ICSV13 - Vienna

The Thirteenth International Congress on Sound and Vibration

Vienna, Austria, July 2-6, 2006

TUNNING STRATEGY OF KOREAN BELL FOR CLEAR BEAT WITH PROPER PERIOD

H. G. Park, J. M. Lee, S. H. Kim, S. W. Kang

[1,2]School of Mechanical and Aerospace Engineerings, Seoul National University, San 56-1, Shinlim-dong Kwanak-gu, Seoul 151-742 301-215
[3]Department of Precision Mechanical Engineering, Kangwon National University
[4]Department of Mechanical Engineering, Hansung University
E-mail : kilbun@nate.com

Abstract

Clear beat with proper period from low mode sound is very critical factor in vibration and sound characteristics of Korean bell. The clearance and period of beat depend on the position of striking point and asymmetry of bottom of the bell. In this research we expressed Korean bell as a circular ring with multiple point masses and suggested tuning strategy of Korean bell for clear beat with proper period. For tuning, we composed simple equivalent circular ring model with 2 point masses which behave equally to the circular ring model with multiple point masses in mode shapes and natural frequencies of (0,2) and (0,3) mode. And we used that for making clear beat and proper period in Korean bell.

INTRODUCTION

Korean bells, manufactured since the ancient Silla dynasty (57 B.C.}A.D. 935), are unique in shape and have beautiful sounds and appearances. The natural frequencies of bell vibration modes are basically determined by the main dimensions of the nearly axisymmetric body and the material properties of the bell. However, sculptures, carved figures on the body and casting irregularities cause slight asymmetry generating beating sounds which are deemed to be beautiful and desirable in Korean bells. People feel the depth and the liveliness of a bell sound by this beating sound. A clear beat with a proper period is one of critical factors in the appreciation of the Korean bell sound[1]. Low mode vibration characteristics of bell type structure with asymmetry can be analysed by thin circular ring with asymmetry. Low mode moving radial direction dominates sound and that frequency and beat characteristics are determined by thick portion of bottom of the bell. Especially vibration motions of (0,2) and (0,3) mode are rarely not changed by axial length of the bell.[2] So for watching the tendency of mode shape and frequency change of the bell type structure, it is efficient to use circular ring with asymmetry which is similar to low thick portion of the bell in vibration characteristics. Hong and Lee analysed circular ring with asymmetry and then they obtained precise solution considering local mass and stiffness deviations as heavy side step function.[3] They clarified mode separation phenomenon from additional point mass and stiffness reduction by local cut. Fox analyzed mode shape and natural frequency of circular ring with multiple point masses.[4] He used Rayleigh-Ritz method and assumed mode shape didn't change. This method is very efficient in solving inverse problem which can be applied for trimming problem because of relatively simple numerical expression.[5] If we apply equivalent imperfection mass method by fox, it is possible to compose simple equivalent model from circular ring with multiple point masses. On the other hand Kim and Lee acquired vibration wave distribution and draw beat map by analyzing impulse response of circular ring with asymmetry.[2] In this research we made circular ring model with multiple point masses similar to low portion of the Korean bell in vibration motion. We composed simple equivalent circular ring model with 2 point masses which is equal to original model in (0,2) and (0,3) mode in mode shape and natural frequency. Also we suggested new tuning strategies of Korean bell for clear beat with proper period.

CONSIDERATION POINTS IN TUNNING KOREAN BELL

clear beat

When the striking point is placed in the center of low mode and high mode, 2 modes are equally excited and then clear beat is occurred.[6] But it is often that unexpectable asymmetry in casting makes the striking point positioned at one nodal line and produces unclear beat. In this case we should change structure which makes low and high mode nodal lines move and then the striking point can be placed in the center of low and high mode nodal lines. It means when the anti-node of (0,2) mode is placed at 22.5 or -22.5 degree from striking point, clear beat is occurred.[2] We can clarify that by next equation.

$$\begin{aligned}\ddot{u}_{3mn}(x,\theta,t) = -e^{-\zeta_{mna}\frac{\omega_{mnL}+\omega_{mnH}}{2}t}[&\cos n(\theta^{*}-\phi_L)\cos n(\theta-\phi_L)\sin(\omega_{mnL}t) \\ &+\cos n(\theta^{*}-\phi_H)\cos n(\theta-\phi_H)\sin(\omega_{mnH}t)]\end{aligned} \quad 1)$$

(ζ_{mna} : average value of damping, ω_{mnH} : natural frequency of high mode, ω_{mnL} : natural frequency of low mode, θ^{*} : position of striking point, ϕ_L : position of anti-node of low mode, ϕ_H : position of anti-node of high mode)

In Korean bell hum, the first vibration sound lasts the longest that is why its beat characteristic is the most important. And beat characteristic of fundamental, second vibration sound is also significant because it determines striking sound tone and lasts after the hum. So if the striking point is placed in the center of low and high mode nodal lines of first and second mode, both hum and fundamental produce clear beat.

proper beat period

In Korean bell beat period is also important with the clearance of the beat. When the beat period is too long, it makes the feeling of existence of beat weaken. When the beat period is too short, it makes the sound so warble. People can feel the vitality of the sound naturally when beat period is 3 to 4 seconds like period of human breathing.[7] So we should consider the beat period together with the position of nodal line for clear beat sound.

MODELING KOREAN BELL INTO CIRCUALR RING WITH MULTIPLE POINT MASSES

Fox and Rourke acquired mode shapes and natural frequencies of the circular ring with multiple point masses.[5]

$$\tan 2n\psi_n = \frac{\sum_i m_i \sin 2n\phi_i}{\sum_i m_i \cos 2n\phi_i} \tag{2}$$

$$\omega_{n1,2}{}^2 = \omega_{0n}{}^2 \left(\frac{1+\alpha_n{}^2}{(1+\alpha_n{}^2) + \sum_i m_i[(1+\alpha_n{}^2) \pm (1-\alpha_n{}^2)\cos 2n(\phi_i - \psi_n)]/M_0} \right) \tag{3}$$

(n : n-th mode, m_i : i-th point mass, Φ_i : position of i-th point mass, ψ_n : anti-node of low n-th mode, $\alpha_n \approx n$, M_0 : mass of perfect circular ring)

We can express circular ring model with multiple point masses which has same diameter and similar natural frequencies with Korean bell using equation 2) to 3). And we consider this model as the Korean bell.

Table 1. Information of considered circular ring

Information of circular ring	M_0=1684.5kg, ρ=8700kg/m^3, R=1.012m, E=5.6e10Pa, h=0.203m, d=0.15m, m_i=[3, 1, 4, 6, 4]kg, Φ_i=[0, 35, 125, 260, 300]°			
Mode	ω_0	ω_1	ω_2	ψ_1
N=2	64.9503	64.5684	64.6437	14.4887
N=3	183.6178	182.2465	183.0453	7.9993

Table 1 shows mode shapes and natural frequencies of the circular ring model made by those equations. If we know M, total imperfection masses, it is possible to make simple circular ring model with 2 point masses directly. But we first considered complex circular ring model with 5 point masses as a medium stage for checking equivalence. Figure 1(We set up figure 1 at 7th page for comparison with beat map after the tuning) expresses vibration wave distribution and nodal lines of (0,2) low and (0,2) high mode[2]. The center of (0,2) low and (0,2) high mode is positioned by -8 degree from the striking point, so two modes are not excited equally. As the result we can not feel clear beat at striking point and the strength is not strong even at the clear beat zone(4,6,10,12,…). And beat period of hum is about 13.3 second which is too long. So we should grind proper position and move the nodal lines to increase clearance of the beat and shorten the period of the beat. But it is very difficult to find proper grinding position in complex circular ring model with 5 point masses. Simple equivalent model which has same vibration characteristics makes it easy to find proper grinding position. Arrange

SIMPLE EQUIVALENT MODEL WITH 2 POINTS MASSES

Equivalent imperfection masses method by Fox is used for making equivalent model with 2 point masses.

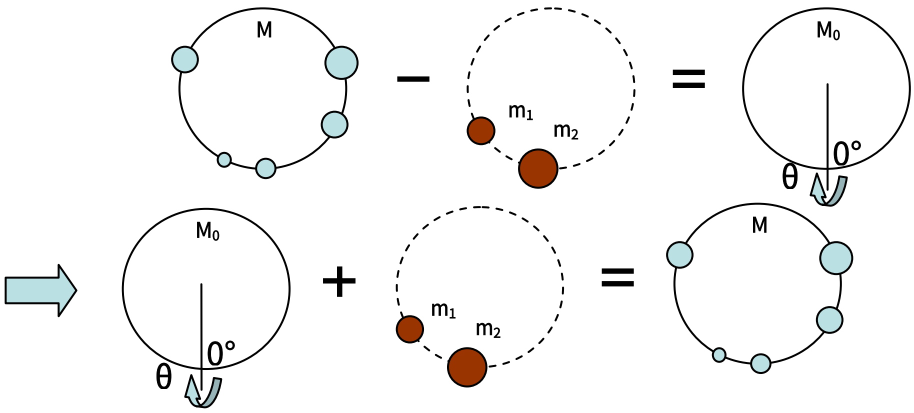


Figure 2. Concept of equivalent modelling using equivalent imperfection masses

In Figure 2, firstly we search for m_1 and m_2 which are subtracted from original circular ring model with 5 point masses for trimming. We can make new model composed of $M_0(= M - \Sigma\, m_i)$ and m_1, m_2. It becomes equivalent model which follows the same mode shapes and natural frequencies with original circular ring model with 5 point masses in (0,2) and (0,3) mode. Here m_1, m_2 are acquired by equation 4) to 7).

$$\lambda_n = \frac{(\omega^2_{n1} - \omega^2_{n2})(1+\alpha^2_n)}{(\omega^2_{n1} + \omega^2_{n2})(1-\alpha^2_n)}, \qquad \delta_1 = -\left(\frac{n_1\psi_{n1} + n_2\psi_{n2}}{n_1 + n_2}\right), \qquad \delta_2 = \frac{n_1\psi_{n1} - n_2\psi_{n2}}{n_2 - n_1} \qquad 4)$$

$$\tan((n_1+n_2)(\phi_1+\delta_1))\tan((n_2-n_1)(\phi_2+\delta_2)) = \frac{\lambda_{n2} - \lambda_{n1}}{\lambda_{n2} + \lambda_{n1}} \qquad 5)$$

$$\tan((n_1+n_2)(\phi_2+\delta_1))\tan((n_2-n_1)(\phi_1+\delta_2))=\frac{\lambda_{n2}-\lambda_{n1}}{\lambda_{n2}+\lambda_{n1}} \qquad 6)$$

$$m_1=M\lambda_{n1}\frac{\sin 2n_1(\phi_2-\psi_{n1})}{\sin 2n_1(\phi_2-\phi_1)},\quad m_2=M\lambda_{n1}\frac{\sin 2n_1(\phi_1-\psi_{n1})}{\sin 2n_1(\phi_1-\phi_2)} \qquad 7)$$

Table 2.Equivalent model with 2 point masses

Information of circular ring	M_0=1689.2kg, ρ=8700kg/m^3, R=1.012m, E=5.6e10Pa, h=0.203m, d=0.15m, m_i=[8.4138, 5.1138]kg, Φ_i=[13.03, 57.0815] °		
Mode	ω_1	ω_2	ψ_1
N=2	64.5679	64.6432	14.4887
N=3	182.2449	183.0439	7.5692

Table 2 shows the acquired the equivalent circular ring model with 2 point masses. For checking equivalence, we compared tendency of change in mode shape and natural frequency for additional point masses between original model and equivalent model. Table 3 shows the results that 2 point masses(4kg, 3kg) are added to same positions(50° , 200°) of each circular ring

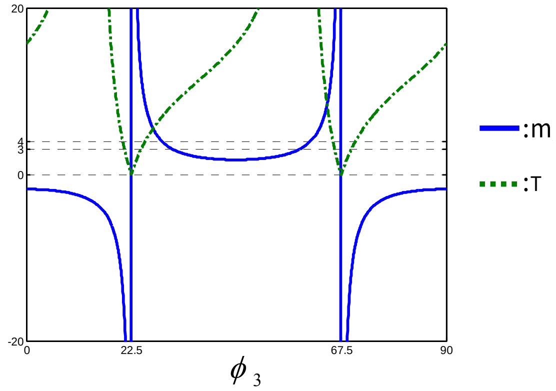
Table 3. The comparison of trend for additional point masses between original model and Equivalent model

Information of circular ring	Original model m_i =[3, 1, 4, 6, 4, 4, 3]kg Φ_i=[0,35,125,260,300,50,200]°			Equivalent model m_i =[8.4138, 5.1138 4 3]kg Φ_i =[13.03, 57.0815 50 200]°		
Mode	ω_1	ω_2	ψ_1	ω_1	ω_2	ψ_1
N=2	64.4215	64.5259	-17.8838	64.4209	64.5254	-17.8837
N=3	181.8836	182.6593	6.5664	181.8820	182.6580	6.5654

In Table 3, two circular ring models show same results in mode shapes and frequencies of (0,2) and (0,3) modes. So circular ring model with 2 point masses is equivalent to the original circular ring model with 5 point masses. It is fine to use simple equivalent circular ring model with 2 point masses instead of complex original model for tuning only (0,2) and (0,3) modes.

MODE TUNNING FOR CLEAR BEAT WITH PROPER PERIOD

Firstly we should think about what position and how much amount we grind for clear beat with proper period. Now the position of anti-node of (0,2) low mode is 14.4887 degree and the beat period of (0,2) mode is 13.3 second which is too long. So we should move the position of anti-node to 22.5 degree or -22.5 degree and shorten beat period to 3 to 4 seconds. For satisfying these two requirements we added the third point mass and find the relation between the magnitude of m_3 and ϕ_3, position of m_3 using equation 1) to 3).



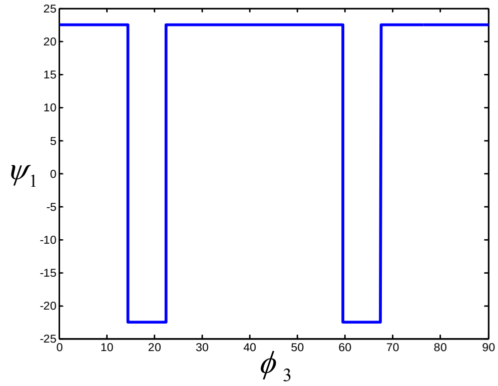


Figure 3. ϕ_3 vs. m_3 and beat *period(T)* *Figure 4.* m_3 vs. antinode of (0,2) low mode

Figure 3. shows the relation between the magnitude of m_3, Φ_3 and beat period(T). Figure 4. shows the relation between Φ_3 and the position of anti-node of (0,2) low mode. In figure 3, we consider the section that m_3 is positive, 22.5 to 67.5 degree. The reason is that grinding is unique tuning technology in reality and the grinding effect is similar to positive point mass effect. In Figure 3 the sections that beat period is 3 to 4 seconds are about 24 to 26 degree and 65 to 66 degree. From figure 4, we can know the anti-node of (0,2) mode moves for 22.5 degree when m_3 is positioned in 24 to 26 degree section. Also it moves for -22.5 degree when m_3 is positioned in 65 to 66 degree section. That is why at the moment anti-node of (0,2) low mode is 14.4887 degree which is closer to 22.5 degree than -22.5 degree. And then less magnitude of m_3 is required when m_3 is positioned in 24 to 26 degree section than m_3 is positioned in 65 to 66 degree section as we can see in Figure 3.

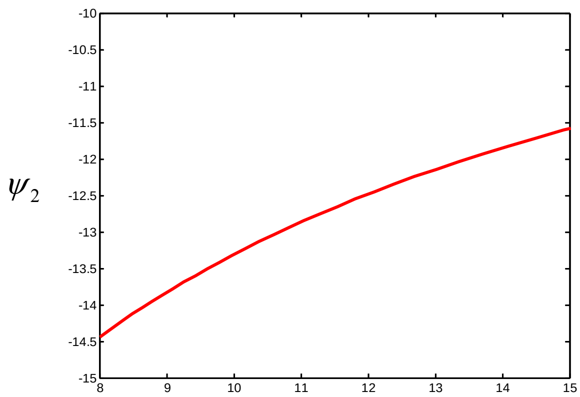


Figure 5. m_3 vs. ψ_2, anti-node of (0,2) mode

Figure 5. shows the relation between m_3 and the position of anti-node of (0,3) low mode considering the result from Figure 3. Finally if we choose the solution that makes beat

period of (0,2) mode 3 to 4 seconds and striking position placed close to the center of nodal lines of both (0,2) and (0,3) modes, it becomes m_3=10.17kg, $\Phi_3=25^\circ$. This solution is applied to simple equivalent circular ring model with 2 point masses as third point mass. Table 4 shows the results.

Table 4. Information of considered circular ring after tuning

Information of circular ring	M_0=1689.2kg, ρ=8700kg/m^3, R=1.012m, E=5.6e10Pa, h=0.203m, d=0.15m, m_i=[8.4138, 5.1138, 10.17]kg, Φ_i=[13.03, 57.0815, 25]			
Mode	ω_1	ω_2	ψ_1	Beat period
N=2	64.2700	64.5578	22.5	3.47
N=3	181.5974	182.6068	-13.2315	0.99

Figure 6 shows vibration wave distribution chart of (0,2) mode after tuning. Compared to Figure 1 beat becomes totally clear around the bell. Especially beat at striking position became very clear and the beat period of (0,2) mode is 3.47 seconds and very identical. In case of (0,3) mode, striking point is positioned in the center of nodal lines of (0,3) low and (0,3) high mode which produces clear beat. Figure 7 shows vibration wave distribution chart of (0,3) mode after tuning.

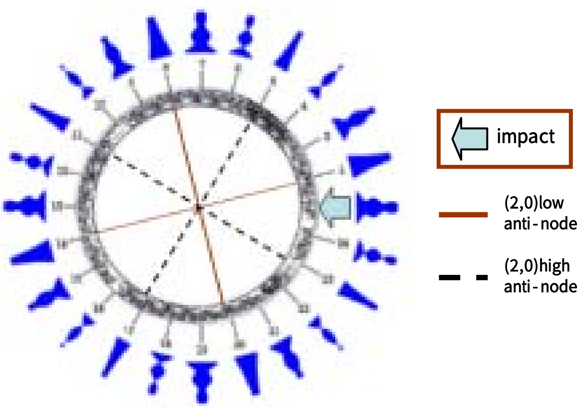


Figure 1. Vibration wave distribution charts of (0,2) mode(before tunning, for 30s)

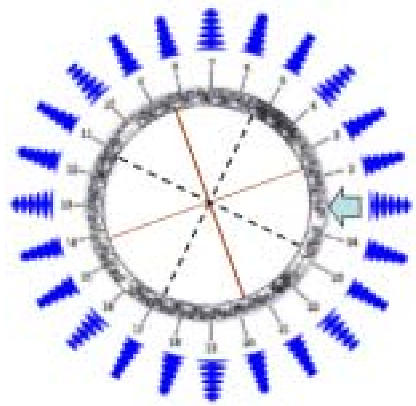


Figure 6. Vibration wave distribution charts of (0,2) mode (after tuning, for 20s)

In these process we can determine the magnitude and the position of additional point mass for clear beat with proper period. In the contrast to point mass, in case of giving cut and reducing the stiffness, it gives the same effect to the movement of nodal line with additional positive point mass. According to theory[2] if we increase the magnitude of additional point mass the nodal line of high mode(anti-node of the low mode) moves towards that position. Similarly if we give a cut to the same position and reduce the stiffness, the nodal line of high mode(anti-node of the low mode) moves towards that position. So tuning strategies with point mass can be used in case of grinding low portion of the bell.

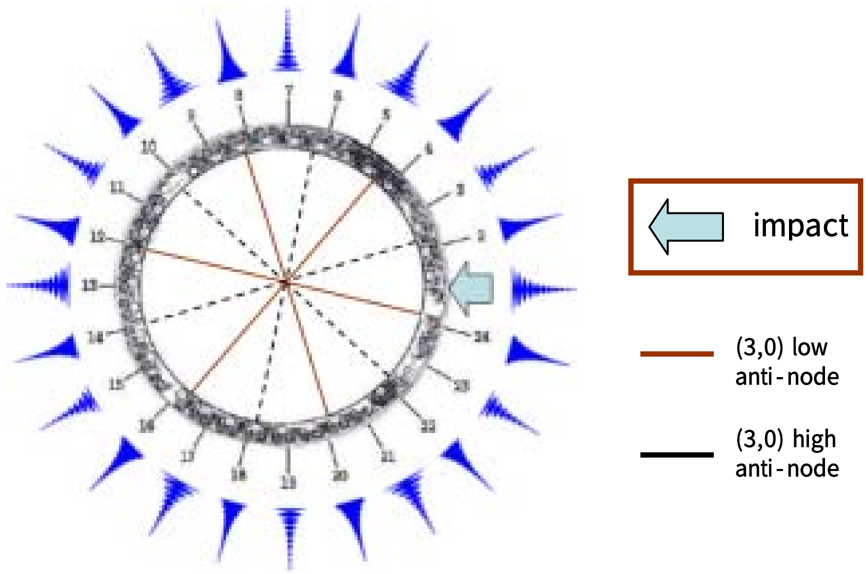


Figure 7. Vibration wave distribution charts of (0,3) mode (after tuning, for 10s)

CONCLUSIONS

The Korean bell is expressed by a circular ring with multiple point masses, similar mode shapes and natural frequencies. From this, a simple equivalent circular ring model with 2 point masses is developed and using this equivalent model we suggested tuning strategy for clear beat with proper period of Korean bell.

REFERENCES

[1| J.M.Lee, S.H.Kim, S.J.Lee, J.D.Jeong, H.G.Choi, "A study on the vibration charactereistics of a large size Korean bell", Journal of Sound and Vibration. 257(4), 779-790 (2002)

[2] S.H.Kim, C.W.Lee, J.M.Lee, "Beat characteristics and beat maps of the King Seong-deok Divine Bell", Journal of Sound and Vibration. 281,21-44 (2005)

[3] J.S.Hong, J.M.Lee, "Vibration of circular rings with local deviation", Journal of Applied Mechanics. 61,317-322 (1994)

[4] C.H.J.Fox, "A simple theory for the analysis and correction of frequency splitting in slightly imperfect rings", Journal of Sound and Vibration. 142(2), 227-243 (1990)

[5] A.K.Rourke, S.McWilliam, C.H.J.Fox, "Multi-mode trimming of imperfect rings", Journal of Sound and Vibration. 248(4), 695-724 (2001)

[6] S.H.Kim, W.Sodel, J.M.Lee, "Analysis of the beating response of bell type structures", Journal of Sound and Vibration. 173(4),517-536 (1994)

[7] Y.H.Yum, "A study on the Korean bell", Research Institute of Korean Spirits and Culture Research Report 84-14 (1984)

[8] J.D.Jeong, J.M.Lee,H.G.Choi, "A study on the vibration characteristics of cylindrical shells with local thickness deviations", Tenth international congress on sound and vibration 7-10 July (2003)