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BIFURCATION ANALYSIS OF A FLEXIBLE ROTOR IN SQUEEZE-FILM DAMPERS WITH RETAINER SPRINGS

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Abstract

Squeeze-film dampers are commonly used in conjunction with rolling-element or hydrodynamic bearings in rotating machinery. Although these dampers serve to provide additional damping to the rotor-bearing system, there have however been some cases of rotors mounted in these dampers exhibiting nonlinear behavior. In this paper a numerical study is undertaken to determine the effects of design parameters, i.e., gravity parameter (W), mass ratio (α) and stiffness ratio (K), on the bifurcations in the response of a flexible rotor mounted in squeeze-film dampers with retainer springs. The numerical simulations were undertaken for a range of speed parameter (Ω) between 0.1 and 5.0. Numerical results showed that increasing K causes the onset speed of bifurcation to increase, whilst an increase of α reduces the onset speed of bifurcation. For a specific combination of K and α values, the onset speed of bifurcation appeared to be independent of the gravity parameter, W. The instability of the rotor response at this onset speed was due to a saddle-node bifurcation for all the parameter values investigated in this work with the exception of the combination of α=0.1 and K=0.5, where a secondary Hopf bifurcation was observed. The speed range of non-synchronous response was seen to decrease with the increase of α; in fact non-synchronous rotor response was totally absent for α=0.4. With the exception for the case of α=0.1, the speed range of non-synchronous response was also seen to decrease with the increase of K. The numerical results presented in this work were obtained for an unbalance parameter (U) of 0.1, which is considered as the upper end of the normal unbalance range of practical rotor systems. These results provide some insights into the range of design parameters of squeeze-film dampers to ensure synchronous rotor response within a specified operating speed range.

INTRODUCTION

The application of squeeze-film dampers are commonly found in aircraft gas turbine engines, whereby these dampers provide additional external damping to the rotor-bearing system for the purpose of reducing the synchronous response of the rotor especially while traversing critical speeds. There are two basic configurations of these dampers, which are the dampers with retainer springs and those without retainer springs. They differ in the way the rotor finds its position in the damper clearance circle. In the damper without retainer spring, the journal that usually lies at the bottom of the clearance circle when the rotor is at rest, is lifted when sufficient imbalance is generated during running conditions of the rotor. In the damper with retainer spring, the journal is fitted with a spring, which often takes the form of a thin ribbed cylinder known as a squirrel cage. The retainer spring is fixed, at one end, to the damper’s journal, whilst the other end is fixed to the damper’s housing. A centralizing mechanism is occasionally used in conjunction with the retainer spring for the purpose of centering the journal in the damper clearance circle.

The stability and unbalance response of a flexible rotor mounted in centrally preloaded squeeze-film dampers has been theoretically and experimentally investigated by Rabinowitz and Hahn [1-3], whereby bistable operation of the rotor was found at certain values of design and operating parameters. Nikolajsen and Holmes [4] observed non-synchronous whirl orbits of period-3 and period-4 in the experimental response of a flexible rotor supported by journal bearings in series with squeeze-film dampers with retainer springs. The effect of fluid inertia for cavitated dampers operating at moderately large squeeze-film Reynolds number has been theoretically observed by San Andres and Vance [5] to possibly reduce or totally eliminate bistable operation and jump phenomena in the response of a flexible rotor mounted in centrally preloaded squeeze-film dampers. Zhao [6] has shown the occurrence of jump phenomena and quasi-periodic motion in the concentric operation of squeeze-film dampers supported flexible rotor. For eccentric operation of the damper, sub-synchronous motion of period-2 was also observed in addition to the jump phenomena and quasi-periodic motion.

In the response of a flexible rotor in squeeze-film dampers without retainer springs, theoretically investigated by Cookson and Kossa [7], it was found that for poorly designed dampers, the maximum force transmitted to the bearing support can be significantly greater than would have been the case if the dampers were not fitted. Non-synchronous response of the rotor was however not observed for the range of parameters investigated in their work. The occurrence of quasi-periodic vibrations has been observed by Rezvani and Hahn [8] in the experimental response of a flexible rotor in squeeze-film dampers without retainer springs. Inayat-Hussain [9-10] has numerically investigated the response of a flexible rotor supported by squeeze-film dampers without retainer springs for a range of practical design and operating parameters. Period-2, period-4 and quasi-periodic vibrations were observed in the response of the rotor. The present paper, which extends the work reported in [9-10], aims to investigate the bifurcations in the response of a flexible rotor mounted in squeeze-film dampers fitted with retainer springs.

THEORETICAL TREATMENT

A flexible rotor system with a mass of $2m_D$ in its mid-span and a mass of m_J at each of its bearing stations is considered in this work. The rotor is mounted on two identical ball bearings that are supported on squeeze-film dampers with retainer springs. The outer race of the ball bearing serves as the journal of the squeeze-film damper. The retainer spring is fixed, at one end, to the damper's journal, whilst the other end is fixed to the damper's housing. The journal is therefore prevented from rotating while still able to perform orbital motion due to the flexibility of the retainer spring. In the derivation of the equations of motion of a flexible rotor in squeeze-film dampers with retainer springs, the following assumptions are made: (i) rotor is symmetric with part of its mass lumped at the rotor mid-span with the remainder at the bearing stations; (ii) rotor speed is constant; (iii) rotor and support stiffness are radially symmetric; (iv) damping force acting on the disc at rotor mid-span due to air dynamics is viscous; (v) unbalanced is defined in a single plane on the disc at the rotor mid-span; (vi) gyroscopic effect is neglected; (vii) retainer springs are linear; (viii) rolling element bearings are rigid and do not introduce significant excitation forces; (ix) short damper approximation based on Reynold's equation for incompressible flow is valid; (x) cavitation is modeled as π-film and therefore the contribution below ambient pressure to the oil-film force is neglected.

It is sufficient to only consider one-half of the system due to it being symmetric. The motion of the system can be described by the displacements $\bar{x}_D$ and $\bar{y}_D$ of the geometric center of the rotor mid-span, and $\bar{x}_J$ and $\bar{y}_J$ of the geometric center of the journal. With the external forces acting on the rotor mid-span and damper journal that include the imbalance force, shaft elastic force, viscous damping force, oil-film forces (F_{D_X} and F_{D_Y}), retainer spring force and gravity, the equations of motion become:

$$\begin{aligned}
m_D \ddot{\bar{x}}_D &= -c_R \dot{\bar{x}}_D - k_R\left(\bar{x}_D - \bar{x}_J\right) + m_D u \omega^2 \cos \omega t \\
m_D \ddot{\bar{y}}_D &= -c_R \dot{\bar{y}}_D - k_R\left(\bar{y}_D - \bar{y}_J\right) - m_D g + m_D u \omega^2 \cos \omega t \\
m_J \ddot{\bar{x}}_J &= F_{D_X} - k_R\left(\bar{x}_J - \bar{x}_D\right) - k_S \bar{x}_J \\
m_J \ddot{\bar{y}}_J &= F_{D_Y} - k_R\left(\bar{y}_J - \bar{y}_D\right) - k_S \bar{y}_J - m_J g
\end{aligned} \tag{1}$$

The pressure (P) distribution in a cavitated (π-film) short squeeze-film damper derived from the Reynold's equation is given in rotating coordinates as follows [10]:

$$P(\theta, z) = \frac{6\mu}{c^2}\left(z^2 - \frac{L^2}{4}\right)\frac{\left(\varepsilon\dot{\phi}\sin\theta + \dot{\varepsilon}\cos\theta\right)}{\left(1+\varepsilon\cos\theta\right)^3} \tag{2}$$

ε and $\dot{\varepsilon}$ respectively denote the displacement and velocity of the journal in the radial direction, and $\dot{\phi}$ denotes the journal's angular velocity. θ is the angular coordinate measured from the position of maximum film thickness in the direction of rotor angular speed. L denotes the length of the damper, c the damper's radial clearance, z the position in the axial direction of the damper, and μ the dynamic viscosity of the lubricant. The resulting oil-film forces due to the motion of the journal in a squeeze-film damper are usually expressed naturally in polar coordinates. The forces in the polar coordinates can be easily transformed into the Cartesian coordinates by the following equations.

$$\begin{aligned} F_{D_X} &= F_r \cos\phi - F_t \sin\phi \\ F_{D_Y} &= F_r \sin\phi + F_t \cos\phi \end{aligned} \tag{3}$$

where F_r and F_t are the radial and tangential oil-film forces, respectively. These forces are obtained by integrating the pressure distribution, given in Eq. (2), over the entire damper surface, and can be expressed as functions of $(\varepsilon, \dot{\varepsilon}, \dot{\phi})$ by Eq. (4).

$$\begin{aligned} F_r &= -\frac{\mu R L^3}{c^2}\left[I_1 \dot{\varepsilon} + I_2 \varepsilon \dot{\phi}\right] \\ F_t &= -\frac{\mu R L^3}{c^2}\left[I_2 \dot{\varepsilon} + I_3 \varepsilon \dot{\phi}\right] \end{aligned} \tag{4}$$

where,

$$\begin{aligned} I_1 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\cos^2\theta}{(1+\varepsilon\cos\theta)^3} d\theta, \qquad & I_2 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\sin\theta\cos\theta}{(1+\varepsilon\cos\theta)^3} d\theta \\ I_3 &= \int_{\theta_1}^{\theta_1+\pi} \frac{\sin^2\theta}{(1+\varepsilon\cos\theta)^3} d\theta, \qquad & \theta_1 &= \tan^{-1}\left(-\frac{\dot{\varepsilon}}{\varepsilon\dot{\phi}}\right) \end{aligned} \tag{5}$$

The integrals I_1, I_2 and I_3 can be evaluated analytically in closed form [11]. θ_1 denotes the angular position of the start of positive pressure region measured from θ. Inserting Eqs. (4) and (5) into Eq. (3), and the result into Eq. (1), and dividing the resulting equation by $m_D \omega_n^2 c$, and substituting the appropriate non-dimensional parameters into the final equation, we obtain the non-dimensional governing equations for a flexible rotor mounted in cavitated squeeze-film dampers with retainer springs.

$$
\begin{aligned}
x_D'' &= -2\zeta x_D' - \left(x_D - x_J\right) + U\Omega^2 \cos\Omega\tau \\
y_D'' &= -2\zeta y_D' - \left(y_D - y_J\right) - W + U\Omega^2 \sin\Omega\tau \\
x_J'' &= -B\left[\frac{x}{\varepsilon}\left[I_1\varepsilon' + I_2\varepsilon\phi'\right] - \frac{y}{\varepsilon}\left[I_2\varepsilon' + I_3\varepsilon\phi'\right]\right] - \frac{1}{\alpha}\left(x_J - x_D\right) - \frac{K}{\alpha}x_J \\
y_J'' &= -B\left[\frac{y}{\varepsilon}\left[I_1\varepsilon' + I_2\varepsilon\phi'\right] + \frac{x}{\varepsilon}\left[I_2\varepsilon' + I_3\varepsilon\phi'\right]\right] - \frac{1}{\alpha}\left(y_J - y_D\right) - \frac{K}{\alpha}y_J - W
\end{aligned}
\tag{6}
$$

The bearing parameter (B) is a measure of the amount of damping that a squeeze-film damper can provide. The unbalance parameter (U), which is a measure of the rotor unbalance, is defined as the ratio of the eccentricity (u) of the rotor center of gravity (G) from its geometric center of rotation, to the radial clearance of the damper. The gravity parameter (W) represents the unidirectional static force acting on the disc at the rotor mid-span, and on the damper journal. K is the ratio of the retainer spring stiffness (k_S) to the shaft elastic stiffness (k_R). α is the ratio of the journal mass (m_J) to the half-mass of the disc at the rotor mid-span (m_D). Ω, the speed parameter, is the ratio of the rotor operating speed (ω) to the rotor pin-pin critical speed (ω_n). ζ is half the viscous damping ratio on the disc at the rotor mid-span and τ is the non-dimensional time.

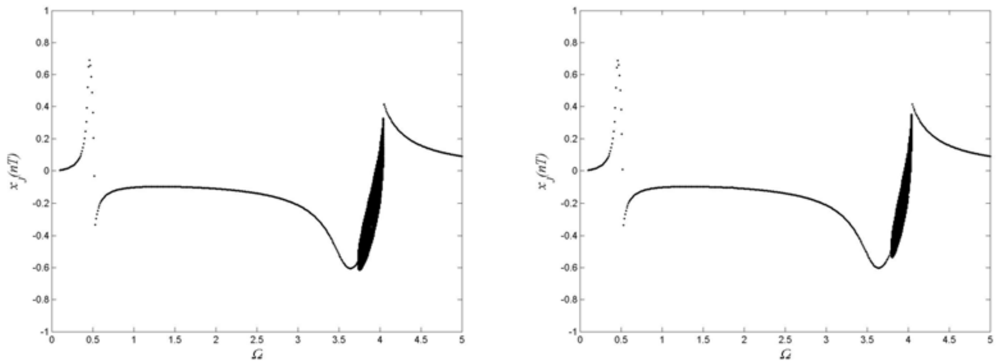
NUMERICAL RESULTS AND DISCUSSION

Direct numerical integration of Eq. (6) was carried out using the MATLAB software package, which utilizes a variable-step continuous solver based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. The results of the numerical simulation are illustrated using bifurcation diagrams. In order to generate the bifurcation diagram, the Poincaré map is first obtained by sampling the trajectory of the journal whirl orbit at constant interval of the forcing period of $T = 2\pi/\Omega$ and projecting the outcome on the $x_J(nT)$ versus $y_J(nT)$ plane. The bifurcation diagram represents the response of the rotor at the bearing station in the X-direction against the variation of the speed parameter (Ω). Ω is then varied at a constant increment of 0.01 and the state variables at the end of the integration are used as the initial values for the solution at the next speed parameter. This ensures a tendency for the integration to follow a single response curve. The variation of $x_J(nT)$ in the Poincaré map with Ω is then plotted to form the bifurcation diagram. The combinations of parameters W, α and K used in the numerical simulation are given in Table 1. Ω is varied from 0.1 to 5.0, whilst parameters U, B and ζ are respectively fixed to 0.1, 0.05 and 0.001. The magnitude of U=0.1 represents the upper end of the acceptable range of balance quality specifications for practical rotors, and the values of the parameters W, α and K used in this work are representative of actual rotating machinery installations [1, 5].

Table 1 – Combinations of parameters W, α and K

W	α	K
	0.1	0.1
0.0	0.2	0.3
	0.4	0.5
	0.1	0.1
0.015	0.2	0.3
	0.4	0.5
	0.1	0.1
0.03	0.2	0.3
	0.4	0.5

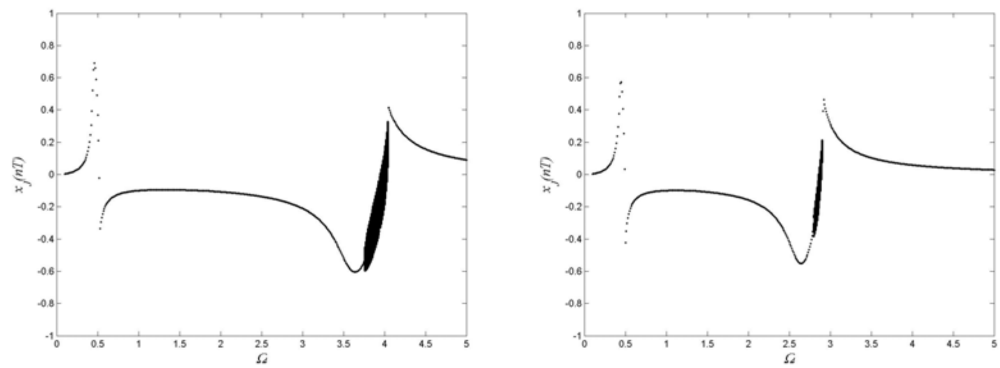
The effects of W on the onset speed of bifurcation and the extent of non-synchronous response within the speed parameter range $0.1 \le \Omega \le 5.0$ is first considered. The numerical results did not show a clear trend of the effect of W on the onset speed of bifurcation. It also appeared that the variation of the values of W had negligible effect on the extent of non-synchronous vibrations within the speed parameter range $0.1 \le \Omega \le 5.0$. It is interesting to note that for the case of a flexible rotor response in squeeze-film dampers without retainer springs, reported in [9-10], an increase in the extent of non-synchronous vibrations was observed as the magnitude of W was increased. A comparison of the rotor response for W=0.0, α=0.1, K=0.3, and W=0.03, α=0.1, K=0.3 is illustrated in the bifurcation diagrams of Figure 1.



(a) (b)

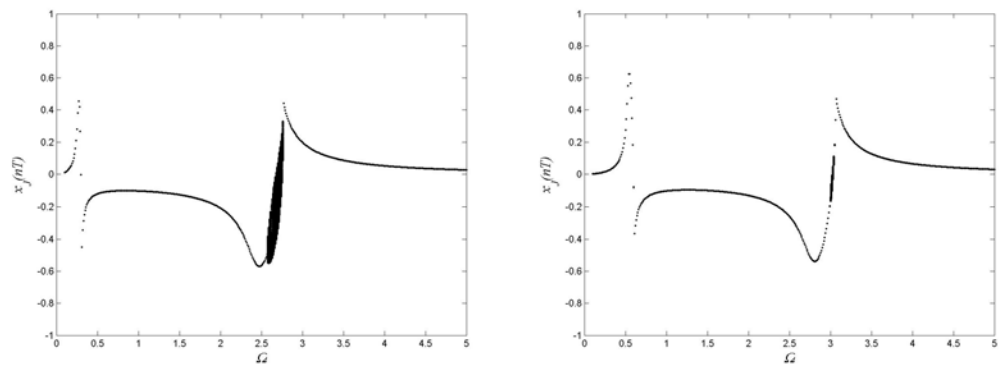
Figure 1 – Bifurcation diagrams for α =0.1 and K =0.3; (a) W =0.0, (b) W =0.03

Figure 2 exhibits the bifurcation diagrams, which compares the rotor response for the case of α=0.1 with that of α=0.2. The values of W and K are 0.015 and 0.3, respectively. An increase in the magnitude of α is seen to cause a reduction in the extent of non-synchronous vibrations within the speed parameter range investigated. An increase in the magnitude of α is also seen to cause the onset speed of bifurcation to decrease, contrary to the case of squeeze-film dampers without retainer springs, where an increase of the magnitude of α resulted in the increase of the onset speed of bifurcation [9-10]. It is also noted that for the case of α =0.4, the response of the rotor was synchronous, irrespective of the magnitudes of W and K.



(a) (b)

Figure 2 – Bifurcation diagrams for W =0.015 and K =0.3; (a) α =0.1, (b) α =0.2



(a) (b)

Figure 3 – Bifurcation diagrams for W =0.0 and α =0.2; (a) K =0.1, (b) K =0.5

A comparison of the rotor response for K =0.1 and K =0.5 is illustrated in the bifurcation diagrams of Figure 3. The values of W and α are 0.0 and 0.2, respectively. It is seen in this figure that the effect of increasing K is to increase the onset speed of bifurcation. Increasing the magnitude of K is also seen to reduce the extent of non-synchronous vibration within the range $0.1 \le \Omega \le 5.0$, with the exception for the case of α =0.1. Secondary Hopf bifurcation was observed at the onset speed for the combination of α =0.1 and K =0.5, irrespective of the magnitude of W, resulting in quasi-periodic vibrations in the response of the rotor. For all other combinations of the parameters W, α and K, saddle-node bifurcation was observed at the onset speed, resulting in the occurrence of the jump phenomena in the rotor response. The numerical results further revealed that the onset speed of bifurcation was largely determined by the parameters α and K; for a specific combination of α and K, the onset speed was found to be independent of the magnitude of W.

SUMMARY

The effects of the gravity parameter (W), mass ratio (α) and stiffness ratio (K) on the bifurcations in the response of a flexible rotor mounted in squeeze-film dampers with retainer springs have been numerically investigated in this work. The results showed that increasing K causes the onset speed of bifurcation to increase, whilst an increase of α reduces the onset speed of bifurcation. For a specific combination of K and α magnitudes, the onset speed of bifurcation appeared to be independent of W. The instability of the rotor response at this onset speed was due to a saddle-node bifurcation for all the parameter values investigated in this work with the exception of the combination of α=0.1 and K=0.5, where a secondary Hopf bifurcation was observed. The speed range of non-synchronous response was seen to decrease with the increase of α; synchronous rotor response was observed for α=0.4 irrespective of the magnitudes of W and K. With the exception for the case of α=0.1, the speed range of non-synchronous response was also seen to decrease with the increase of K. The numerical results presented in this work were obtained for an unbalance parameter (U) of 0.1, which is considered to be at the upper end of the acceptable range of balance quality specifications for practical rotor systems.

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