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IDENTIFICATION OF ROTOR UNBALANCE IN MINIMAX STATEMENT

Yuri Menshikov

Fuculty of Mechanics and Mathematics, Dnepropetrovsk University, Nauchnaja st. 13, 49050, Dnepropetrovsk, Ukraine
du-mmf@ff.dsu.dp.ua

Abstract

The fast algorithm of unbalance evaluation of rotor which is based on the solution of recognition inverse problem in minimax statement is suggested. The vibrations of rotor supports (accelerations or velocities or displacements) in two mutually perpendicular directions during the work for a few rotor rotations as the initial information are used. Tikhonov regularization method is applied for solution of these ill-posed (unstable) problems taking into consideration the error of mathematical model. The method of choice of special mathematical model is proposed. The numerical calculation of example is given to illustrate suggested approach.

INTRODUCTION

The dynamical strain which is excited by unbalance is one of the main factors to restrict use of rapidly rotating rotors [2],[11]. The two principal problems are included into balancing of deformable rotor:

– the definition of unbalance value and its location;
– the unbalance removal by the system of plummets.

The main tendencies of balancing methods development are connected with ways development of coefficient influence definition.

But current methods of unbalance definition of machine rotor in its own bearings are not effective in case when the unbalance arises in the machine work process because they demand the special conditions of work or installation of trial plummets [1],[11]. Besides, these methods do not give the complete information about the position of unbalance if the rotor has a large length along the axis of rotation. The active testing of expensive rotating machines is not always possible or it is connected with great financial expenditures.

The suggested algorithms of unbalance evaluation use the experimental data about accelerations of rotor supports in two mutually perpendicular directions during the work during a few rotor rotations as the initial information. These algorithms do not demand of special conditions of work or the installation of trial plummets.

PROBLEM DEFINITION

The motion of rotor on two non-rigid supports is described by system of ordinary differential equations of 18th order [2],[4],[5]. Unbalance of rotor is modelled by some external load (EL). The value of this EL and the place of its action is it necessary to find. It is assumed that the vibrations of rotor supports in two mutual perpendicular directions are obtained from experiment. Let us suppose that the functions $z_1(t), z_2(t), z_3(t)$ characterize the unbalance of rotor

$$z_1(t) = m_r r \varphi^2 \sin(\vartheta + \varphi),\ z_2(t) = m_r r \varphi^2 \cos(\vartheta + \varphi),$$
$$z_3(t) = h m_r r \varphi^2 \sin(\vartheta + \varphi)\ ;$$

where r is the radius of rotor, m_r is the mass of unbalance reducing to a surface of rotor, φ is the angular velocity of rotation, h is unbalance arm, ϑ is angular deviation of the factor of EL with respect to correction plane. If the unbalance is absent then the functions $z_1(t), z_2(t), z_3(t)$ will be equal to zero. We suppose that with the help of acceleration transducers the function have been recorded ($\ddot{x}_A(t), \ddot{x}_B(t)$ are the acceleration of supports in horizontal direction, $\ddot{y}_A(t), \ddot{y}_B(t)$ are the acceleration of supports in vertical direction). As an example, we consider the equation for the unknown function $z_1(t)$ only.

Then the problem of unbalance recognition is reduced to the solution of integral equations of Volterra first kind

$$\int_0^t (t - \tau)\, z_1(\tau) d\tau = u_1(t), \quad t \in [0, T];$$

or

$$A z = B_p \vec{x}_\delta = u_\delta , \tag{1}$$

where A is a linear integral operator $(A : Z \to U)$, z is the searched characteristic of EL, B_p is a linear irreversible operator $(B_p : X \to U)$ depending on vector parameters of mathematical model (MM) of "rotor-supports" system $p = (p_1, p_2, \ldots, p_n)^T$ $((\cdot)^T$ is the sign of transposition); $\vec{x}_\delta$ is the vector-function of initial data. Subjective factors influence on the definition of parameters of system "rotor-supports" MM and therefore the parameters are supposed to have their values within certain limits: $p^0 \le p \le \hat{p}$, $1 \le i \le n$. In this way the vector p can be changed inside the known closed region $p \in D \subset R^n$.

The equations for required functions $z_2(t), z_3(t)$ will be similar to the equation (1).

For a rotor on two support for function $z_1(t)$ the vector – parameter $p \in D \subset R^{11}$ of MM has a kind

$$p = (E, J, m, m_A, m_B, c_A^{x}, c_B^{x}, b_A^{x}, b_B^{x}, a, b)^T,$$

where E is module of Jung of rotor material, m is the mass of rotor, m_A is the mass of the A support, m_B is the mass of the B support; c_A^{x}, c_B^{x} are the stiffness of supports A and B with respect to the horizontal and vertical direction; b_A^{x}, b_B^{x} are the coefficients of external friction; a is the distance of gravity centre of rotor to the A support, $a + b = l$ is the shaft length of rotor.

The vector-function $\vec{x}_{\boldsymbol{d}}$ is obtained using the experimental data (vibrations of supports) where the noise is present. Therefore it is convenient to think that each component of vector-function $\vec{x}_{\boldsymbol{d}}$ and function $u_{\boldsymbol{d}}$ belongs to L_2 [0,T]. Under this conditions the problem of equation (1) solution belongs to ill-posed problems if the searched functions $z_1(t)$ belong to C[0,T] as the operator A in (1) is completely continuous [12].

The value of function deviation $u_{\boldsymbol{d}}$ from the exact function u_T is:

$$\| u_{\boldsymbol{d}} - u_T \|_U = \left\| B_p \vec{x}_{\boldsymbol{d}} - B_p \vec{x}_T \right\|_U \le \boldsymbol{d}_0,$$

where

$$\boldsymbol{d}_0 = \boldsymbol{d}\, b_0 + d \,\| \vec{x}_{\boldsymbol{d}} \|_X, \quad b_0 = \sup_{p \in D} \left\| B_p \right\|_{X \to U}, \left\| B_p - B_T \right\|_{X \to U} \le d, \; \| \vec{x}_{\boldsymbol{d}} - \vec{x}_T \|_X \le \boldsymbol{d};$$

$\vec{x}_T$ is the exact vector- function of initial data; B_T is the exact operator; $\boldsymbol{d}$, b_0, d are given values. Let us suppose that such exact solution of equation (1) z_T $(A z_T = u_T = B_T \vec{x}_T)$ belongs to the function space W_1^2 [0,T] [12].

MINIMAX STATEMENT OF IDENTIFICATION PROBLEM

Let us consider the set of possible solutions of equation (1) with account of whole error of initial data

$$Q_{d,\boldsymbol{d}} = \{z : z \in Z,\ \left\| Az - B_p \vec{x}_{\boldsymbol{d}} \right\|_U \le \boldsymbol{d}_0\}.$$

The set $Q_{d,\boldsymbol{d}}$ is unbounded for any $\boldsymbol{d}_0 > 0$ as this problem is ill-posed [12].

For definition of stable approximate solution is used the regularization method of Tikhonov [12]. This way is based on the search of following extreme problem solution:

$$\Omega[z_0] = \inf_{z \in Q_{d,d}} \Omega[z], \tag{2}$$

where $\Omega[z]$ is the stabilizing functional which is defined on Z_1 (Z_1 is the everywhere dense set into Z).

From the practical point of view the function z_0 gives a guaranteed estimation from below sizes of real of a rotor in sense of functional $\Omega[z]$. If $\Omega^0 \le \Omega[z_0]$ (Ω^0 there is known limiting an allowable size for the given type of rotor machine) then the rotor is working in emergency operation with guarantee.

If the inequality $\Omega[z_0] < \Omega^0$ is carried out, then no objective conclusions can be made. Will be name the solution of extreme problem (2) as estimation from below of real unbalance.

However at realization of such approach there are large difficulties at definition of size *d*, as the absolute exact operator B_T is unknown and basically it cannot be constructed. Therefore size *d* is determined with the large overestimate and in set $Q_{d,\boldsymbol{d}}$ the "extraneous" functions get, that considerably reduces accuracy of the regularized solution.

Let us consider the sets:

$$Q_{p,\boldsymbol{d}} = \{z : z \in Z,\ \left\| Az - B_p \vec{x}_{\boldsymbol{d}} \right\|_U \le \boldsymbol{d} \left\| B_p \right\|_{X \to U}\}.$$

It is not difficult to show, that the solution of an extreme problem (3) always exists [12].

Let us consider the following extreme problem [6[,[7]:

$$\Omega[\tilde{z}_p] = \sup_{p \in D} \inf_{z \in Q_{p,d}} \Omega[z]. \tag{3}$$

It is obvious that $\Omega[z_0] \le \Omega[\tilde{z}_p]$. If $\Omega^0 \le \Omega[\tilde{z}_p]$, then the machine *probably* works in emergency operation. If $\Omega[\tilde{z}_p] \le \Omega^0$, then no certain conclusions can be made.

It is possible to show that the solution of an extreme problem (3) always exists.

T h e o r e m . The solution $\tilde{z}_p \xrightarrow{Z_1} z_T$ by $d \to 0$ and by $\boldsymbol{d} \to 0$.

For the realization of such approach it is necessary to choose within the vectors $p \in D$ some vector $p^1 \in D$ such that

$$\Omega[A^{-1} B_{p^1} \vec{x}_d] \ge \Omega[A^{-1} B_p \vec{x}_d]$$

for all possible $\vec{x}_d \in X$ and all $p \in D$. The operator B_{p^1} with parameter $p^1 \in D$ will be called the special maximal operator. Appropriate to this operator the model is named as the special maximal MM [6],[7].

If the special maximal MM exists then the solution of an extreme problem (3) will coincide with the solution of the following more simple extreme problem:

$$\Omega[\tilde{z}_p] = \inf_{z \in Q_{p^1,d}} \Omega[z]. \qquad (4)$$

For our case the special maximal MM exists, unique and corresponds to a vector

$$p_0 = (\hat{E}, \hat{J}, m^0, m^0{}_A, m^0{}_B, \hat{c}_A^x, \hat{c}_B^x, \hat{b}_A^x, \hat{b}_A^x, a^0, b^0)^T.$$

TEST CALCULATION

As the test example of unbalance identification was choice the problem of modelling of fluctuations in ventilator of the furnace [9]. The parameters of MM are equal

$$m^0 = 0{,}84 \cdot 10^4 kg, \hat{m} = 0{,}9 \cdot 10^4 kg,\ E^0 = \hat{E} = 2 \cdot 10^{11} H/\text{ì}^2,\ J^0 = 3 \cdot 10^{-3} \text{ì}^4,$$

$$\hat{J} = 3.1 \cdot 10^{-3} \text{ì}^4,\ m_A^0 = m_B^0 = 0{,}34 \cdot 10^5 kg, \hat{m}_A = \hat{m}_B = 0{,}35 \cdot 10^5 kg,$$

$$c_A^0 = c_B^0 = 0{,}65 \cdot 10^9 H/\text{ì},\ \hat{c}_A = \hat{c}_B = 0{,}7 \cdot 10^9 H/\text{ì},\ b_A^0 = b_B^0 = 0,$$

$$\hat{b}_A = \hat{b}_B = 100\, Hs/\text{ì}^2,\ a^0 = 5\text{ì}, \hat{a} = 5{,}1\text{ì},\ b^0 = 3\text{ì}, \hat{b} = 3{,}1\text{ì}.$$

For suggested algorithm examination of unbalance characteristics evaluation there was calculated the case when functions $\ddot{x}_A(t), \ddot{x}_B(t), \hbar_A(t), \hbar_A(t)$ are the results of mathematical simulation of rotor vibrations with given unbalance. The parameters of rotor unbalance were chosen as:

$$m_r = 0.5\,kg \text{ by } r = 0.25\,\text{ì}, \ \text{h} = 0.25\text{ì}, \ \boldsymbol{J} = 0.5\text{rad}.$$

The values of initial data inaccuracy were chosen after filtering as the following:

$$\left\| \ddot{x}_A(t) - \ddot{x}_{AT}(t) \right\|_C \le \boldsymbol{d}_1 = 0.08\text{ì/s}^2, \ \left\| \ddot{x}_B(t) - \ddot{x}_{BT}(t) \right\|_C \le \boldsymbol{d}_2 = 0.1\text{ì/s}^2,$$
$$\left\| \hbar_A(t) - \hbar_{AT}(t) \right\|_C \le \boldsymbol{d}_3 = 0.1\text{ì/s}^2, \ \left\| \hbar_B(t) - \hbar_{BT}(t) \right\|_C \le \boldsymbol{d}_4 = 0.1\text{ì/s}^2.$$

The whole inaccuracy of function $u_\delta(t)$ in equation (1) is $\boldsymbol{d} = 0.266 \cdot 10^4$ by chosen inaccuracy of initial data. The discrepancy method defined the parameter of regularization α [12].

The results of identification as solution of extreme problem (2) are followings:

$$m_r = 0.02\,kg, \ h = 0.01\text{ì}, \ \boldsymbol{J} = 0.0\,\text{rad}.$$

The results of identification with use the special maximal MM are followings:

$$m_r = 0.39\,kg, \ h = 0.22\,\text{ì}, \ \boldsymbol{J} = 0.45\,\text{rad}.$$

Efficiency of suggested algorithm was also shown on other tests [8],[9].

SUMMARY

In article consider minimax statements of unbalance identification. This approach can be used for technical diagnostics of unbalance and for balancing of rotors in their own bearings. The method can be adapted to those cases when measuring velocity or displacement of supports.

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