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COMPARISON OF THE VIBROACOUSTIC BEHAVIOUR OF A RECTANGULAR THIN PLATE EXCITED BY A DIFFUSE SOUND FIELD AND A TURBULENT BOUNDARY LAYER

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Abstract

In the context of understanding and predicting airborne sound transmission through aircraft structures typically a diffuse sound field excitation is used. Numerous studies on the vibroacoustic behaviour of aeronautical structures under a diffuse sound field excitation or a turbulent boundary layer have been published but have been limited to a frequency range not including or going above the structural critical frequency f_c. For some structural components of the cockpit the critical frequencies are around 2000 Hz, means the sound transmission characteristic at and above critical frequency can have an impact on the overall sound quality of the cockpit.

In this paper we therefore focus on the vibroacoustic response of a rectangular plate, using the two types of excitations, up to $3f_c$. A study is made, using a modal expansion to describe the vibrations of the plate.

The diffuse field excitation is obtained through the standard summation of plane waves, and the turbulent boundary layer excitation is based on Corcos' model with Davies' approximation. The focus is on the physical mechanisms producing vibration leading to noise radiation. One can see that the insertion loss obtained with diffuse field excitation is different to the insertion loss obtained with a turbulent boundary layer excitation, even if the insertion loss curve shapes are quite similar particularly at and above the critical frequency f_c. Another important result is that the plate structural damping transmission loss sensitivity varies from one case to the other below f_c but remains the same above it. One can conclude that using the real excitation is compulsory to make precise assessment of interior noise due to turbulent boundary

layer excitation below f_c. Consequently the identified dependency on a simplified case has to be verified on more complex structures and to be compared against tests.

INTRODUCTION

Several authors studied the vibroacoustic behaviour of plates under a diffuse sound field excitation and more recently GRAHAM [1], MAURY & al [2] used a wave-number approach and a turbulent boundary layer excitation. At low frequency, the main physical mechanisms involved in the acoustic transmission through the plate were studied. The wave number approach was applied to a limited frequency range.
Through calculation below and above the critical frequency f_c, this paper compares the vibroacoustic behaviour of a plate excited alternately with a diffuse field and a turbulent boundary layer. The main objectives are to compare the physical phenomena governing sound transmission in order to determine how valid are reverberant room experiments for acoustic aircraft design.

THEORETICAL MODEL

Model description

In this study we consider a thin rectangular plate of length b and width a simply supported along its edges. For the plate motion, bending, membrane and transverse shear stresses are considered. Pressurization effects are not accounted for. Consequently there are no in-plane tensions.
The plate is set in a rigid baffle and is surrounded by two semi-infinite light fluid media. The plate is assumed to be weakly coupled to the surrounding media, which means that its motion is not affected by the pressures generated by its vibrations.
Two excitation fields will be considered: a diffuse sound field and a turbulent boundary layer fully developed when exciting the plate. The classical Corcos model [4] will be used for the latter. We assume that the wall-pressure fluctuations are not modified by the plate radiation for both cases, which means that these fluctuations will be those observed on a rigid wall (blocked pressure).
We will first assess the sound power radiated on the reception medium due to the excitation field in the incident fluid, and then define a vibroacoustic indicator for analysis and comparison.

Acoustic radiated power calculation

Plate modal expansion

The radiated acoustic power is first studied. The x and y axis are located along the edges of the plate, and the z axis is normal to the plate. The classical modal expansion is applied to calculate the displacement w(x,y) along the z axis. The response is given by equation (1) where $w_{mn}(x,y)$ is the modal shape of the eigen-mode (m,n), and $a_{mn}(\omega)$ the modal amplitude. $a_{mn}(\omega)$ corresponds to the ratio of $F_{mn}(\omega)$ the modal force and $Z_{mn}(\omega)$ the modal impedance.

$$W(x,y,\omega) = \sum_m \sum_n a_{mn}(\omega) w_{mn}(x,y)\,,\ a_{mn}(\omega) = \frac{F_{mn}(\omega)}{Z_{mn}(\omega)} \quad (1)$$

In far field, the radiated power Π^{rad} is driven by equation (2) where $Z^{rad}{}_{mn}$ is the modal impedance of radiation given by WALLACE [3] and N_{mn} the eigen-mode squared norm. As it is generally accepted, the cross-terms due to the acoustic coupling are neglected in this case of light fluid. As in our case $N_{mn} = S/4$, one obtains equation (3) for radiated sound power.

$$\Pi^{rad} = ½\, \omega^2\, \Sigma_{mn}\, N_{mn}\, Z^{rad}{}_{mn}(\omega)\, |a_{mn}(\omega)|^2 \quad (2)$$

$$\Pi^{rad} = ½\, S/4\ \ \omega^2\, \Sigma_{mn}\ \ Z^{rad}{}_{mn}(\omega)\ \ |Z_{mn}(\omega)|^{-2}\ \ |F_{mn}(\omega)|^2 \quad (3)$$

One can see that Π^{rad} is directly linked to the squared modal force $|F_{mn}(\omega)|^2$, given by equation (4), $S_{pp}(x,y,w)$ being the power cross-spectrum density of the pressure on the plate surface. The radiated power is thus dependant of the excitation field through the modal force.

$$\left|F_{mn}\right|^2 = F_{mn}\ F^*_{mn} = \int_S S_{pp}(x,y,\omega)\ \ W_{mn}(x,y)\ \ W^*_{mn}(x,y)\ \ dxdy \quad (4)$$

Modal force calculation – Diffuse sound field excitation

The diffuse field is defined as a sum of independent plane waves coming from all the directions of the half space with the same energy. This formulation is not presented here and one refers to the general courses.

The overall modal force for all the plane waves is obtained through an integration, cf. equation (5), each modal force for (θ,φ) incidence being given by equation (6). Note that Spp(w) is the power spectral density of the pressure on the plate surface, identical for each plane wave.

$$\left|F_{mn}\right|^2 = \int_\theta \int_\varphi \left|F_{mn}(\theta,\varphi)\right|^2 \sin\theta d\theta d\varphi \quad (5)$$

$$\left|F_{mn}(\theta,\varphi)\right|^2 = \int_x\int_y S_{pp}(\omega)\, e^{-j\frac{\omega}{c}\sin\theta\cos\varphi x}\, e^{-j\frac{\omega}{c}\sin\theta\sin\varphi y}\, e^{\left(-j\frac{\omega}{c}\sin\theta\cos\varphi x\right)^*}\, e^{\left(-j\frac{\omega}{c}\sin\theta\sin\varphi y\right)^*}\, dxdy \quad (6)$$

Modal force calculation – Turbulent boundary layer excitation

One considers that the flow direction is parallel to the x axis. Using the CORCOS's model [4] and the approximations of DAVIES [5], the cross-spectrum density of wall pressure is given by equation (7).

$$S_{pp}(x,y,\omega) = S_{pp}(\omega) A\left(\frac{\omega x}{U_c}\right) B\left(\frac{\omega x}{U_c}\right) e^{-i\frac{\omega x}{U_c}} \quad (7)$$

$$\text{where } A\left(\frac{\omega x}{U_c}\right) = \exp\left(-\frac{1}{\alpha_1}\left|\frac{\omega x}{U_c}\right|\right) \quad \text{and} \quad B\left(\frac{\omega y}{U_c}\right) = \exp\left(-\frac{1}{\alpha_2}\left|\frac{\omega y}{U_c}\right|\right) \quad (8)$$

Using equations (8) and (4) one obtains the modal force given in equation (9) where k_n and k_m are the wave numbers of the plate : $k_n = \frac{n\pi}{a}$ and $k_m = \frac{m\pi}{b}$. Although this expression is rough at low frequency, it is often used because only four experimental data are needed: the convection speed U_c, the sound power spectral density $S_{pp}(\omega)$, the two constants α_1 and α_2.

$$\left|F_{mn}\right|^2(\omega) =$$

$$\frac{ab}{4}\,\frac{S_{pp}(\omega)}{\omega^2}\,\alpha_1\,\alpha_2\,U_c^2\,\frac{1}{1+\left(\frac{\alpha_2 U_c}{\omega}k_n\right)^2}\left[\frac{1}{1+\alpha_1^2\left(1-k_m\frac{U_c}{\omega}\right)^2}+\frac{1}{1+\alpha_1^2\left(1+k_m\frac{U_c}{\omega}\right)^2}\right] \quad (9)$$

Insertion loss definitions

For a diffuse sound field excitation, the classical insertion loss is the Transmission Loss TL defined by equation (10), Π^{inc} being the incident acoustic power. Π^{inc} corresponds to the power in the reverberant room, far away from the plate surface. According to Sabine's theory, Π^{inc} depends on S_p (Pa^2/Hz), the power spectral density of the acoustic pressure, c the sound celerity (m/s), ρ_0 (kg/m^3) the fluid density, S the plate surface (m^2).

$$TL(dB) = 10\log_{10}\left(\frac{\Pi^{inc}}{\Pi^{rad}}\right) \text{ with } \Pi^{inc} = \frac{S_p}{4\rho_0 c} S \tag{10}$$

For a turbulent boundary layer, there is no incident acoustic pressure in far field. Nevertheless, to compare both excitation fields, we define an equivalent incident power $\Pi^{incidequ}$, which will be the one of a diffuse sound field having a sound power spectral density equal to S_{pp} on the plate surface. This way, the equivalent incident intensity is given by Id = S_p / (4 ρ_0 c) = ½ S_{pp} / (4 ρ_0 c), the equivalent incident power and insertion loss by equations (11) and (12).

$$\Pi^{incidequ} = S_{pp} / 8\, \rho_0\, c \; * \; S \tag{11}$$

$$TLequ = -10\ Log(\ \Pi^{rad} / \Pi^{incidequ}) \tag{12}$$

Characteristic frequencies

We define two characteristic frequencies, which will be useful to analyze the results:

- the aerodynamic coincidence frequency f_{ca}, when the velocity of the flexural waves is identical to the convection velocity U_c (F_{mn} is maximum), $f_{ca} = \frac{1}{2\pi} U_c^2 \sqrt{\frac{M}{D}}$;
- the critical frequency f_c, where the radiation efficiency of the plate is maximum $f_c = \frac{1}{2\pi} c^2 \sqrt{\frac{M}{D}}$.

  Note that M is the surface density of the plate, D its bending stiffness.

EXCITATION EFFECT

Calculation was made for a plate made of duralumin characterized by its Young's modulus E, Poisson ratio ν and mass density ρ. The plate length is equal to 1 m (flow direction in case of turbulent boundary layer excitation), its width is equal to 0.8 m, with a thickness of h.

For the diffuse sound field, we consider standard air characterized by its mass density ρ_0 and a speed of sound c. The turbulent boundary layer is characterized by the convection velocity is U_c and the constants α_1 and α_2. For both cases the modal expansions are made with 138 x 112 modes.

First we consider for the plate a structural damping loss factor of $\eta 1$ for both excitations. The results are shown on figure 1. The characteristic frequencies, when the excitation is maximum (f_{ca}=1200 Hz), and when the radiation is maximum (f_c=6000 Hz) are also indicated.

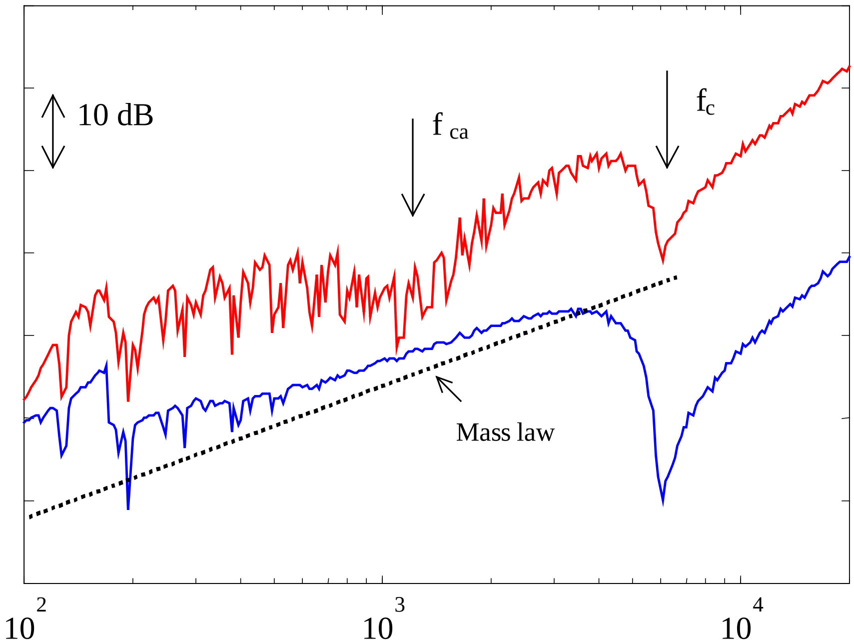

Figure 1 :Influence of the excitation on the Insertion Loss (dB) versus frequency (Blue curve) Diffuse field (Red curve) Turbulent boundary layer

The plate insertion loss under diffuse sound field excitation is very classical: it is driven by mass (non-resonant structural modes) and increases with a 6 dB/octave slope below f_c (mass law), then presents a deep gap at f_c and increases with a 9 dB/octave slope above it. Comparing these results to the ones under turbulent boundary layer excitation, the main results are the following:

- the insertion loss under turbulent boundary layer excitation is far higher to the one under diffuse sound field excitation, the discrepancy is up to 25 dB;
- below $0.5f_c$, and because f_{ca} is lower than f_c, the curves shapes are different due to the minimum at f_{ca} for the turbulent boundary layer excitation;
- above $0.5f_c$, the two curves have the same shape and a constant shift of approximately 25 dB is observed.

The first result is similar to the one obtained by Maury [1], and is due to the fact that the injected power with the turbulent boundary layer is very small compared to the injected power by a diffuse field. The two other results are rather new, Maury's study [2] was limited to low-frequency analysis.

STRUCTURAL DAMPING LOSS FACTOR EFFECT

To better understand sound transmission mechanisms, we made calculation for three different damping loss factors η_1, $\eta_2=\eta_1/10$ and $\eta_3=\eta_1*5$. The results are shown on figures 2 and 3.

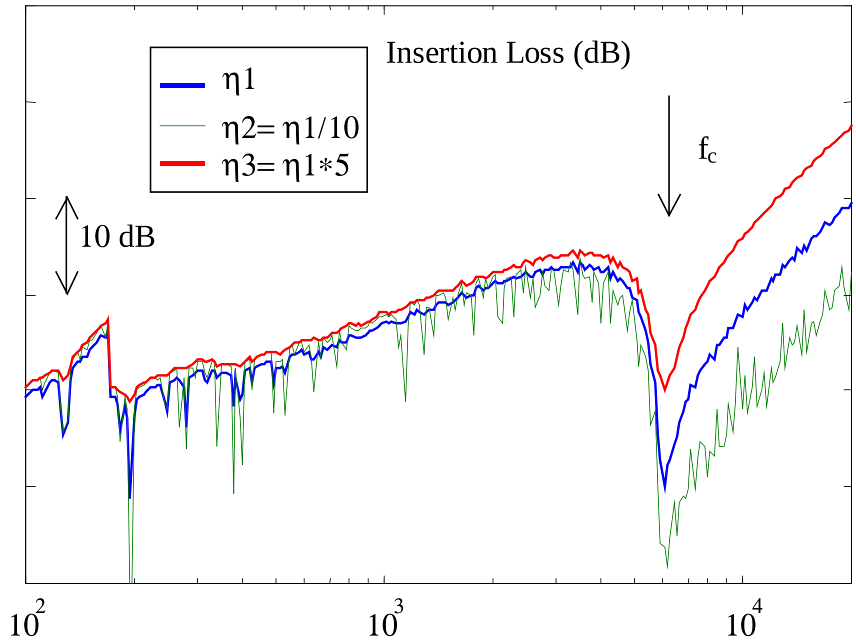

Figure 2: Influence of the structural damping loss factor – Diffuse sound field

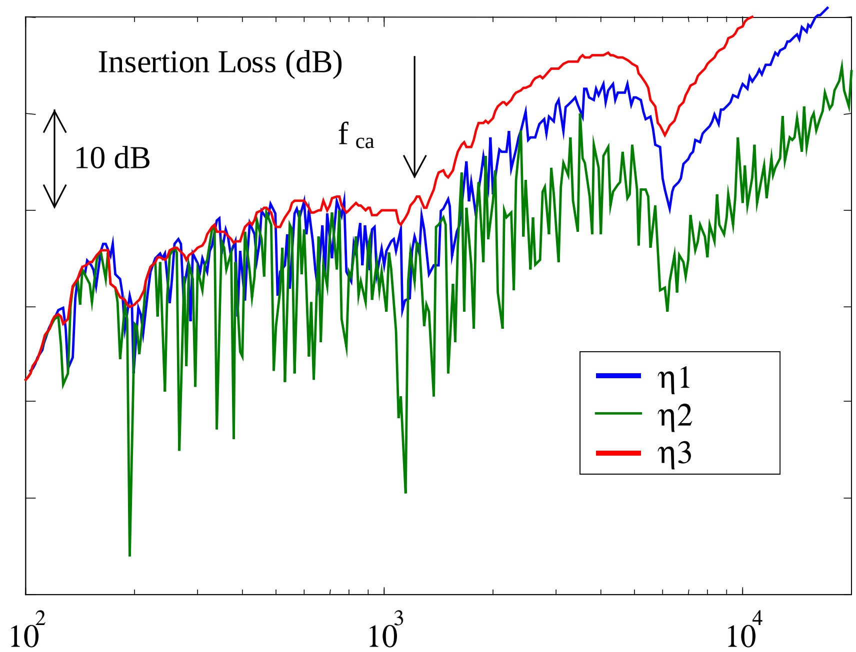

Figure 3: Influence of the structural damping loss factor – Turbulent boundary layer

The main conclusions are:

- below f_c:
    - under diffuse sound field excitation, the structural damping loss factor has a poor effect on the insertion loss, because the sound transmission is driven by modes of low orders, non-resonant, but strongly excited by the acoustic waves having the same wave number;
    - under turbulent boundary layer excitation, the structural damping loss factor has an important effect at and above f_{ca}, indicating that sound transmission is mainly driven by resonant modes;
- above f_c, the structural damping loss factor has the same effect for both excitations, indicating phenomena are similar: resonant modes are dominant in both cases.

CONCLUSIONS

The insertion loss of a plate excited with a diffuse sound field was compared to an equivalent insertion loss when a turbulent boundary layer was used for excitation. The insertion loss curve shapes are quite similar particularly at and above the critical frequency f_c whilst a constant shift (approximately 25 dB) was observed. This shift is due to a low value of the injected power by a turbulent boundary layer compared to the injected power by a diffuse sound field with the same sound power spectral density S_{pp}. At low frequency the transmission mechanisms differ. Below f_c, a diffuse field excites mainly the non-resonant modes with large wave number, when a turbulent boundary layer excites plate resonant modes. Consequently, with a turbulent boundary layer excitation the plate structural damping seems efficient to increase the insertion loss below f_c contrary to the observations made under diffuse field excitation. One can conclude that using the real excitation is compulsory to make precise assessment of interior noise due to turbulent boundary layer excitation below f_c. Nevertheless this result should be confirmed by tests and studies on more realistic configurations.

REFERENCES

[1] W.R. GRAHAM, "Boundary layer induced noise in aircraft", Part 1, Journal of Sound and Vibration 192, 101-120 (1996).

[2] C. MAURY, P. GARDIONO, S.J. ELLIOTT, "A wave number approach to modelling the response of a randomly excited panel", Part 1, Journal of Sound and Vibration 252(1), 83-113 (2002).

[3] C.E. WALLACE, "Radiation resistance of rectangular panel", Journal of Acoustical Society of America, **53**, 946-953 (1972).

[4] G. M. CORCOS, "Resolution of pressure in turbulence", Journal of Acoustical Society of America **35**(2), 192-199 (1963).

[5] H. G. DAVIES, "Sound from turbulent boundary layer excited panels", Journal of Acoustical Society of America, **49**(3 – part 2), 878-889 (1971).