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A NEW ACOUSTIC MODEL FOR PULSEJETS WITH VALVES

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Abstract

Principles of valved pulsejet operation were examined using a commercially available, but fully instrumented, 50 cm pulsejet. A variety of fuels were used, but primarily ethanol. Measurements providing thrust, pressure, temperature, velocity, and acoustic information were carried out. These experimental results were used in the validation of a numerical model. The pulsejet operation cycle can be divided into eleven steps involving a complicated interaction between fluid mechanics, acoustics, and chemical kinetic phenomena. The cycle consists of a high-pressure portion and a sub-atmospheric portion caused by the reflected waves. Combustion chamber analysis indicates a strong vortex which enhances turbulence and thus increases the reaction rate.

An acoustics laboratory was used to obtain pulsejet resonance frequencies experimentally. Acoustic models were developed analytically using the one-dimensional wave equation applied to vibrations in tubes, with appropriate boundary conditions selected at the open end and the combustion chamber end of the pulsejet. These models were compared to the experimental data, leading to a number of conclusions regarding valved pulsejet acoustic characteristics. The pulsejet's first resonance frequency was found to be best described not as a fourth-wave tube, as typically done, but as a sixth-wave tube, due to the combustion chamber boundary condition.

Important in the understanding of the pulsejet is that when heat is added at the moment of greatest condensation or removed at the moment of greatest rarefaction, then the "vibration is encouraged" [1,5]. Thus, for the proper operation of the pulsejet with valves, there must be coincidence between the acoustic resonance frequency and the fluid dynamic characteristic frequency, which is a function of the heat release.

INTRODUCTION

The pulsejet engine with valves is one of the simplest propulsion devices, with passively moving valves being the only moving parts. The pulsejet has been described as a ¼ wave tube, and clearly acoustics play an important role in their operation. The pulsejet is based on the Humphrey thermodynamic cycle, where isochoric heat addition (combustion) follows an isentropic compression and isobaric heat rejection follows an isentropic expansion. However, since the wave compression is weak, the thermodynamic efficiency is low, especially when compared with the Brayton cycle, where mechanical compression offers very high thermodynamic efficiency.

Recently, there has been a renewed interest in pulsejets and their principles of operation due to their potential applications for miniaturized propulsion devices. It has been found [3,4] that because of the pulsejet's simplicity, efficiency losses increase at a much slower rate than other more complex engines as the characteristic length dimension is reduced. In order to optimize and determine the scalability of these propulsion devices, a better understanding of the interaction between the acoustics, fluid mechanics, and chemical kinetics was needed.

The pulsejet engine used in this investigation is a direct descendent from the German World War II aero-pulse powered Vergeltungswaffe-1 (V-1) "buzz bomb". Currently, hobbyists use a variant of this engine, on the 50 cm scale, to power radio-controlled aircraft and hydroplanes. For this investigation, a 50 cm pulsejet was fully instrumented for measurements of thrust, pressure, temperature, velocity, and acoustics. This investigation concentrates on the pressure and acoustics experimental results in developing an analytical model of the important coincidence between the acoustic resonance frequency and the fluid dynamic characteristic frequency.

EXPERIMENTS

The 50-centimeter pulsejet geometry and dimensions are shown in Fig. 1. Three ports were welded on to the pulsejet in the axial direction for diagnostic measurements during hot operations. The port locations can be seen in Fig. 1. Only Port 1 data was used in this analysis, where the instantaneous static combustion chamber pressure was measured using an Omega DPX-101 high-speed pressure transducer linked to an Agilent oscilloscope. Type B thermocouples were also used to measure exhaust tube temperatures at the tube inlet and exit. The complete hot pulsejet experimental setup is shown in the photo in Fig. 2.

Figure 1
Pulsejet Geometry and Dimensions

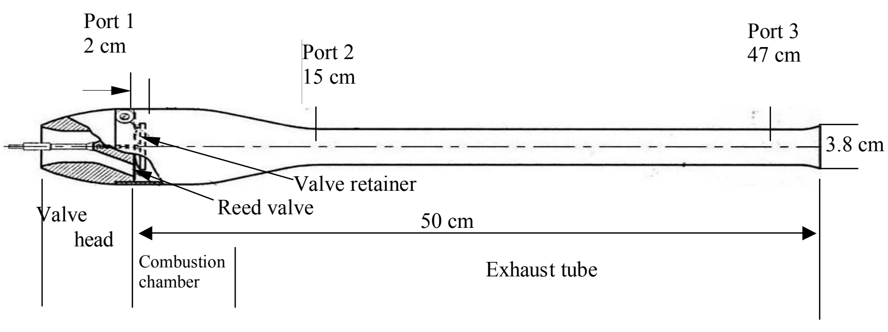
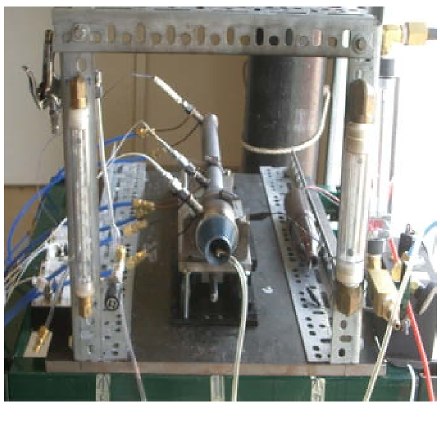


Figure 2
Pulsejet Experimental Setup



The data gathered during the operating pulsejet experiments [3,4] that was applicable to this analysis included:

- It was determined from inspection of the Port 1 combustion chamber pressure, that over the 360 degree operating cycle of the pulsejet, which begins with maximum combustion chamber pressure, the reed valves open at 120 degrees and close at 240 degrees. Therefore, during one-third of the operating cycle, the reed valves are open. This time of valve opening was confirmed by high-speed imaging of the reed valves during operation.
- Average exhaust tube inlet temperature was 1400 ^oC and average exhaust tube exit temperature was 800 ^oC, when the pulsejet fuel was ethanol.
- Approximately 22 Newtons of thrust was produced.
- Operating frequency was 232 Hz, based upon combustion chamber pressure.
- Ratio of combustion chamber pressure to ambient pressure was approximately 2.0.

The pulsejet acoustic experimental setup used to determine the cold pulsejet resonance frequencies included a microphone, sound level meter, amplifier and an HP/Agilent 35665A Signal Analyzer. This experiment found the first two resonance frequencies of the hobby scale 50 cm pulsejet to be

$$f_1 = 124 \text{ Hz} \qquad f_2 = 462 \text{ Hz}$$

THEORY

The one-dimensional acoustic wave equation describing plane waves is

$$d^2p/dt^2 = c_o^2(d^2p/dx^2) \tag{1}$$

Where p is the acoustic pressure (a function of axial direction x and time t only), c_o is the speed of sound and is a constant for a cold pulsejet, and the motion of every particle is parallel to a fixed line and is the same in all planes perpendicular to that line. Substituting a solution of the form

$$p = e^{i(kx - \omega t)} \tag{2}$$

where the wave number $k = 2\pi/\lambda$, ω is the radian frequency, and f is the circular frequency, yields

$$c = \omega/k = f\lambda \tag{3}$$

Substituting $p = \cos(kx)$ and applying the boundary conditions

velocity	$v = 0$ at $x = 0$	tube closed end		(4)
pressure	$p = 0$ at $x = L$	tube open end		(5)

where L is the tube length, yields the characteristic equation

$$f_n = (2n+1)\, c/4L \quad \text{where } n = 0,1,2\ldots \tag{6}$$

This solution describes the pulsejet as a ¼ wave model.

For a pulsejet at room temperature of T = 22 °C or 77 °F, the speed of sound is c = 345 m/sec and length L = 48 cm (19 in). These values yield the two resonance frequencies of:

$$f_1 = 180 \text{ Hz} \qquad \text{and} \quad f_2 = 539 \text{ Hz}$$

However, when these frequencies are compared to the experimental results of

$$f_1 = 124 \text{ Hz} \qquad \text{and} \quad f_2 = 462 \text{ Hz}$$

it becomes apparent that a better model is needed than a tube with constant cross-sectional area. This can be accomplished by specifying a more appropriate boundary condition at the combustion chamber end, which considers the combustion chamber to be a volumetric chamber and provides a compliance boundary condition at the upstream end of the constant cross-section exhaust tube. The improved boundary conditions are

$$d^2p/dt^2 = (c^2 A_t / V_c)\, dp/dx \qquad \text{at } x = 0 \tag{7}$$

$$p = 0 \qquad \text{at } x = L' \tag{8}$$

where A_t is the area of the tube and V_c is the volume of the combustion chamber and L' is the length of the exhaust tube only (less than the total jet length, L). The small amount of kinetic energy in the combustion chamber is neglected. Now, solving the one-dimensional acoustic wave equation with these new boundary conditions yields the characteristic equation

$$\mathrm{ctn}(kL') = kL'\, (V_c / L'A_t) \tag{9}$$

For this pulsejet geometry, $V_c = (\pi/4)\,(6 \text{ cm})^2\,(9 \text{ cm}) = 250 \text{ cm}^3$, L' = 39 cm and $A_t = (\pi/4)\,(3 \text{ cm})^2 = 7 \text{ cm}^2$. Substituting in the above characteristic equation gives

$k_1L' = 0.9$ and thus the fundamental frequency of

$$f_1 = c_o k_1/(2\pi) = 127 \text{ Hz}$$

This predicted value of f_1 = 127 Hz is very close to the experimentally recorded f_1 = 124 Hz. Finally, the wavelength is now

$$\lambda = 2\pi/k_1 = 2\pi L'/(0.9) = 7L' = 5.7L \sim 6L$$

Thus, through this model analysis, the pulsejet with valves can be approximately described as a 1/6 wave tube.

APPLYING THE MODEL TO OPERATING PULSEJETS

Since this 1/6 wave model is based upon a cold, non-operating pulsejet, adjustments must be made to the model to account for: (1) valve openings and (2) higher operating temperatures.

From the operating pulsejet experiments, it was determined that the reed valves are open for one-third of the operating cycle [4] and therefore the pulsejet acoustic model must be adjusted to accommodate for this time of valve openings. An oscillator may be used to model this frequency adjustment by noting that the frequency of an oscillator may be calculated by equating the potential and kinetic energies, averaged over the cycle. For mass M on a spring with constant K, and with deflection δ,

$$K\delta^2/2 = M(2\pi f\delta)^2/2 \tag{10}$$

Solving for frequency f,

$$f = 1/2\pi(K/M)^{1/2} \tag{11}$$

The open reed valves reduce the average potential energy of the cycle by one-third because the gas in the combustion chamber and exhaust pipe does not expand or compress during that one-third of the cycle. From our equation for frequency above, it is clear that this reduces the frequency by a valve opening correction factor of

$$c_v = (2/3)^{1/2} = 0.82.$$

It is also necessary to adjust the 1/6 wave model for the higher temperatures of the operating pulsejet. Using $T_{\text{operating jet}} = 1100\ ^oC$ as the operating pulsejet temperature, calculated from the average of the approximate combustion chamber (1400 ^{o}C) and exhaust tube exit (800 ^{o}C) gas temperatures, and applying this to the same energy correction for this hot pulsejet, the frequency temperature correction factor is

$$c_t = (1373K/295K)^{1/2}.$$

Applying both frequency correction factors yields the final 1/6 wave pulsejet model operating frequency of

$$f_{\text{operating jet}} = f\, c_t c_v \tag{12}$$

and for the pulsejet used in this analysis,

$$f_{\text{operating jet}} = 127\text{ Hz }(1373\text{K}/295\text{K})(0.82)$$

$$f_{\text{operating jet}} = 225\text{ Hz}$$

From the experimental results, the operating frequency of the hot, operating pulsejet was f_{actual} = 232 Hz, which is very close to the predicted value of 225 Hz (3% error).

COINCIDENCE OF ACOUSTIC AND FLUID DYNAMIC FREQUENCIES

For the proper operation of the pulsejet with valves, there must be coincidence between the acoustic resonance frequency and the fluid dynamic characteristic frequency. The acoustic resonance governs the oscillation of the pressure in the combustion chamber, which provides the timing for the valves to periodically open and close. Coincident to this, the fluid dynamic characteristic frequency governs the discharge, replenishing, and preheating of the fuel and air mixture in the combustion chamber. The momentum of the exhaust outflow helps sustain the combustion chamber negative pressure and the associated inflow of fresh charge through the open valves. Subsequently, the exhaust backflow, which is clearly visible in all numerical simulations, provides the preheating of the fresh charge via mixing with the residual hot gases from the previous cycle.

Time-Average Net Mach Number of Exhaust

This coincidence of the acoustic resonance frequency ($f_a = c/6L$) and the fluid dynamic characteristic frequency ($f_d = V/2L$), where

- c = space-time–average speed of sound in the exhaust tube, which is averaged over the length of the tube and the period.
- V = time-average fluid velocity in the exhaust tube. The flow is assumed to be slug-like, so that the fluid velocity is constant over the length of the tube, but may vary with time during the cycle. The velocity used to calculate the net Mach number is averaged over the entire period, including outflow and backflow.
- L = exhaust tube length

dictates an exhaust time-average net Mach number of: $M = V/c = 0.33$

The average exit Mach number calculated with the CFD numerical model is approximately M = 0.3. The average exit Mach number computed from Laser Doppler Velocimetry (LDV) and thermocouple measurements, for speed of sound, also indicate an approximate M = 0.3. [4]

Reference [2] calculates that for a Mach number of M = 0.3, the ratio of combustion chamber pressure to ambient pressure must be approximately two. Indeed, the pressure ratios of the pulsejet of this analysis and the German World War II "buzz bomb" are approximately two.

Time-Average Outflow and Backflow Mach Numbers

The time-average Mach number of the exhaust outflow (M_o) is of course higher than that of the exhaust backflow (M_b), otherwise, there would be no thrust. M_o and M_b may be estimated as follows from the calculated time-average net Mach number and from the measured hobby scale pulsejet thrust of 22 Newtons.

$$(M_o + M_b)/2 = \text{time-average net Mach number} = 0.33 \qquad (13)$$

$$(M_o^2 - M_b^2)\rho_o c^2 A = 22 \text{ Newtons} \qquad (14)$$

$$\text{or} \quad (M_o - M_b)(M_o + M_b)\ )\rho_o c^2 A = 22 \text{ Newtons}$$

$$\text{or} \quad (M_o - M_b)(M_o + M_b) = 0.20 \qquad (15)$$

Solving Equations (13) and (15) simultaneously,

Outflow Mach, $M_o = 0.48$ and Backflow Mach, $M_b = 0.18$

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CONCLUSIONS

Acoustic models were developed analytically using the one-dimensional wave equation applied to vibrations in tubes, with appropriate boundary conditions selected at the open end and the combustion chamber end of the pulsejet. These models were compared to the experimental data, leading to a number of conclusions regarding valved pulsejet acoustic characteristics:

- A simple quarter-wave tube of constant cross-section is not a good model of a cold pulsejet with valves closed, due to the combustion chamber volume.
- The first resonance frequency of a cold pulsejet with valves closed was found to be best described as a sixth-wave tube ($\lambda \sim 6L$), due to the

need for a more complex compliance boundary condition at the combustion chamber end of the pulsejet.

- To adequately predict the operating frequency of a hot pulsejet with valves, the potential energy must be reduced by 33% due to the time of valve openings at the combustion chamber boundary, which were found to occur from 120^o to 240^o of the total operating cycle.
- Using Raleigh's criterion [1], it was calculated that a pulsejet with valves will have a net Mach number of approximately 0.33, which requires a net combustion chamber-to-ambient pressure ratio of approximately 2.0.
- The average Mach numbers of the exhaust gases during the outflow and backflow portions of the cycle are 0.48 and 0.18, respectively, calculated from the measured 5 pounds of thrust.

REFERENCES

[1.] Rayleigh J.W.S., "The Theory of Sound", 2nd ed.. (Dover Books, New York, NY, 1945. Originally published 1877).
[2.] Guggenheim Aeronautical Laboratory and Jet Propulsion Laboratory, "Jet Propulsion", California Institute of Technology, Air Technical Service Command (1946).
[3] Applied Energy Research Laboratory, "Development and Demonstration of a Micro-Pulsejet", North Carolina State University (2005)
[4] T. Geng, A. Kiker, Jr., R. Ordon, A. V. Kuznetsov, T. F. Zeng and W. L. Roberts, "Combined Numerical and Experimental Investigation of a Hobby-Scale Pulsejet," Journal of Propulsion and Power (submitted).
[5] T. Rossing, N. Fletcher, "Principles of Vibration and Sound", 2nd ed. (Springer-Verlag, New York, NY, 2004)