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APPLICATION OF PERIODIC STRUCTURES IN MEMS

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Abstract
Periodic structures can be applied as a MEMS(micro-electro-mechanical system) sensor or actuator due to low energy loss and wideband frequency response. The dynamic behavior of a mistuned periodic structure is dramatically changed from that of a perfectly tuned periodic structure. The effects of mistuning, coupling stiffness, and driving point on the forced vibration responses of a simple periodic structure are investigated through numerical simulations. On the basis of that, one can design effective and reliable MEMS components using periodic structures.

INTRODUCTION

Periodic structures with cyclic symmetry can be found in several engineering systems. Aircraft rotor, turbine blades, satellite antenna dishes, and large space structures are some examples of these structures [1-4]. Recently, such structures are applied as micromechanical devices for communications due to high quality factor Q and broadband characteristics as shown in Fig.1 [5,6]. Practically, the structural identity in substructures is often destroyed by small differences that result from manufacturing and material tolerances. This deviation from the ideal has been observed to cause localization. When localization occurs, energy is confined near the disordered substructure and the dynamic behavior of the structure is drastically changed. In general, most of the previous studies show that mistuning may result in undesirable effects and investigated at the viewpoint of modal response of the overall structure [1-3]. However, when a periodic structure is applied in MEMS, the dynamic behavior of each substructure is more important than that of the whole structure.

In this paper, the combined effects of mistuning and coupling stiffness on vibration behavior of each substructure are investigated. The primary objective of the

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present study is to investigate the effects of mistuning, coupling stiffness, and driving point on the forced vibration response of each mass in the structure. Therefore, it can be used as valuable guidelines for effective and reliable designs of general periodic structures applied in MEMS.

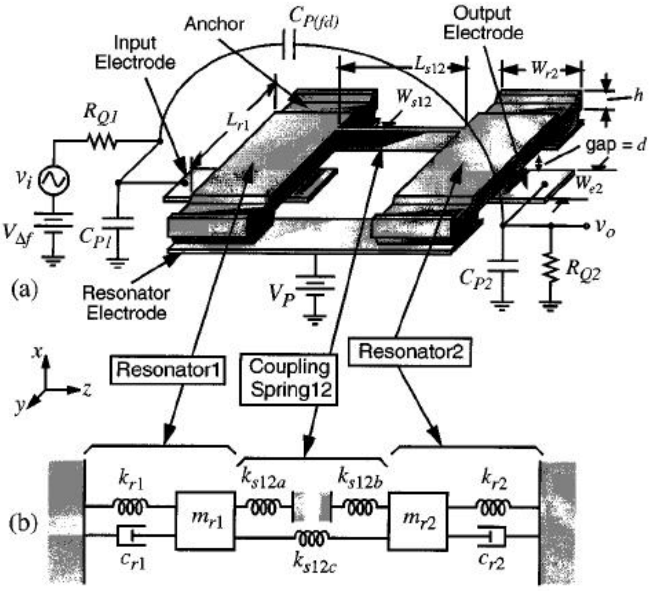

Figure 1 – (a) Schematic diagram of a MEMS filter

(b) Equivalent mechanical model

A SIMPLE MODEL OF A PERIODIC STRUCTURE

A periodic structure consists of identical substructures that are joined to each other in a consistent manner to form the overall structure. Practically, ideal periodicity of the overall structure is often influenced by small differences in the substructures that result from manufacturing and material tolerances. Fig. 2 shows a simple example of a 2 degree of freedom near-periodic structure. Each substructure has the same mass, slightly different stiffness, and coupled by identical coupling stiffness.

The equation of motion of the structure is

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1 + k_c & -k_c \\ -k_c & K_2 + k_c \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{1}$$

Eq. (1) can be rewritten with parameters as follows.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \omega_{n1}^2 \begin{bmatrix} 1+\alpha & -\alpha \\ -\alpha & \beta+\alpha \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2}$$

where $\omega_{n1} = \sqrt{\dfrac{K_1}{m}}$, $\alpha = \dfrac{k_c}{K_1}$, $\beta = \dfrac{K_2}{K_1}$.

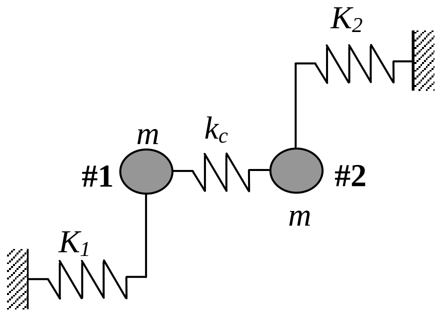

Figure 2 – A 2-degree-of-freedom near-periodic structure

The natural frequency ratios of Eq.(2) are

$$r_1 = \frac{\omega_1}{\omega_{n1}} = \left[\frac{1}{2}\{1+\beta+2\alpha-\delta\}\right]^{\frac{1}{2}} \tag{3}$$

$$r_2 = \frac{\omega_2}{\omega_{n1}} = \left[\frac{1}{2}\{1+\beta+2\alpha+\delta\}\right]^{\frac{1}{2}} \tag{4}$$

where $\delta = \sqrt{4\alpha^2 + (1-\beta)^2}$, ω_1 and ω_2 are natural frequencies of the system. The modal matrix is

$$U = \begin{bmatrix} 1 & 1 \\ \dfrac{1}{2\alpha}(1-\beta+\delta) & \dfrac{1}{2\alpha}(1-\beta-\delta) \end{bmatrix}. \tag{5}$$

The columns of the modal matrix are the actual mode shapes.

Modal characteristics of the tuned system

If substructures are exactly same (β=1), then natural frequency ratios are

$$r_1 = 1 \tag{6}$$

$$r_2 = (1+2\alpha)^{\frac{1}{2}} \tag{7}$$

The modal matrix is

$$U = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{8}$$

Eq. (6) and (7) show that the first natural frequency ratio is independent of the coupling stiffness ratio α and the space between two natural frequencies depends on α. Eq. (8) means that, in the first mode, both masses vibrate in the same amplitude and same direction whereas each mass vibrates in the same amplitude and opposite direction in the second mode.

Modal characteristics of the mistuned system

In this section, the effect of mistuning on the dynamic characteristics of the 2 degree-of-freedom system is examined. The sensitivity of the natural frequency ratios to stiffness imperfection ratio β are obtained by taking the derivative of r^2 with respect to β for the simplicity.

$$\frac{\partial(r_1^2)}{\partial\beta} = \frac{1}{2} + \frac{1}{2}\frac{(1-\beta)}{\delta} = \frac{1}{2} + \frac{1}{2}\frac{(1-\beta)}{|1-\beta|\sqrt{1+\left(\frac{2\alpha}{1-\beta}\right)^2}} \tag{9}$$

$$\frac{\partial(r_2^2)}{\partial\beta} = \frac{1}{2} - \frac{1}{2}\frac{(1-\beta)}{|1-\beta|\sqrt{1+\left(\frac{2\alpha}{1-\beta}\right)^2}} \tag{10}$$

In case of $\beta \rangle 1$ and $\frac{2\alpha}{\beta-1} \langle\langle 1$, $\frac{\partial(r_1{}^2)}{\partial\beta} \cong 0$ and $\frac{\partial(r_2{}^2)}{\partial\beta} \cong 1$, which mean that the square of the first natural frequency ratio doesn't change as the stiffness imperfection ratio changes, whereas the square of the second natural frequency ratio changes as much as the stiffness imperfection ratio changes. In case of $\beta < 1$ and $\frac{2\alpha}{1-\beta} \langle\langle 1$, $\frac{\partial(r_1{}^2)}{\partial\beta} \cong 1$ and $\frac{\partial(r_2{}^2)}{\partial\beta} \cong 0$, which are the opposite situation.

The sensitivity of the mode shapes to stiffness imperfection ratio β are

$$\frac{\partial(u_{21})}{\partial\beta} = \frac{1}{2\alpha}\left(-1-\frac{1-\beta}{|1-\beta|\sqrt{1+\left(\dfrac{2\alpha}{1-\beta}\right)^2}}\right) \tag{11}$$

$$\frac{\partial(u_{22})}{\partial\beta} = \frac{1}{2\alpha}\left(-1+\frac{1-\beta}{|1-\beta|\sqrt{1+\left(\dfrac{2\alpha}{1-\beta}\right)^2}}\right). \tag{12}$$

In case of $\beta \rangle 1$ and $\frac{2\alpha}{\beta-1} \langle\langle 1$, $\frac{\partial(u_{21})}{\partial\beta} \cong 0$ and $\frac{\partial(u_{22})}{\partial\beta} \cong -\frac{1}{\alpha}$, which mean that the amplitude ratio of each masses at the first mode doesn't change as the stiffness imperfection ratio changes, whereas at the second mode, the amplitude ratio changes $\frac{1}{\alpha}$ times as much as the stiffness imperfection ratio changes. In case of $\beta < 1$ and $\frac{2\alpha}{1-\beta} \langle\langle 1$, $\frac{\partial(u_{21})}{\partial\beta} \cong -\frac{1}{\alpha}$ and $\frac{\partial(u_{22})}{\partial\beta} \cong 0$, which are the opposite situation.

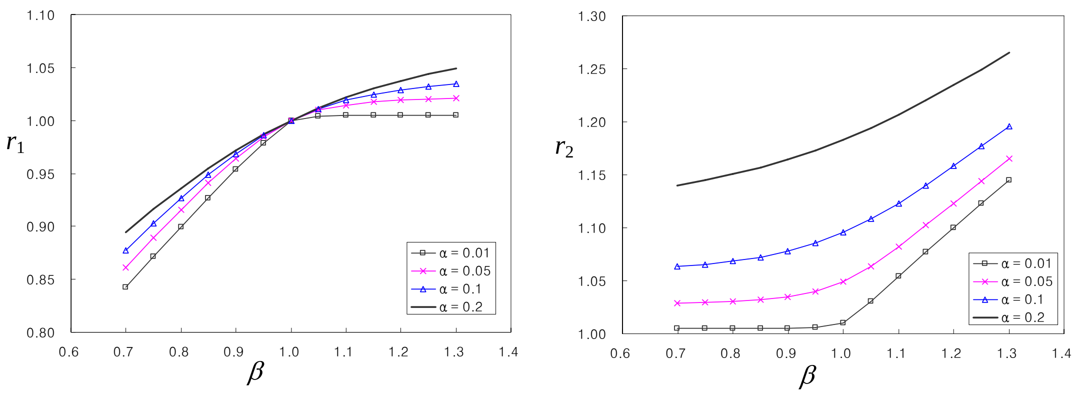

Figure 3 – Frequency ratios r due to β

Fig. 3 shows the variations of natural frequency ratios as α and β change. As we expect from Eq. (9) and (10), when β<1 we get a linear relationship between the r_1 and β with small α(=0.01), whereas, when β>1, the r_1 doesn't change with small α. The r_2 shows similar characteristics with r_1. The mode shapes in Fig.4 shows similar characteristics as α and β change.

Consequently, if we apply this characteristics as a sensor in which one of substructure stiffness values varies with a physical input, we can measure the physical phenomena by measuring changes in the natural frequency and response amplitude.

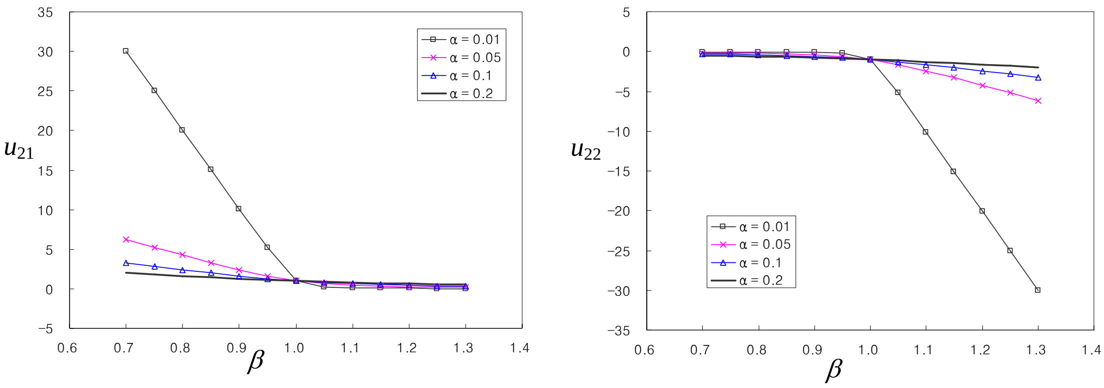

Figure 4 – Mode shapes due to β

FORCED RESPONSE OF THE SIMPLE MODEL

When we consider damping C and force F in substructures, the equation of motion of Fig. 2 is

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1 + k_c & -k_c \\ -k_c & K_2 + k_c \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{13}$$

The steady-state response of Eq. (13) due to harmonic forces of frequency ω can be expressed as follows.

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{bmatrix} (1+\alpha) - r^2 + j2\varsigma r & -\alpha \\ -\alpha & (\beta+\alpha) - r^2 + j2\varsigma r \end{bmatrix}^{-1} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \tag{14}$$

where $\varsigma = \dfrac{C}{2\sqrt{mK_1}}$, $f_{1,2} = \dfrac{F_{1,2}}{K_1}$, $r = \dfrac{\omega}{\omega_{n1}}$ and X is the response amplitude.

The tuned system

The effects of the coupling stiffness on the forced vibration response of the tuned system are investigated. Fig. 5 shows the amplitude of each mass, when a harmonic force is subjected to #1 mass (f_1=1, f_2=0) with the damping factor ζ=0.01. It shows that as α increases from 0, the maximum amplitude of #1 mass goes down whereas that of #2 mass goes up, which means that input energy cannot transfer from #1 mass to #2 mass in case that coupling stiffness is weak comparing to damping factor.

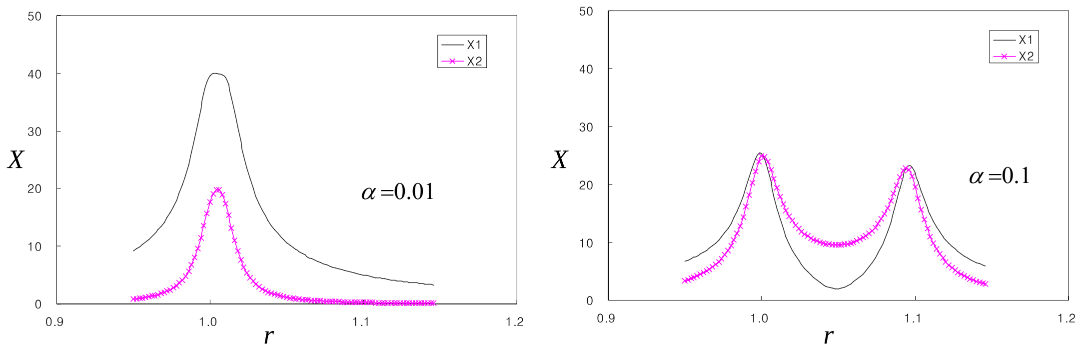

Figure 5 – Forced responses of the tuned system

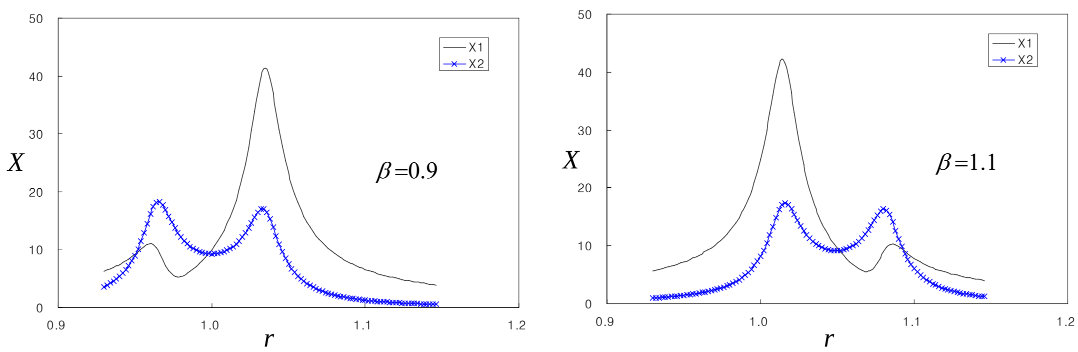

Figure 6 – Forced responses of the mistuned system with α =0.05

The mistuned system

The effects of mistuning on the forced vibration responses are investigated. Fig. 6 shows amplitudes of each mass, when a harmonic force is subjected to #1 mass. The coupling stiffness ratio α=0.05 and damping factor ζ=0.01 are used for numerical simulation.

When β=1.1, the amplitude of mass #1 is a little higher than that of #2 because all the motional energy of #1 can not transfer to #2 due to damping. When β=1.1, the natural frequency ratios are 1.015 and 1.082 from Eq. (3) and (4), and u_{21}= 0.4, u_{22}= –2.4 from Eq. (5). At the first mode, the amplitude ratio between masses satisfies Eq. (5) whereas the second mode doesn't satisfy it. It means that if the exciting frequency is not close to the natural frequency of the #1 substructure, then the vibration energy of #1 mass is not enough to excite #2 mass because of damping. When β =0.9, the figure shows a sort of symmetrical phenomena to the former case.

Consequently, when the forcing frequency is close to the natural frequency of the excited substructure, we can get larger amplitude of the excited mass and the motional energy is enough to excite the entire structure in its mode shape.

CONCLUSIONS

The effects of mistuning, coupling stiffness, and driving point on the forced vibration response of each mass in a periodic structure with 2 degree-of-freedom are investigated.

In case of weak coupling stiffness, there is a linear relationship between variations of substructure stiffness and natural frequency, mode shape. Consequently, this structure can be applied as a MEMS sensor if the substructure whose stiffness varies with the input physical condition is excited as a driving point. Then we can get larger response amplitude in addition to linear variations of the frequency and amplitude than the conventional structures. And it is found that vibration localization can even occur in the perfect periodic system with relatively weak coupling stiffness to damping factor. So, high Q factor is necessary when this structure is used as a MEMS filter.

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