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SENSITIVITY TO THE SECONDARY PATH MODELING ERRORS OF THE BANDWIDTH LIMITED MODIFIED FX-LMS ALGORITHM

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Abstract
One of the major issues in broadband feedforward active noise control is secondary path inversion. The Multi Input Multi Output Inversion Theorem (MINT) states that it is possible to invert non-minimum phase transfer functions, as long as there are more input channels than output channels, and that there are no common zeros in the paths. In active noise control this means that apart from true delays in the system, non minimum phase secondary paths can be inverted as long as there are more anti-noise sources than error sensors. However, this violates a common rule of thumb that is used to guarantee a reasonably large quiet zone: that the number of error sensors should be twice the number of anti-noise sources. In previous papers a new technique that limits the control bandwidth was proposed to prevent the reduction of the quiet zone size for systems with more anti-noise sources than error sensors. In this paper the new technique is analyzed for sensitivity to secondary path modeling errors. For no secondary path modeling errors the effect of the secondary path is actually removed and the algorithm has exact the same convergence properties than the underlying algorithm used to update the control filter, namely the LMS, RLS, LSL or other. It is shown that the new structure is less sensitive to secondary path modeling errors than the FX-LMS, or the MFX-LMS algorithm. In fact it is shown that the allowed secondary modeling errors increase as the control bandwidth decreases. The results are obtained using narrow band analysis which results in a big simplification but still maintains enough information about the dynamics of the system to give useful results. Finally the results are confirmed throw computer simulations.

INTRODUCTION

Active noise control (ANC) system generate sound waves with opposite phase to the noise (anti-noise) that interfere destructively with it to reduce the noise level [1,2] in a given region called "quiet zone". A feedforward active noise control systems are represented in Fig. 1. The reference sensors measure the noise signals, $x_i(t)$, at some points so that it can be used to predict the noise signal at the error sensors. This signal is sampled and digitally processed by a digital signal processor (DSP), which implements the ANC controller. The ANC controller produces the anti-noise signals, $y_j(t)$. These signals are then fed to the anti-noise transducers and travel through the physical medium, usually air, to the error sensors. The error sensors measure the error signals, $e_k(t)$, which are used to adapt parameters of the ANC controller. The path traveled by the anti-noise signal, from the transducers to the error sensors is the cancellation or secondary path, $S(f)$.

In order to achieve larger quiet zones, many active noise control systems are multi-channel, with several reference sensors, anti-noise transducers and error sensors. The system represented in Fig. 1 have N reference sensors, P anti-noise sources and M error sensors. Fig. 1 represents the Modified Filtered X Least Mean Squares algorithm (MFXLMS) [3,4,6].

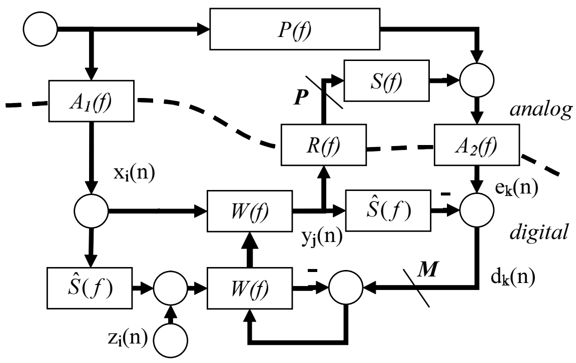

Figure 1: MFXLMS algorithm for feedforward Active Noise Control.

It should be noted that the cancellation path estimate, $\hat{S}(f)$, includes the anti-aliasing and reconstruction filter. In the MFXLMS algorithm the anti-noise signal is subtracted from the error signal $e_k[n]$, resulting in $d_k[n]$, which is then used as a desired signal to the LMS algorithm. Thus, the effect of the cancellation path is actually removed, as along as the estimate is accurate. It will allow the implementation of the anti-noise band limiting action.

It can happen that the quiet zone of an active noise control system degenerates, resulting in very small quiet zone around the error microphones. In order to prevent this many systems use several error sensors per anti-noise source, two as a rule of thumb, so that if one anti-noise source is able to achieve noise reduction in two sensors then it should be creating a fairly large quiet zone. However, in order to achieve better cancellation path inversion, it is desirable to have more anti-noise transducers than error sensors, as stated by the Multi Input Multi Output Inversion Theorem (MINT) [7]. It is a know fact that the dimension of the quiet zone is related to the wavelength of the noise under consideration. For example, in narrowband single-channel active noise control systems in a diffuse sound field the quiet zone has a radius of about one tenth of the wavelength, $\lambda/10$ [8]. So, in order to solve this contradictory requirements, we propose techniques to limit the control bandwidth of the system (the maximum anti-noise frequency), that will prevent quiet zone degeneration while still allowing the application of the MINT theorem.

LIMITING THE ANTI-NOISE FREQUENCY

In order to limit the anti-noise signal frequency we propose to use the modified filtered-x configuration to change the desired signals (which correspond to the desired anti-noise signals) to a band-limited version of the originals. This is presented in Fig. 2. In this configuration the band limiting action is implemented by the controller. It will adapt into a band limiting filter that removes the high frequency components of the anti-noise signal, i.e., the controller will adapt to minimize the error signal,

$$S_{xx}(f)\,P(f)\,A_2(f)\,F(f) - S_{xx}(f)\,A_1(f)\,e^{-\omega D_{\mathrm{F}} j}\,\hat{S}(f)\,W(f). \tag{1}$$

The optimal filter, disregarding causality constrains, is then given by,

$$W(f) = P(f)/\hat{S}(f)F(f)\,e^{\omega D_{\mathrm{F}} j} \tag{2}$$

with $\hat{S}(f) = \tilde{S}(f)$. This is the band limited version of the optimal controller as desired. Note that D_{F} is the group delay of the filter F, and so $F(f)\,e^{\omega D_{\mathrm{F}} j}$ is a filter with no group delay. Variations of this configuration were presented in [9].

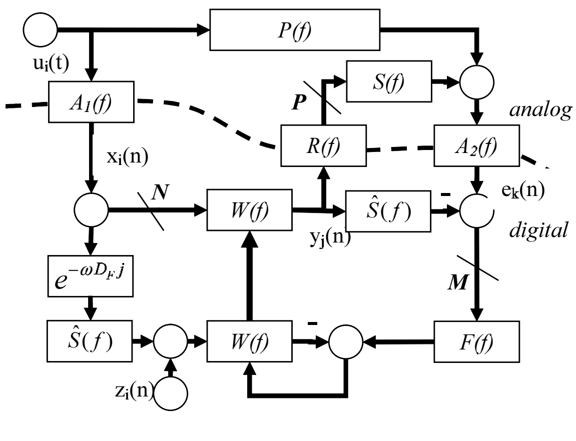

Figure 2: Block diagram of the proposed algorithm, using a filtered desired signal (Configuration I).

EFFECT OF THE CANCELLATION PATH ESTIMATION ERRORS

In order to simplify the results, a one-channel narrowband analysis will be presented. The analysis is similar to the one in [6], but extended to the new algorithms presented in the paper. In frequency-domain analysis [10, 11] only the amplitude and phase response of the system is taken into account. However in active noise control systems, the delay in the cancellation path can have a large influence on the behavior of the systems. In narrowband analysis, the group delay is also taken into account. The signal is taken to be a narrowband amplitude-modulated signal with a given carrier frequency, so the system will alter the carrier phase and delay the modulated signal [12]. More exactly, the secondary path will be defined by amplitude S, phase θ_S and delay d, which correspond to the complex amplitude S_z, and a group delay d, at a given frequency. The band limiting filter will also be defined by the amplitude, phase, and group delay, F, θ_F, D_F. Narrowband analysis results in much simplification, but it still maintains information about the dynamics of the system to provide some useful insight. The equations will be first derived for a sinusoidal input, as follows.

Define the modulated signals, filtered reference signal, $x'[n]$, and innovation signal, $\alpha[n]$, in terms of the baseband signals $x'_z[n] = x'_A[n]\, e^{\phi_x}$ and $\alpha_z[n] = \alpha_A[n]\, e^{\phi_\alpha[n]}$ centered at carrier frequency ω_0,

$$x'[n] = x'_A[n] \cos(\omega_0\, n + \phi_x) \tag{3}$$

$$\alpha[n] = \alpha_A[n] \cos(\omega_0\, n + \phi_\alpha[n]) \tag{4}$$

for all proposed algorithms the L-tap controller filter, $\omega_k[n]$, for k from zero to $L-1$, is updated as,

$$w_k[n] = w_k[n-1] - \mu\, x'[n-k]\, \alpha[n] \tag{5}$$

where μ is the step size of the algorithm. Now we have,

$$x'[n-k]\, \alpha[n] = \frac{1}{2} x'_A[n-k] \alpha_A[n] \left(\cos(\phi_x - \omega_0\, k - \phi_\alpha[n]) + \cos(2\omega_0\, n + \phi_\alpha[n] - \omega_0\, k + \phi_x) \right) \tag{6}$$

the last cosine terms can be neglected for small step sizes, since it is of high frequency and should be filtered by the update equation of the filter. Since the input signal is around frequency ω_0, the controller response at this frequency is of most interest. Calculating the discrete-time Fourier transform of the controller at $z = e^{j\omega_0}$ results in,

$$\hat{w}_z[n] = \sum_{k=0}^{L-1} w_k[n] e^{-j\omega_0 k}, \tag{7}$$

and

$$\hat{w}_z[n] = \hat{w}_z[n-1] - \mu\, \frac{L_x}{2}\, x'^*_z[n - d_L]\, \alpha_z[n], \tag{8}$$

with

$$L_x = e^{j2\phi[n]}\, e^{j\omega_0 (L-1)} \frac{\sin(\omega_0 L)}{\sin(\omega_0)} + L \quad \text{and} \quad d_L = (L-1)/2 \tag{9}$$

For $\omega_0 = k\pi/L$, $L = L_x$. For large L, L_x is approximately real and $L_x \approx L$. This equation was derived taking into account that $x'_A[n]$ is mostly constant. The procedure that follows is similar to the one in [6] with some modifications, namely now taking into account the effect of L_x, d_L, and, most importantly of the band limiting filters. For Configuration I, in Fig. 2, we can write,

$$\hat{w}_z[n] = \hat{w}_z[n-1] - \mu_x\, x_z^*[n - d_L - \hat{d} - D_F]\hat{S}_z^*\alpha_z[n] \tag{10}$$

where $\mu_x = \mu\, L_x/2$. The secondary path model is defined by placing a hat over the corresponding symbols for the secondary path, namely, $\hat{S}$, $\theta_{\hat{S}}$ $\hat{d}$, $\hat{S}_z$. The innovation term is given by,

$$\alpha_z[n] = x_z[n-d-D_F-d_{wo}]w_{zo}S_zF_z + x_z[n-d-D_F-d_w]\hat{w}_z[n-1-d-D_F]S_zF_z$$
$$-x_z[n-\hat{d}-D_F-d_w]\hat{w}_z[n-1-\hat{d}-D_F]\hat{S}_zF_z + x_z[n-\hat{d}-D_F-d_w]\hat{S}_z\hat{w}_z[n-1]+r[n] \tag{11}$$

where d_w is the delay of the controller filter and $r[n]$ is a measuring noise term, which is uncorrelated with the reference signal. In this paper, only the convergence of the mean is going to be studied. Replacing $\alpha[n]$ in (10), taking expected values, and letting $R_{\hat{d}\hat{d}} = E\{x_z^*[n - d_L - \hat{d} - D_F - d_w]x_z[n - \hat{d} - D_F]\}$, $R_{\hat{d}d} = E\{x_z^*[n - d_L - \hat{d} - D_F - d_w]x_z[n - d - D_F]\}$ and taking the Z-transform [13], one obtain,

$$E\{\hat{w}_z(Z)\} = \frac{\mu_x F_z R_{\hat{d}d}\hat{S}_z^* S_z Z}{Z - 1 + \mu_x\, |\hat{S}_z|^2\, R_X}\, w_{zo}(Z) \tag{12}$$

with,

$$R_X = R_{\hat{d}\hat{d}}(1 - F_z Z^{-\hat{d}-D_F}) + F_z R_{\hat{d}d} S_z/\hat{S}_z\, Z^{-d-D_F}. \tag{13}$$

Using this equation, one can express μ_x as a function of Z, $\mu_x = \Gamma(Z)$, where Z is a pole of the system. This means that if a pole of the adaptive filter convergence is known, then it is possible to calculate the step size used by the algorithm. However, in general this function has no inverse, since there are several modes of convergence, or poles, for a given step size. Nonetheless, for the case of very small step sizes, the convergence is dominated by a single pole, near $Z = 1$, and the inverse exists. Using the rule for the inverse of the implicit function, that is,

$$F(\mu_x, Z) = 0 \Rightarrow \frac{\partial Z}{\partial \mu_x} = -\frac{F^1(\mu_x, Z)}{F^2(\mu_x, Z)} \tag{14}$$

one can obtain a linear approximation for very small step sizes. In this equation, $F^1(\mu_x, Z)$ represents the derivative in terms of the first argument and $F^2(\mu_x, Z)$ is the derivative in terms of the second argument of the function. Since, once more, the algorithm is dominated by the pole at $Z = 1$, calculating the partial derivative for $Z = 1$ and $\mu_x = 0$ one gets,

$$Z(\mu_x) \approx Z(0) - \mu_x \frac{\partial Z}{\partial \mu_x} = 1 - \mu_x \left[(1 - F_z)R_{\hat{d}\hat{d}}|\hat{S}_z|^2 + F_z R_{\hat{d}d} S_z \hat{S}_z^*\right]$$

The equation should be analyzed for the case when the signals are inside the noise reduction band, with $F_z = 1$ and outside of the noise reduction band $F_z = 0$. If the reference signal is not narrowband then it can be split into two signals inside and outside of the noise reduction band. We will assume that μ_x is real and greater than zero. This is not actually true because L_x is not always real. The effect of complex valued L_x can be interpreted as reducing the tolerance of the algorithms to phase errors in the cancellation path estimate, in a worst-case scenario, although in practice it can improve or degrade the stability in a rather aleatory way. When the reference signal is outside the noise reduction band, $F_z = 0$, the system is always stable since $R_{\hat{d}\hat{d}}|\hat{S}_z|^2$ is always real and positive. Actually for this signal there is no feedback from the physical system. When the reference signal is inside the noise reduction band, then for the system to be stable $R_{d\hat{d}} S_z \hat{S}_z^*$ must have a positive real component, otherwise the poles will step out of the unit circle. For a narrowband reference system, the values of $x_z[n]$ can be taken to be approximately constant, so $R_{d\hat{d}}$ is approximately equal to R_{dd} and positive real. This implies that the condition for stability is simply that the phase error $\Delta\theta_S = \theta_S - \theta_{\hat{S}}$ must be smaller than 90^o, $|\Delta\theta_S| < 90^o$. This is the same result as the one obtained for the FXLMS algorithm and is in agreement with what would be intuitively expected.

Now it remains to determine the maximum values for the step size, which assures the stability of the algorithm. That is, the value of the step-size that results in the first crossing of the unit circle by a pole. Once again assuming the step size is a real number, a point Z is a pole of the system if $\mu_x = \Gamma(Z)$, as given by (12), is a real number. If the pole is in the unit circle, then, $Z = e^{\theta_Z i}$. The paper now proceeds to determine the values of θ_Z for the poles, and then calculate $\mu_x = |\Gamma(e^{\theta_Z i})|$ to obtain the limiting values of the step-size. To obtain analytical expressions, some simplifications are required, which result in sufficient conditions for stability, but which are not always necessary. It is possible to write,

$$\mu_x \leq \mu_{x\text{MAX}} = \Gamma(e^{\theta_Z i}) = -\frac{2\,j\,\sin(\theta_Z/2)\,e^{\theta_Z/2\,i}}{R_{\hat{d}\hat{d}}\,|\hat{S}_z|^2(1 - F_z\delta_S\,e^{-(\hat{d}+D_F)\,\theta_Z j})} \tag{15}$$

with,

$$\delta_S = 1 - \frac{R_{\hat{d}d}}{R_{\hat{d}\hat{d}}}\frac{S_z}{\hat{S}_z} z^{-\Delta d} \text{ and } \Delta d = d - \hat{d}. \tag{16}$$

Once again for $F_z = 0$ the system is always stable. For $F_z = 1$, a lower bound for the absolute value of $\mu_{x\text{MAX}}$ is,

$$|\mu_{x\text{MAX}}| \geq \frac{2\,\sin(\theta_Z/2)}{R_{\hat{d}\hat{d}}\,|\hat{S}_z|^2(1 + |\delta_S|)}. \tag{17}$$

To determine the values of θ_Z of the poles, one must make the imaginary part of $\mu_{x\text{MAX}}$ equal to zero, as in (18). The smallest values for θ_Z is the one which results in a lower limit for the step-size:

$$\text{Im}\{\mu_{x\text{MAX}}\} = 0 \Leftrightarrow \text{Re}\{(1 - \delta_S\,e^{-(\hat{d}+D_F)\,\theta_Z j})\,e^{-\theta_Z/2\,j}\} = 0 \tag{18}$$

which is equivalent to,

$$\cos\left(\frac{\theta_Z}{2}\right) = |\delta_S|\,\cos(\phi), \quad \phi = \left(\hat{d} + D_F + \frac{1}{2}\right)\frac{\theta_Z}{2}. \tag{19}$$

If $|\delta_S| > 1$ and if d and $\hat{d}$ are large, then this equation has solutions for small values of θ_Z, which limits the step size to very small values, making the algorithm unstable in practice. Otherwise the equation only has solutions for $\theta_Z > \theta_Z{}'$, with,

$$\cos(\theta_Z{}'/2) = |\delta_S| \Leftrightarrow \theta_Z{}' = 2\cos^{-1}(|\delta_S|). \tag{20}$$

Once again, for large d, small changes in θ_Z, result in large changes in ϕ, namely $\cos(\phi)$ goes from -1 to 1 in only $2\pi/(\hat{d} + D_F + 1/2)$, so the first zero crossing can be taken as $\theta_Z \approx \theta_Z{}'$. Replacing (20) in (17), results in,

$$\mu_x \leq \frac{2}{R_{\hat{d}\hat{d}}|\hat{S}_z|^2}\sqrt{\frac{1-|\delta_S|}{1+|\delta_S|}} \leq \mu_{x\mathrm{MAX}}. \tag{21}$$

This equation results in a lower limit for the step size which guarantees stability. The equation only has solutions for $|\delta_S| < 1$. To determine the greatest cancellation path delay estimation error that permits the algorithm to be stable, one must determine Δd so that $|\delta_S| = 1$. This results, for no phase errors, in the equation

$$\left|1 - \cos(2\pi f/f_A \Delta d)e^{2\pi f/f_A \Delta d j}\right|^2 = 1 \Leftrightarrow \sin(2\pi f/f_a \Delta d) = 1 \Leftrightarrow \Delta d = fa/f/4 \tag{22}$$

SIMULATION RESULTS

The algorithm proposed in Figs. 2 was validated through numerical simulations. It proved to work as expected even in the presence of secondary path modeling errors [9].

Sinusoidal Reference

f_c	f	d_M	d_M(T)
1000 Hz	900	0.5	1.11
500 Hz	450	1.4	2.22
500 Hz	800	∞	∞
250 Hz	200	3.2	5.00

White Noise Reference

f_c	d_M	d_M(T)
1000 Hz	1.1	1.0
500 Hz	1.9	2.0
250 Hz	4.1	4.0

Table 1: Comparison of theoretical and simulation results for the sensitivity to secondary path modeling errors.

Simulations for the sensitivity to secondary path modeling errors were made. The secondary path and primary path were modeled as pure delays corresponding to fractional values of the DSP sampling time. In order to simulate these delays the analogue section was simulated in discrete time, but at a higher sampling rate. The sampling frequency was $f_A = 4$ kHz. The maximum error for the delay of the cancellation path estimate, d_M was measured for different anti-noise band limiting frequencies, f_c and reference frequencies, f, for the case of sinusoidal input and compared to the theoretical predicted value, d_M(T), derived from (22). Table 1 shows the results for a $1 \times 3 \times 2$ system with a sinusoidal reference, and for a $2 \times 3 \times 2$ system with a white noise reference. The results are in close agreement with the theory.

CONCLUSION

In broadband active noise control systems, limiting the anti-noise frequency prevents quiet zone degeneration, without the use of several error signals per anti-noise source. In this paper, techniques to do this for multichannel systems are presented. This techniques increase the stability to secondary path modeling errors, allow the use of low-order and delay anti-aliasing and reconstruction filters, and finally, allow systems with more anti-noise sources than error sensors for better cancellation path inversion while still achieving large quiet zones.

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