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# A VARIABLE DAMPING AND STIFFNESS SEMI-ACTIVE VIBRATION ISOLATION SYSTEM USING MAGNETORHEOLOGICAL DAMPERS

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## Abstract
Semi-active systems with variable damping and stiffness have excellent performances. However, conventional devices for controlling variable stiffness are complicated and difficult to implement in most applications. To address this issue, a new configuration that requires two controllable dampers is proposed. The configuration is experimentally implemented using two magnetorheological (MR) dampers for the controllable dampers in one degree-of-freedom (DOF) system. The responses of the system to sinusoidal and random excitations show that the variable damping and stiffness control can be realized by the proposed system, and the system with damping and stiffness on-off control provides the best vibration isolation. To expand this work, a 2 DOF vibration isolation system with variable damping and stiffness is also studied. The responses of the system to a random excitation show that the damping and stiffness on-off control system has the best performance.

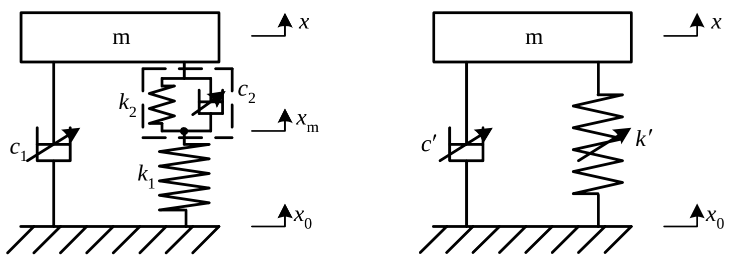
## INTRODUCTION

Traditionally, based on a vibration control system's design and its energy requirements, it can be categorized as: passive, active or semi-active. The semi-active control systems represent a compromise between performance improvement and simplicity of implementation [1-3]. The idea of using variable damping in vibration systems has been shown by many researchers [1-4]. However, because the system stiffness is constant, the performance at the resonance frequency can not be avoided with the variable damping control for a base exciting system. Therefore, the vibration system with variable stiffness control has been proposed by some researchers [5, 6].
Kobori proposed a variable stiffness system to suppress the building's response to earthquakes [6]. Youn and Hac used an air spring to vary the stiffness in a suspension system between three discrete values [5]. However, conventional implementation of

variable stiffness device is complicated and their responses exhibit time delay. The authors had proposed a structure using two Voigt elements in series to realize the variable damping and stiffness [7]. The sinusoidal and random responses of one degree-of-freedom (DOF) and 2-DOF systems showed that the system with damping and stiffness on-off control had a good vibration isolation performance [8].
However, because two controllable dampers were in series in the above system, if one of the two controllable dampers was in the off-sate and the other was on-off controlled, the total damping of the system almost was not be changed. In this paper, a new variable stiffness system using one Voigt element and spring in series is proposed. The performances of the proposed systems are studied with the sinusoidal and random excitations in calculation and experiments.

## A NEW VIBRATION SYSTEM

A new model of 1-DOF vibration isolation system with two controllable dampers (Damping coefficients are $c_1$ and $c_2$.) is proposed and shown in Fig. 1 (a). The variables $x_0$ and $x$ are the base excitation and the response of mass *m*, respectively. The stiffness ($k_1$ and $k_2$) values of the two springs (spring 1 and 2) are not varied. However, the system effective stiffness can be varied by $c_2$. Damper 2 and spring 2 comprise a Voigt element. The Voigt element and spring 1 are in series. The variable $x_{\mathrm{m}}$ is the displacement of the point between Voigt element and spring 1. The system equivalent model is shown in Fig. 1 (b), here $k'$ and $c'$ are the equivalent stiffness and damping coefficient, respectively.



(a) Original model (b) Equivalent model

*Figure 1 A new vibration isolation system.*

If $c_2$ is small, $k'$ is equal to the series of $k_1$ and $k_2$. However, if $c_2$ is large, $k'$ approaches $k_1$. Therefore, damper 2 can be used to realize the variable stiffness for the system. Damper 1 provides variable damping for the system. The equations of motion for the system shown in Fig. 1 (a) are

$$m\ddot{x} = -k_2(x - x_m) - c_2(\dot{x} - \dot{x}_m) - c_1(\dot{x} - \dot{x}_0)\,, \tag{1}$$

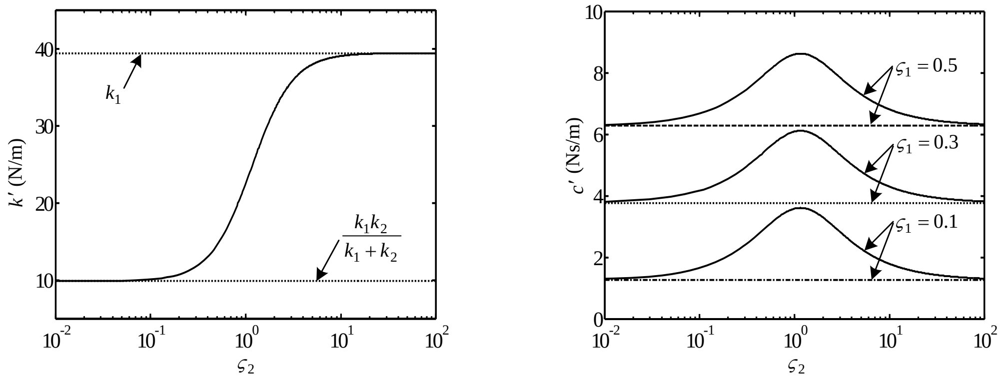
$$k_1(x_m - x_0) = k_2(x - x_m) + c_2(\dot{x} - \dot{x}_m)\,, \tag{2}$$

where $\ddot{x}$ is the acceleration, $\dot{x}_0$, $\dot{x}_m$, and $\dot{x}$ are the velocities of $x_0$, $x_{\mathrm{m}}$, and $x$, respectively. The equivalent stiffness and damping coefficient are

$$k' = k_1 - \frac{k_1^{\,2}(k_1 + k_2)}{(k_1 + k_2)^2 + c_2^{\,2}\omega^2}\,, \tag{3}$$

$$c' = c_1 + \frac{k_1^{\,2} c_2}{(k_1 + k_2)^2 + c_2^{\,2}\omega^2}\,. \tag{4}$$

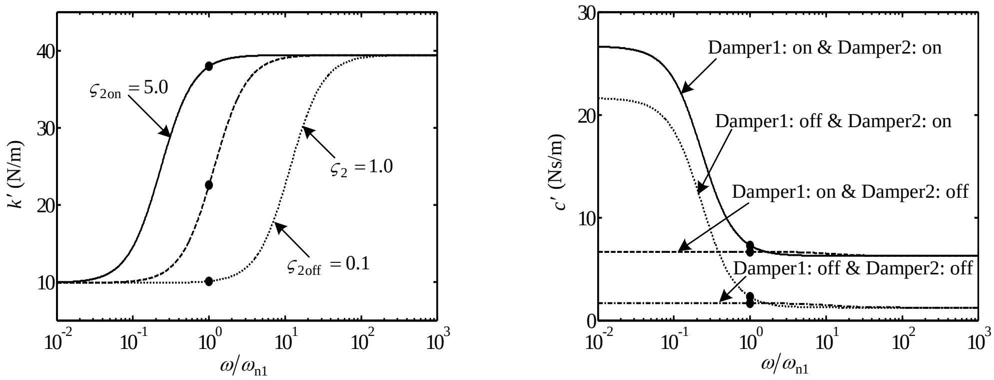
where $\omega$ is the excitation frequency. Based on Eqs. (3) and (4), $k'$ is independent of $c_1$, and $k'$ and $c'$ are influenced by $c_2$. If the parameter values: $k_1 = 4\pi^2$ N/m, $k_2/k_1 = 1/3$, and $m = 1$ kg, Fig. 2 shows the values of $k'$ and $c'$ with different $\varsigma_2$ ($\varsigma_2 = c_2/2\sqrt{mk_2}$) and $\varsigma_1 = 0.1$, 0.3 and 0.5 ($\varsigma_1 = c_1/2\sqrt{mk_1}$). The values of $k_1$, $k_1k_2/(k_1+k_2)$, and corresponding $c_1$ are also shown by dotted lines in the figure. The frequency $\omega = \omega_{n1}$, and $\omega_{n1} = \sqrt{k_1/m}$ in Eqs. (3) and (4).




(a) Equivalent stiffness (b) Equivalent damping

*Figure 2 Equivalent damping and stiffness varied by changing damper 2 ($\omega = \omega_{n1}$).*

According to Fig. 2, $k'$ is changed by damper 2 from $k_1k_2/(k_1+k_2)$ to $k_1$, and $c'$ can reach a maximum value. For Eq. (3), $\partial c'/\partial c_2 = 0$, and $\omega = \omega_{n1}$, the peak is obtained as: $\varsigma_2 = 1.15$ and $c'_{\max} = c_1 + 3\pi/4$. Based on Eqs. (3) and (4), $k'$ and $c'$ are also dependent on $\omega$. If damper 1 has a damping ratio $\varsigma_{1\text{off}} = 0.1$ in the off-sate and $\varsigma_{1\text{on}} = 0.5$ in the on-state, and damper 2 has a damping ratio $\varsigma_{2\text{off}} = 0.1$ in the off-sate and $\varsigma_{2\text{on}} = 5.0$ in the on-sate, $k'$ and $c'$ are shown in Fig. 3. For $\omega = \omega_{n1}$, $k'$ and $c'$ are plotted as dots in Fig. 3. When $\varsigma_2 = 1.0$, the value of $k'$ are also shown in Fig. 3 (a).




(a) Equivalent stiffness (b) Equivalent damping

*Figure 3 Equivalent damping and stiffness of the system with excitation frequencies.*

In Fig. 3 (a), when the excitation frequency is very large, $k'$ reaches the maximum value $k_1$. If $\varsigma_{2\text{on}} = 5.0$ and $\omega = \omega_{n1}$, $k'$ almost reaches the maximum value $k_1$. Moreover, if $\varsigma_{2\mathrm{off}} = 0.1$ and $\omega = \omega_{\mathrm{n1}}$, $k'$ is almost equal to $k_1k_2/(k_1+k_2)$. However, when $\omega$ is very small ($\omega = 0.01\omega_{\mathrm{n1}}$) or very large ($\omega = 100\omega_{\mathrm{n1}}$), $k'$ is almost equal to $k_1k_2/(k_1+k_2)$ or $k_1$, respectively, whatever value of $\varsigma_2$ is. According to Fig. 3 (b), $c'$ is changed by damper 1 in the on and off states. When the excitation frequency is small ($\omega < \omega_{\mathrm{n1}}$) and damper 2 is in the on-state, $c'$ is changed very much by the excitation frequencies.

## THE VIBRATION SYSTEM WITH CONTROL ALGORITHMS

The variable damping logic of damper 1 is

$$f_{\mathrm{d1}} = \begin{cases} -c_{1\mathrm{on}}(\dot{x}-\dot{x}_0) & \text{if} \quad \dot{x}(\dot{x}-\dot{x}_0) > 0 \\ -c_{1\mathrm{off}}(\dot{x}-\dot{x}_0) & \text{if} \quad \dot{x}(\dot{x}-\dot{x}_0) \le 0 \end{cases}, \tag{5}$$

where $f_{\mathrm{d1}}$ is the damping force of damper 1, the damping coefficient $c_1(t)$ is equal to c1on ($c_{1\mathrm{on}} = 2\varsigma_{1\mathrm{on}} m\omega_{\mathrm{n1}}$) in the on-state and c1off ($c_{1\mathrm{off}} = 2\varsigma_{1\mathrm{off}} m\omega_{\mathrm{n1}}$) in the off-state [1]. The control algorithm of damper 2 is

$$f_{\mathrm{d2}} = \begin{cases} -c_{2\mathrm{on}}(\dot{x}-\dot{x}_m) & \text{if} \quad \dot{x}(x-x_0) > 0 \\ -c_{2\mathrm{off}}(\dot{x}-\dot{x}_m) & \text{if} \quad \dot{x}(x-x_0) \le 0 \end{cases}, \tag{6}$$

where $f_{\mathrm{d2}}$ is the damping force of damper 2, the damping coefficient c2(t) is equal to c2on ($c_{2\mathrm{on}} = 2\varsigma_{2\mathrm{on}} m\omega_{\mathrm{n2}}$) in the on-state and c2off ($c_{2\mathrm{off}} = 2\varsigma_{2\mathrm{off}} m\omega_{\mathrm{n2}}$) in the off-state [7]. Five types of control schemes are studied in the following sections. For Type 1, damper 1 is in the off-state and damper 2 is in the on-state (called “Low damping”). For Type 2, damper 1 and 2 are both in the on-state (called “High damping”). In Type 3, damper 1 is on-off controlled as given by Eq. (5) and damper 2 is in the on-state (called “D on-off”). Alternatively, in the Type 4, damper 2 is on-off controlled as given by Eq. (6) (called “S on-off”). For Type 5, damper 1 and 2 are on-off controlled (called “D+S” on-off).Considering a sinusoidal base excitation, the values $X/X_0$ of the system with five control schemes are shown in Fig. 4.

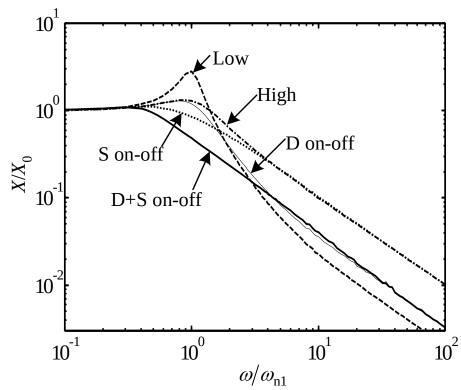


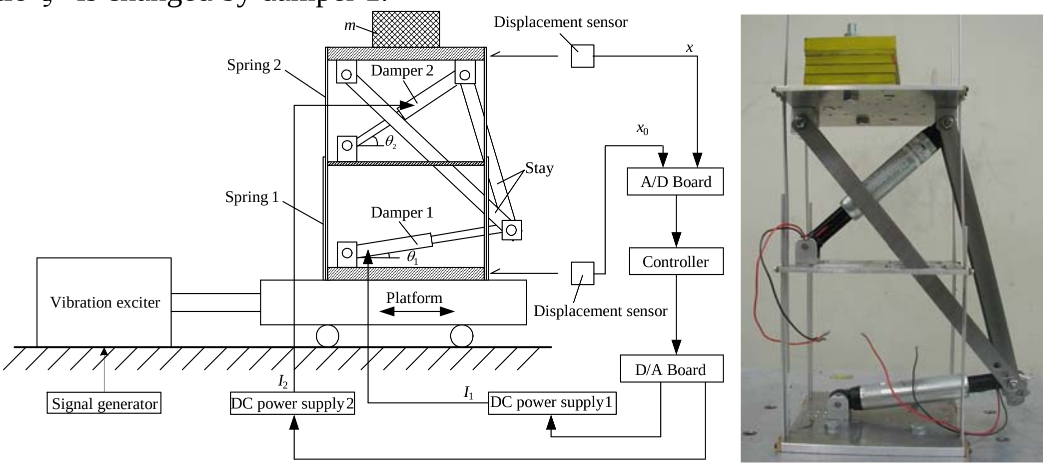

*Figure 4 Frequency responses of the system.*

According to Fig. 4, the system with D on-off control exhibits good vibration isolation performances of low and high damping systems in the resonance and high frequency regions. However, the response of S on-off control system is smaller than that of D on-off control at the resonance. The system with D+S on-off control has the best performance at the resonance frequency and has the same performance of D on-off control in the high frequency region.

# EXPERIMENTS OF THE VIBRATION SYSTEM

## Experimental setup

Figure 5 shows the experimental setup of the system. Two magnetorheological (MR) fluid dampers (RD 1097 made by Lord Cooperation) are used to provide the variable damping. In the experiment, the displacements $x_0$ and $x$ are measured by laser displacement sensors. The accelerations $\ddot{x}_0$ and $\ddot{x}$, and velocities $\dot{x}$ and $\dot{x}_0$ are obtained by differentiating the displacements in a computer. The dampers are oblique to the horizontal plane with angles $\theta_1 = 7.7^\circ$ and $\theta_2 = 40.5^\circ$. The platform is moved in horizontal direction using an electromagnetic vibration exciter. The experimental parameter values are listed in Table 2. The mass of the experimental structure is included in $m$. The applied currents of dampers $I_1$=0.19 A and $I_2$=0.48 A in the on-states and zeros in the off-states. According to the characteristics of MR damper and the responses of the vibration system, the damping coefficients $\varsigma_{1\mathrm{off}}$, $\varsigma_{1\mathrm{on}}$, $\varsigma_{2\mathrm{off}}$, and $\varsigma_{2\mathrm{on}}$ are obtained and also shown in Table 2 [8]. Based on Eqs. (3) and (4), when $\omega = \omega_{\mathrm{n}1}$, the values of $k'$, $c'$, damping ratio $\varsigma'$ and resonance frequency $f_\mathrm{n}$ are shown in Table 3. Here $\varsigma' = c' / 2\sqrt{mk'}$ and $f_\mathrm{n} = \sqrt{k'/m} / 2\pi$. According to Table 3, the maximum value of $k'$ is more than 2.8 times of the minimum value of $k'$. Moreover, the damping ratio $\varsigma'$ is changed by damper 1.




(a) Experimental system (b) Photograph of the model

*Figure 5 Experimental setup.*

*Table 2 Experimental parameter values*

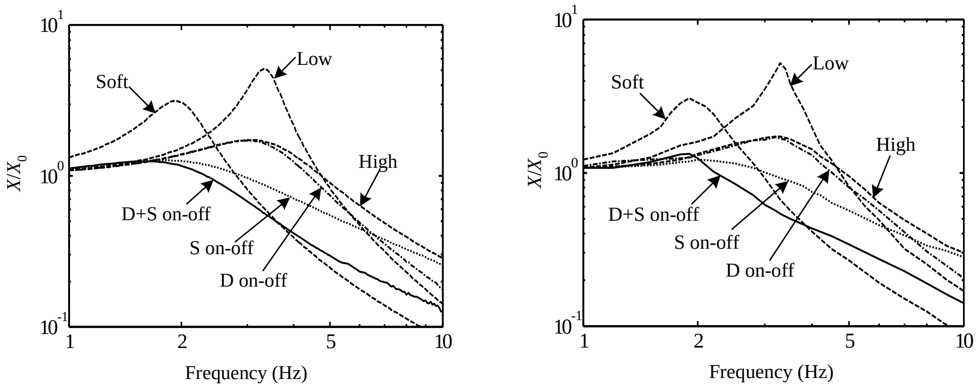
| Parameters | Values | Parameters | Values |
|---|---|---|---|
| $m$ | 10.524 kg | $\varsigma_{1\mathrm{off}}$ | 0.0815 |
| $k_1$ | 4679.60 N/m | $\varsigma_{1\mathrm{on}}$ | 0.3492 |
| $k_2$ | 2506.96 N/m | $\varsigma_{2\mathrm{off}}$ | 0.0619 |
| | | $\varsigma_{2\mathrm{on}}$ | 18.4696 |

*Table 3 Equivalent stiffness and damping*

| Damper 1 | Damper 2 | $k'$ (N/m) | $c'$(Ns/m) | $\varsigma'$ | $f_\mathrm{n}$ (Hz) |
|---|---|---|---|---|---|
| off | off | 1643.02 | 44.69 | 0.1699 | 1.9886 |
| off | on | 4669.80 | 44.37 | 0.1001 | 3.3526 |
| on | off | 1643.02 | 163.50 | 0.6217 | 1.9886 |
| on | on | 4669.80 | 163.18 | 0.3680 | 3.3526 |

## Frequency responses to a sinusoidal base excitation

When the amplitude $x_0$ is 5 mm and the frequency is from 1 Hz to 10 Hz of sinusoidal wave, the steady state responses of $X/X_0$ are shown in Fig. 6. Here $X_0$, and $X$ are the steady state amplitudes of the displacements of $x_0$ and $x$. Because of the ability of electromagnetic vibration exciter, the experimental results are obtained only from 1 Hz to 10 Hz in the following sections. According to Fig. 6, the calculation results are similar to those of the experiment. The response $X/X_0$ of the S on-off control is smaller than that of D on-off in the resonance frequency region, but it is larger than that of D+S on-off control in the high frequency region. The system with D+S on-off control has a good performance in the resonance and high frequency region.

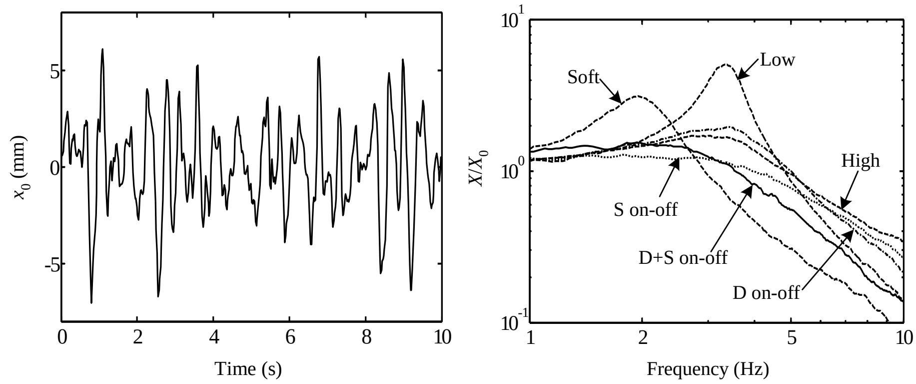



(a) Calculation (b) Experiment

*Figure 6 Frequency responses of the mass with a sinusoidal excitation.*

## Responses to a random base excitation

Figures 7 show the frequency responses of the control systems with a random base excitation. The values of $X/X_0$ are derived from the PSD values of $X$ divided by the PSD values of $X_0$. The PSD values are obtained by averaging the experiment data over 15 minutes. The input signal is also shown in the figure. The responses of the systems with the random input are similar to those of the system with the sinusoidal input shown in Fig. 6.




(a) Input signal (b) Frequency responses

*Figure 7 Frequency responses of the system to a random excitation.*

## A 2-DOF VIBRATION SYSTEM

Figure 8 shows a 2-DOF vibration system with the proposed configuration. The two controllable dampers have damping coefficients of $c_{21}$ and $c_{22}$, respectively. Here, $m_1$ and $m_2$ are masses; $k_{21}$ and $k_{22}$ are stiffnesses; $k_1$ is the bottom spring stiffness; $x_0$, $x_1$, and $x_2$ are the displacements of a base excitation, $m_1$, and $m_2$, respectively. In this section, the parameter values are: $m_1/m_2 = 0.1$, $k_{21}/m_2 = 4\pi^2$ $[N/(m \times \mathrm{kg})]$, $k_{21}/k_{21} = 1/3$, $k_1/k_{21} = 10$, $m_2$=1kg, $\omega_{\mathrm{n}21} = \sqrt{k_{21}/m_2}$, $\omega_{\mathrm{n}22} = \sqrt{k_{22}/m_2}$, $c_{21} = 2\varsigma_{21} m_2 \omega_{\mathrm{n}21}$, $c_{22} = 2\varsigma_{22} m_2 \omega_{\mathrm{n}22}$, $\varsigma_{21\mathrm{off}} = 0.1$, $\varsigma_{21\mathrm{on}} = 0.5$, $\varsigma_{22\mathrm{off}} = 0.1$, and $\varsigma_{22\mathrm{on}} = 5.0$. These values give the first natural frequency of about 0.5 Hz ( $f_{\mathrm{n}22} = \sqrt{k_{21}k_{22}/m_2(k_{21}+k_{22})}/2\pi$ ) and 1 Hz ( $f_{\mathrm{n}21} = \omega_{\mathrm{n}21}/2\pi$ ), and the second natural frequency of about 11 Hz. The control algorithms are same as those in the 1 DOF system.

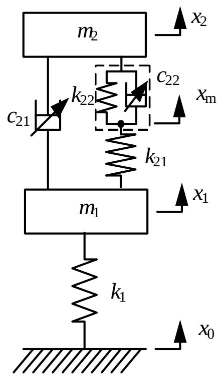
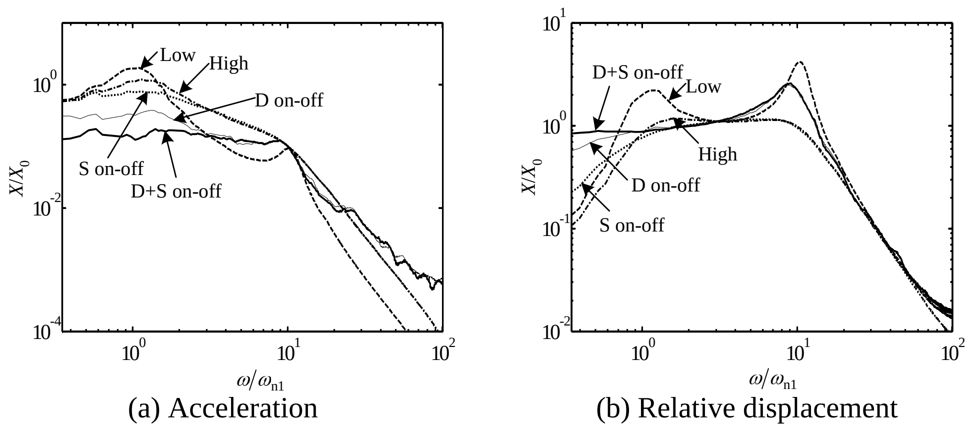



*Figure 8 A 2 DOF vibration isolation system.*

Figure 9 shows the responses of $\ddot{X}_2/\ddot{X}_0$ and $(X_2 - X_1)/X_0$ to a random excitation. Comparisons of the responses show that the D+S on-off control system has a good performance of acceleration in the low frequency region. In the high frequency region, the responses of D+S on-off are similar to those of D on-off control. At frequencies above 20 Hz, there is an increase in the acceleration responses of D on-off and D+S on-off control. That was caused by the shock force at switching time [9]. The relative displacement of D+S on-off control is small at the first resonance.




(a) Acceleration (b) Relative displacement

*Figure 9 Responses of acceleration and relative displacement to a random excitation.*

## CONCLUSIONS

A new configuration of the variable damping and stiffness system was proposed by using two controllable dampers. A controllable damper and a constant spring comprised a Voigt element. The Voigt element and a spring were in series. The stiffness of the whole system was varied by controlling the damper in the Voigt element, and the other damper provided variable damping for the system.

The experiments and theoretical calculations were carried out and the results were in good agreement. The responses to sinusoidal and random inputs were studied for five types of the control systems: low damping, high damping, damping on-off, stiffness on-off, and damping and stiffness on-off control systems. According to the experimental and calculation results, the variable damping and stiffness was implemented by the proposed system, and the system with damping and stiffness on-off control had excellent performances. In the two degree-of-freedom system, the system with damping and stiffness on-off control system also had good performances.

## ACKNOWLEDGEMENTS

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