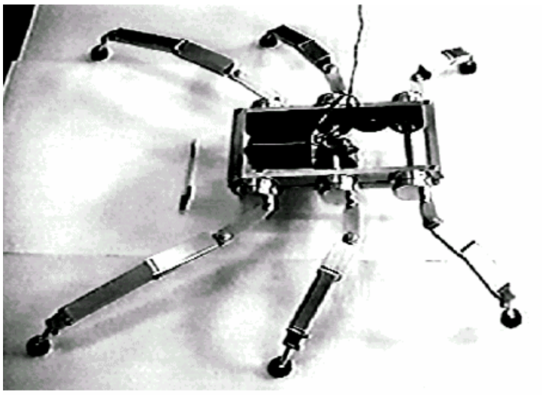

Fig.3. The experimental model

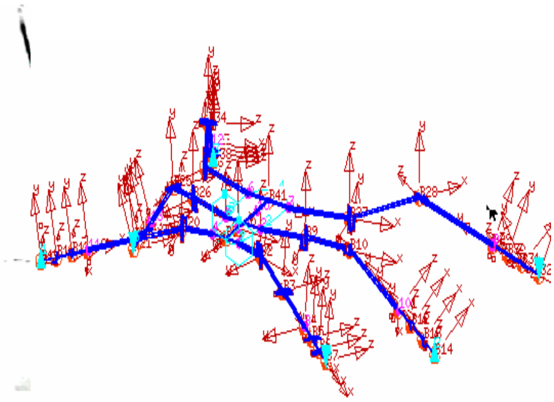

Fig.4 The dynamic model of the hexapod mechanism

The experimental model in figure 4 will be equipped with sensors for researches in a dynamic rating. The experimental model follows accurately the same structure and geometrical-kinematic parameters that characterize the walking process of a hexapod insect.

6. Processing the mathematical models

The paper presents a part of the variation laws of the kinematic parameters and of the liaison forces in the robot's joints.

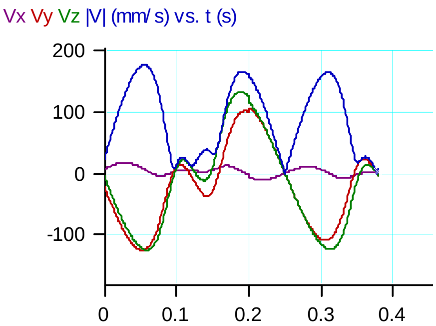

Fig.5 Speed for element 6

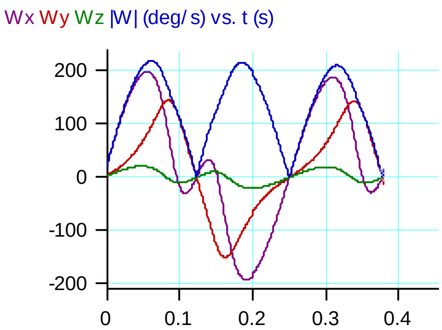

Fig.6 Angular speed for element 6

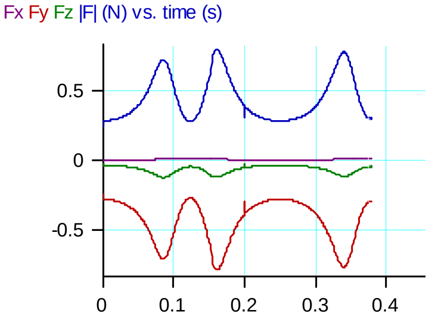

Fig.7 Liaison force variation F_{21}

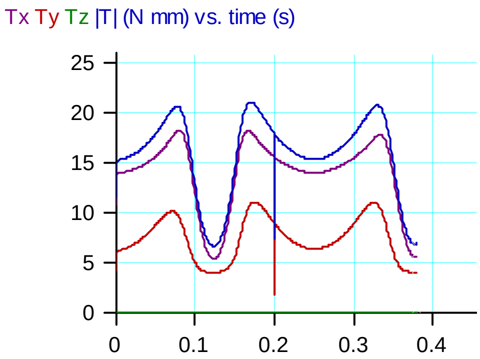

Fig.8 Moment variation in coupling A_s

7. CONCLUSIONS

The Newton-Euler formalism, completed by the method of Lagrange multipliers and the method of kinematic studio, presented in the first part of the paper, led to mathematical models with a flexible character, easy to repair and to implement on the computer.

The soft which was elaborated to process these models allows the movement analysis, for a movement sequence, in varied conditions (moving on horizontal ground in a straight line, in a curve etc).

We have designed an interface between the results of the numerical processing and a modeling and three-dimensional simulation program (NISA), in order to validate the mathematical models.

By modeling and simulating the functioning of the experiment virtual model, we have obtained the necessary data for equipping the experimental model in order to operate and control the movement in a dynamic regime.

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