

OPTIMAL DISTRIBUTION OF SURFACE IMPEDANCE FOR REDUCING WING NOISE

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Abstract

This paper describes optimal distribution of surface impedance for aerodynamic noise from a thin wing. Incidence turbulence noise generated by flow-wing interaction and its reduction by utilizing finite impedance are discussed in this paper. Green's function is introduced to formulate the dipole sound field of wing with perforated surface. The experimental data is utilized as the adequate surface impedance for the wing surface with distributed apertures of small diameter but moderately low impedance ratio. In order to prove the effectiveness of finite surface impedance realized by the appertures, the experiment on a circular cylinder in low Mach number flow is briefly shown. After the effect of finite impedance is formulated for the wing noise generated by incident turbulence flow, numerical examples of the effect are shown finally, which prove the effectiveness of the optimization of impedance distribution for reducing the wing noise.

1 INTRODUCTION

Sound propagating within a duct is known to be very effectively absorbed when the duct walls are treated with sound-absorbent material. This method is finding increasing application in various schemes to control the noise of machineries. Incidentally sound field caused by flow turbulence-body interaction is affected by conditions of surface impedance. When apertures are furnished with large diameter, Reynolds number of which is high enough to be assumed potential flow, the efficiency of sound generation decreases substantially with the reduced effect of added mass[1]. This effect in wing is investigated for reducing the blade-vortex interaction noise through porous leading edge[4] and trailing edge noise through porous surface[3]. Contrally perforated surface can increase the sound radiated by adjacent turbulence, which is caused by a moving stream on the surface[5]. It is known that additional sound can be generated by wall linings in these circumstances so that practical schemems must maximize sound absorption simultaneously with a control of the new source mechanism. Additionally perforated shell structure in wing induces flow through the wing, which causes the reduced arodynamic loading. The fact suggests it favourite to bore apertures with small diameter in wing surface. Apertures with small diameter induce generally low noise and diminished reduction of loading, but they represent the high suface impedance and comparably high efficiency of aerodynamic sound generation. Optimization of properties of the aperture must be pursued, but resolution of the problem demands several new subjects. How can we predict the addional sound generation? How can we predict the reduced efficiency of sound generation? How can we predict the acoustic properties of the surface with ditributed apertures? and so on.

Let's suppose the distributed apertures of small diameter, for example $d \leq 0.1$ (mm) are introduced to a circular cylinder of diameter D = 10(mm) and span length 2L = 0.2(m)immersed in low Mach number flow $M \equiv U/c = 20/340 < 0.1$. The Karman vorex frequency $f_k = 400$ (Hz) and radiated sound pressure level $SPL_k = 55$ (dB) are obserbed in this case [7]. The Reynolds number based on the diameter of the aperture $Re \equiv Ud/\nu \sim$ 133 suggests the laminar flow through the aperture. When dipole sound is supposed to be radiated to the point r = 1.5 (m) apart from the the cylinder by the fluctuating force $\tilde{F} = \tilde{p}_s L d$, the induced normal velocity $\tilde{v}_n = \tilde{p}_s / \rho c |\alpha| \sim 1.2 \times 10^{-2} (\text{m/s})$ is deduced assuming the impedance ratio $|\alpha| = 0.3$. This means $\bar{v}_n / U = 0.6 \times 10^{-3}$ averaged with the cylinder surface. When area ratio of the aperture to the surface is assumed 0.01, the local velocity ratio at the aperture will be $v_n/U = 0.06$. This ratio means the influence of the induced normal velocity to the boundary layer flow is not so remarkable. Therefore the reaction of the normal velocity to the boundary layer turbulence can be neglected, which is basic idea of this paper. The porous surace with apertures of small diameter and low area ratio is supposed. As the impedance ratio $|\alpha| = |Z|/\rho c$ is moderate, reducing effect of sound generation efficiency is expected. Since the diameter of the aperture is small enough, generation of additive sound by the ineraction between flow turbulence and the apertures is neglected.

This paper describes optimal distribution of surface impedance for reducing wing noise. Generally two kinds of aerodynamic noise related with interaction between flow turbulence and wing are imagined; incident turbulence flow noise and trailing edge noise. The former is discussed in this paper. Green's function is introduced to formulate the dipole sound field of wing with porous surface. When the reduced efficiency of aerodynamic sound generation for a wing with apertures is discussed, the surface impedance is essential as the acoustic property of the surace. The experimental data is utilized to indicate the surface impedance for the wing surface with distributed apertures with small diameter but moderately low imedance ratio. In order to prove the effectiveness of finite surface impedance realized by appertures with very small diameter, the experiment on a circular cylinder in low Mach number flow is briefly shown. After the effect of finite impedance is formulated for the wing noise generated by incident turbulence flow, numerical examples of the effect are shown finally.

2 FAR FIELD FROM A COMPACT BODY AFFECTED BY PER-FORATED SURFACE

Let's suppose a body is immersed in the uniform and low Mach number flow $\mathbf{U} = U\mathbf{i}$. Votices are shed in the wake of the body and generates aerodynamic sound as shown in Figure 1. The equation describing sound field generated by the flow turbulence-body



Figure 1: Body immersed in a uniform flow and vortices shed in the wake

interaction is shown as follows basing on the theory of 'vortex sound'[2].

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)(HB) = -\operatorname{div}(B\nabla H) + \nabla H \cdot \frac{\partial \mathbf{v}}{\partial t} + \operatorname{div}(H\omega \times \mathbf{v}) - \nu \operatorname{div}(\nabla H \times \omega) \quad (1)$$

where \mathbf{v} and ω denote the velocity vector and the vorticity vector respectively, and *B* does the total enthalpy defined by the following.

$$B = \int \frac{dp}{\rho} + \frac{v^2}{2} \tag{2}$$

where p denotes acoustic pressure, ρ does density, v does magnitude of particle velocity of the sound, and variables H and f are defined as shown in the followings.

 $H \equiv H(f)$; Heaviside function, $f(\mathbf{y}) = 0$ represents the body surface

Position vectors are classified in the paper as follows; \mathbf{x} means the sound field and \mathbf{y} does the source field. To find the solution of Equation (1), Green's function satisfying following equation is introduced.

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)(HB) = \delta(\mathbf{x} - \mathbf{y})\delta(t - \tau), \quad \text{where } G = 0 \text{ for } t < \tau$$
(3)

When Green's function satisfying $\frac{\partial G}{\partial y_n}(\mathbf{x}, \mathbf{y}, t - \tau) = 0$ on the body surface is selected, the following solution of B is derived.

$$B(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \left\{ -\int_{V} (\omega \times \mathbf{v}) \cdot \nabla G d^{3}\mathbf{y} + \nu \oint_{S} \omega \times \nabla G \cdot \mathbf{n} dS(\mathbf{y}) + \oint_{S} G \frac{\partial v_{n}}{\partial \tau}(\mathbf{y},\tau) dS(\mathbf{y}) \right\}$$
(4)

Now we focus our attention on the far field, where $|\mathbf{x}| \ll 2l$ and 2L (2l means stream-wise length of the body and 2L does span-wise length) is valid. Also we suppose the condition for the Reynolds number as $Re \equiv 2lU/\nu > 10^4 \sim 10^5$. In this condition we can assume the variable B as follows; $B \approx \frac{1}{\rho_0} p(\mathbf{x}, t)$. Moreover we assume $|\mathbf{x}| \to \infty$, then the Green's function $G(\mathbf{x}, \mathbf{y}, \mathbf{t} - \tau)$ can be replaced by Compact Green Function shown as follows[2],

$$G(\mathbf{x}, \mathbf{y}, \mathbf{t} - \tau) = \frac{1}{4\pi |\mathbf{X} - \mathbf{Y}|} \delta(t - \tau - \frac{|\mathbf{X} - \mathbf{Y}|}{c_0})$$
$$\approx \frac{1}{4\pi |\mathbf{x}|} \delta\left(t - \tau - \frac{|\mathbf{x}|}{c_0}\right) + \frac{x_j Y_j}{4\pi c_0 |\mathbf{x}|^2} \frac{d}{dt} \delta\left(t - \tau - \frac{|\mathbf{x}|}{c_0}\right)$$
(5)

where ρ_0 denotes the density of undistrubed flow, c_0 does the velocity of sound in undisturbed flow, and repeated subscript j in the same term means summation with $j = 1 \sim 3$, and Y_j does jth-component of 'Kirchhoff vector **Y**', defined by the following,

$$Y_j(\mathbf{y}) \equiv \mathbf{y}_j - \psi_j^*(\mathbf{y}) \qquad (\mathbf{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}) \tag{6}$$

where ψ_j^* means the instantaneous velocity potential of an incompressible flow past body surface S. Since the high Reynolds number flow is assumed, the viscous term in Equation (4) is neglected. After Equations (5) and (6) are introduced into Equation (4), the following equation for acoustic pressure $p(\mathbf{x}, \mathbf{t})$ is derived.

$$p_{a}(\mathbf{x}, \mathbf{t}) \equiv \mathbf{p}(\mathbf{x}, \mathbf{t}) = \frac{\rho_{0}}{4\pi |\mathbf{x}|} \left[\oint_{S} \frac{\partial v_{n}}{\partial t}(\mathbf{y}, \mathbf{t}) \mathbf{dS}(\mathbf{y}) \right] \\ -\frac{x_{j}}{4\pi |\mathbf{x}|^{2}} \left[\frac{1}{c_{0}} \oint_{S} n_{j} \frac{\partial p_{s}}{\partial t}(\mathbf{y}, \mathbf{t}) \mathbf{dS}(\mathbf{y}) + \frac{\rho_{0}}{c_{0}} \oint_{S} \frac{\partial^{2} \mathbf{v}_{n}}{\partial t^{2}}(\mathbf{y}, \mathbf{t}) \mathbf{Y}_{j}(\mathbf{y}) \mathbf{dS}(\mathbf{y}) \right]$$
(7)

Here v_n means the normal component of the surface velocity induced by finite impedance, and we should note that no translational velocity of the body is supposed. After Fourier transformation is applied to Equation (7), the equation for harmonic component of acoustic pressure $\tilde{p}_a(\mathbf{x}, \omega)$ is derived as follows.

$$\tilde{p}_{a}(\mathbf{x},\omega) = \frac{\rho_{0}e^{ik_{0}|\mathbf{x}|}}{4\pi|\mathbf{x}|} \oint_{S} i\omega \tilde{v}_{n}(\mathbf{y},\omega) \mathbf{dS}(\mathbf{y}) + \frac{e^{ik_{0}|\mathbf{x}|}x_{j}}{4\pi|\mathbf{x}|^{2}} \left\{ \oint_{S} \frac{n_{j}}{c_{0}} i\omega \tilde{p}_{s}(\mathbf{y},\omega) \mathbf{dS}(\mathbf{y}) - \frac{\rho_{0}\omega^{2}}{c_{0}} \oint_{S} \tilde{\mathbf{v}}_{n}(\mathbf{y},\omega) \mathbf{Y}_{j}(\mathbf{y}) \mathbf{dS}(\mathbf{y}) \right\}$$
(8)

where $k_0 = \omega/c_0$ is assumed, and \tilde{p}_s and \tilde{v}_n denote the harmonic component of $p_s(t)$ and $v_n(t)$ respectively. Here we introduce the surface impedance defined by the following.

$$\frac{\tilde{p}_s}{\tilde{v}_n} \equiv Z_n(\mathbf{y}, \omega) \tag{9}$$

We can eliminate $\tilde{v}_n(\omega)$ by introducing $Z_n(\omega)$ into Equation (8) as follows.

$$\tilde{p}_{a}(\mathbf{x},\omega) = -\frac{ik_{0}e^{ik_{0}|\mathbf{x}|}}{4\pi|\mathbf{x}|} \oint_{S} \frac{x_{j}n_{j}}{|\mathbf{x}|} \tilde{p}_{s}(\mathbf{y},\omega) \mathbf{dS}(\mathbf{x}) -\frac{i\omega\rho_{0}e^{ik_{0}|\mathbf{x}|}}{4\pi|\mathbf{x}|} \oint_{S} \frac{\tilde{p}_{s}(\mathbf{y},\omega)}{Z_{n}(\mathbf{y},\omega)} n(\mathbf{y}) \mathbf{dS}(\mathbf{y}) - \frac{\rho_{0}\mathbf{k}_{0}\omega}{4\pi|\mathbf{x}|} \oint_{S} \frac{\mathbf{x}_{j}}{|\mathbf{x}|} \mathbf{Y}_{j}(\mathbf{y}) \frac{\tilde{\mathbf{p}}_{s}(\mathbf{y},\omega)}{Z_{n}(\mathbf{y},\omega)} \mathbf{n}(\mathbf{y}) \mathbf{dS}(\mathbf{y})$$
(10)

The first term in the right side of Equation (10) represents the dipole sound independent on the impedance, and on the contrary the second and third terms represent the effect of the normal velocity on the surface induced by the finite impedance.

3 IMPEDANCE OF SURFACE WITH DISTRIBUTED APERTURES OF SMALL DIAMETER

In this paper very small diameter of the apertures is supposed, for which viscosity effect prevails at the flow through the aperture. The studies on the unsteady flow for this case are reported in literatures[9][10], but experimental approvals for these are not found. In this paper measurement of the impedance conducted by Yahathugoda[11] is shown in Figure 2. The measurement was conducted on the plate of thickness 0.25(mm) with distributed apertures of diameter, $50 \sim 65$ (micron meter), backed with space of depth 0.8(mm). Imaginary part of the impedance ratio $Z_n(\omega)/\rho_0 c_0$ is nearly 3, about ten times



Figure 2: Measurement of impedance on perforated surface with very small diameter

higher than real part of that in the frequency range of 800(Hz). On the other hand Yoda's measurement[12] on a perforated plate with thickness of 0.53(mm) and diameter of 0.2(mm) presents that absorption coefficient of normal incidence α_n is roughly 0.8 in the frequency range of 1000(Hz). Let's define the impedance raio of the surface by $Z_n(\omega)/\rho_0 c_0 \equiv \zeta_n \equiv \alpha_r + i\alpha_i$, then the complex reflection coefficient R is expressed as $R = (\zeta_n - 1)/(\zeta_n + 1)$. Absorption coefficient of normal incidence α_n is written as $\alpha_n =$ $1 - |R|^2 = 4\alpha_r/[(1 + \alpha_r)^2 + \alpha_i^2]$ by using above expression. Using these relationships we can guess that $\alpha_r = 2.619$ is possible for $\alpha_n = 0.8$ assuming $\alpha_i = 0$ for the plate above mentioned. We also guess that $\alpha_r = \alpha_i = 1/\sqrt{2}$ is possible for $\alpha_n = 0.8284$.

4 EXAMPLE OF THE FINITE IMPEDANCE EFFECT AT A CIR-CULAR CYLINDER

Yahathugoda and the author[7] conducted an experiment for examining the effect of the finite impedance to the circular cylinder under U = 20 (m/s) and $Re \equiv 2lU/\nu = 14,200$ applying the cylinder surface with apertures of diameter 50,55,60,65(micron meter). The picture of the apertures is shown in Figure 3, where the separations of the apertures are ten times the diameter. An example of the spectrum of the sound observed at 1.0(m) apart from the cylinder is shown in Figure 4, where solid line denotes spectrum of the cylinder as shown in Figure 3. We can find that the SPL of the cylinder with rigid surface (solid line) is higher by $3 \sim 6(\text{dB})$ than the SPL of that with perforated surface (dotted line) in the frequency range of 200 ~ 800(Hz). This measurement suggests the effectiveness of the finite impedance realized with apertures of very small diamenters.

5 SOUND FIELD FROM THE WING WITH FINITE IMPEDANCE

We suppose a thin wing to which the linear wing theory is applied in this paper. The configuration of the wing is represented by a flat plate with chord length 2a and span length 2L as shown in Figure 5, where the coordinate system is defined. We now discuss the Compact Green's function $Y_j(\mathbf{y})$ in Equation (10). The function is written as follows[2],



Figure 3: Picture of holes

Figure 4: Spectrum of sound from cylinder



Figure 5: Definition of coordinate system for a thin wing

when the complex function is applied at the complex plane $z = y_1 + iy_2$.

$$Y_1 = y_1, \qquad Y_2 = \operatorname{Re}(-i\sqrt{z^2 - a^2}), \qquad Y_3 = y_3$$
 (11)

We now suppose the observation point at $\mathbf{x} = (x_1, x_2, 0)$, $|\mathbf{x}| \equiv R = \sqrt{x_1^2 + x_2^2}$, and the conditions, $2a \ll R$ and $2L \ll R$, are assumed, where the wing spans in $-L \leq y_3 \leq L$. Then we adopt the following approximation at the integration.

$$\oint_{S}(\cdot)dS(\mathbf{y}) = 2L \int_{-a}^{a}(\cdot)dy_1 \tag{12}$$

At integration $\mathbf{n} = (0, n_2, 0)$ and $n_{2u} = -n_{2l} = 1$ are valid on the upper (subscript u) and lower surface (subscript l). Henceforth we assume $Z_{nu}(y_1, \omega) = Z_{nl}(y_1, \omega) \equiv Z_n$ for the surface impedance. Then the following integration is derived from Equation (10).

$$\tilde{p}_a(R,\phi,\omega) = \frac{i2k_0 L e^{ik_0 R}}{4\pi R} \int_{-a}^{a} \left\{ \Delta \tilde{p}_s(y_1,\omega) \sin\phi - \left(\rho_0 c_0 - i\rho_0 \omega y_1 \cos\phi\right) \left(\frac{\Delta \tilde{p}_s(y_1,\omega)}{Z_n}\right) \right\} dy_1 \tag{13}$$

where $\Delta \tilde{p}_s(y_1, \omega) = \tilde{p}_{su} - \tilde{p}_{sl}$ is applied.

To complete integration in Equation (13), the pressure distribution $\Delta \tilde{p}_s(y_1, \omega)$ must be specified. We now suppose only the unsteady pressure fluctuation caused by incidence turbulence flow to the wing. In the paper the linear unsteady wing theory[13] is applied to derive distribution of the pressure fluctuation on the wing surface. When the wing is displaced translationally along y_2 -axis sinusoidally with angular frequency ω or the sinusoidal incidence flow component with small amplitude is added to the main flow U all over the chord length, the following pressure distribution is derived.

$$-\frac{\Delta \tilde{p}_s(x^*)}{\rho_0 U^2} = \tilde{\alpha}(\omega) \left\{ [1 - C(\nu)] \sqrt{\frac{1 - x^*}{1 + x^*}} - \frac{2(1 - x^*)^{\frac{3}{2}}}{\sqrt{1 + x^*}} - \frac{i\nu}{\pi} \left[(1 - x^{*2}) \ln \left| \frac{x^* - 1}{x^* + 1} \right| - 4x^* \right] \right\}$$
(14)

where $\nu \equiv \omega a/U$ and $x^* \equiv y_1/a$ are assumed, $\tilde{\alpha}$ means the converted sinusoidal angle of attack to the wing from the incidence turbulence flow. $C(\nu)$ is defined by $C(\nu) = H_1^{(2)}(\nu)/(H_1^{(2)}(\nu) + iH_0^{(2)}(\nu))$, $H_n^{(2)}(\nu) = J_n(\nu) - iY_n(\nu)$, here $J_n(\nu)$ and $Y_n(\nu)$ mean Bessel and Neumann function of n-th order respectively. After giving the distribution of $Z_n(x^* = y_1/a)$, $\tilde{p}_a(R, \phi, \omega)$ is derived by inserting Equation (14) to Equation (13).

When $Z_n(\omega) = \infty$ is assessed for a wing with rigid surface, the following non-dimensional acoustic pressure of complex quantity is derived.

$$\tilde{p}_a^*(R,\phi,\omega) \equiv \frac{\tilde{p}_a}{\rho_0 U^2} = \frac{i(2k_0 aL)e^{ik_0 R}}{4R} \tilde{\alpha}(\omega)[2+C(\nu)]\sin\phi$$
(15)

Now let's suppose the finite constant impedance along the chord as follows,

$$Z_n(x^*) = -i\rho_0 c_0(\alpha_1 + i\alpha_2) \tag{16}$$

then the following expression for the non-dimensional pressure $\tilde{p}_a^*(R,\phi,\omega)$ is derived.

$$\frac{\tilde{p}_a^*}{\tilde{\alpha}(\omega)} = \frac{(2k_0 a)Le^{ik_0 R}}{4R} \left\{ i[2+C(\nu)] \left(\sin\phi - \frac{1}{\alpha_1 + i\alpha_2} \right) + \left(\frac{a\omega}{c_0} \right) \left(\frac{3}{2} + i\frac{8\nu}{3\pi^2} \right) \frac{\cos\phi}{\alpha_1 + i\alpha_2} \right\}$$
(17)

Finally we should consider the optimal distribution of the impedance along the chord. Optimal impedance should have cancelling effect of the pressure distribution along the chord. Equation (14) indicate that the pressured caused by incidence flow turbulence increases infinitely at the leading edge and it decreases with x^* coordinate to the trailing edge. This suggests the optimal impedance may increase the magnitude with the chord length, therefore the following distribution of the impedance is considered in this paper.

$$Z_n(x^*)/\rho_0 c_0 \equiv \zeta_n(x^*) = -i(1+\beta x^*)(\alpha_1 + \alpha_2), \quad 1/\zeta_n(x^*) \approx i(1-\beta x^*)/(\alpha_1 + i\alpha_2)$$
(18)

The approximation in the second equation is appropriate when $\beta \ll 1$ is valid. The additional term must be added to \tilde{p}_a^* in Equation (17) with the effect of β , but it is neglected in this paper.

6 EXAMPLES OF PREDICTED SOUND FIELD

As a typical case of rotating wing noise, following situation is considered for an example. $U = 30 (\text{m/s}), 2a = 0.1 (\text{m}), f = 240 (\text{Hz}), S_t \equiv 2af/U = 0.8, \nu = 2.513, \omega a/c_0 = 0.2215.$ At first, $\alpha_1 = \alpha_2 = 3.0$ is adopted, which is realistic when remembering the discussion at setion 3, because $\alpha_2 = \alpha_r = 2.619$ is possible for measured normal absorption coefficient $\alpha_n = 0.8$. We should add $\alpha_1 = \alpha_2 = 3.0$ is compatible to $\alpha_n = 0.48$. Secondly $\beta = 0.3$ is adopted as an optimal impedance distribution. Figure 6 shows the distribution of impedance for this case. Finally Figure 7 shows the directivities of the wing noise for three impedance (broken line). We can find remarkable effect of the finite impedance for the dotted line and the broken line comparing the solid line.



Figure 6: Distribution of impedance

Figure 7: Directivity of wing sound

7 CONCLUSIONS

The effect of the surface impedance for reducing the wing noise due to incidence flow turbulence was formulated. Numerical examples were shown to prove the effectiveness of the finite impedance realized by apertures of small diameter.

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