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ON NUMBER AND POSITION OF THE EQUIVALENT SOURCES IN SCATTERING BY SIMPLE GEOMETRY BODIES

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Abstract
The present paper investigates properties of the solutions obtained with the equivalent sources method in scattering problems with the aim of identifying suitable source configurations. Simple geometry scatterers are considered — a cube and parallelepipeds with different aspect ratios — and four source supports are tested: linear, 'double linear', circular and elliptical. It was found that the supports providing best solutions differ according to the body geometry and the incidence angle of the impinging wave. Moreover, in the situations in which the other supports fail, the double linear one provides satisfactory solutions with a minimum number of sources and a thumb rule for monopole positioning is proposed in this case. Also, a rule that furnishes, for the cube, the optimal number of sources as a function of *kL*, is given for the circular and the simple linear supports.

INTRODUCTION

Because of its low computational cost, the equivalent sources method (*ESM*) is an interesting alternative to boundary elements method (*BEM*), yielding an approximate solution with a small number of sources. It substitutes the real body by a set of sources located in its interior chosen in order to satisfy the appropriate boundary condition [1-2]. The method main drawback lies in the fact that the solution quality depends strongly on the source location [3], a handicap that can be overcome by using, for instance, a global search tool like genetic algorithms [4]. The aim of the present paper is to point out suitable source configurations for simple geometry bodies, what should allow a simpler and safer use of the *ESM*.

BACKGROUND

The scattering problem due to the impinging of an acoustic wave on a closed surface S surrounded by a region Ω_E with uniform mean density ρ_0, can be described as follows [1]: the complex scattered pressure $p_{sc}(\mathbf{x})$ has to satisfy, in Ω_E, the Neumann boundary value problem described by the Helmoltz equation, the Sommerfeld radiation condition and the prescription, on S, of the normal velocity u_n, expressed by $u_n = - v_n^{inc}$, where v_n^{inc} is the normal velocity that would be generated by the incident wave in the absence of the body. As *ESM* substitutes the body by a set of M sources at points $\mathbf{y}_m$, located inside the body, the sound field due to these sources is expressed in terms of their unknown complex amplitudes, A_m, and of a function, G, describing their radiation. The scattered pressure and velocity at $\mathbf{x}$ can be written as

$$p_{sc}(\mathbf{x}) = \sum_{m=1}^{M} A_m \; G(\mathbf{x},\mathbf{y}_m); \quad v_n^{sc}(\mathbf{x}) = \frac{-1}{\mathrm{i}\omega\rho_0} \sum_{m=1}^{M} A_m \, \partial G(\mathbf{x},\mathbf{y}_m)/\partial n \qquad (1, 2)$$

where $\omega = kc_0$ is the angular frequency, k is the wave number and c_0 is the speed of sound. Since only monopoles will be considered here, G is the free-space Green function, given by $e^{-ikr}/4\pi r$. In most cases, the equivalent source set does not satisfy exactly the boundary condition, a local boundary error ε_v being generated, given by the difference between the velocity due to the sources and the theoretical values. The solution $\{A_m\}$ is obtained by minimizing the global velocity error on S. The minimization technique used here is the least squares method.

NUMERICAL EXPERIMENT AND RESULTS

General description

The results presented below are relative to the scattering of a plane wave by a cube and by a parallelepiped. For the cube, with the edge L given by n times the wavelength λ, different frequencies are investigated (for $n = 0.5$, 1, 1.5 and 2), corresponding to values of kL from π to 4π. For the parallelepiped, with dimensions given by $(\eta \mathrm{x} 1 \mathrm{x} 1)\lambda$, different cases are investigated, according to the aspect ratio η and the incidence angle considered, $\Phi_{inc} = 0$ and $\Phi_{inc} = \pi/2$ (see Figure1). Three simple source supports located in the $z = 0$ plane, which is the scattered field symmetry plane, are used: linear ($\mathcal{L}$), circular ($\mathcal{C}$) and elliptical ($\mathcal{E}$). Their center are coincident with the body geometric center and their size is obtained by multiplying the maximum size acceptable by a *reduction factor* a, $0 < a < 1$. The ratio of the axes of the elliptical support, which is employed only for parallelepipeds, is given by η. Orientation for $\mathcal{L}$ and for the wave vector $\mathbf{k}$ is given by the angle with the x-axis, respectively, Φ_L and Φ_{inc}.

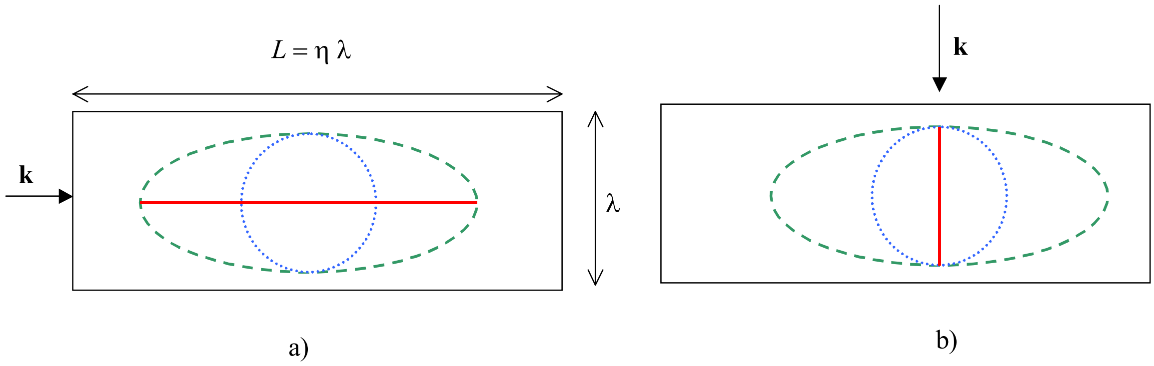

Figure 1: Representation of the parallelepiped and the three supports for the cases Φ_{inc} = 0 (a) and Φ_{inc} = π/2 (b).

For each case (i.e., a given frequency, a support type and size), 24 solutions are computed, corresponding to $M = 2$ to 25. The solution quality is evaluated through the 'boundary error', e_{BC}, defined, for N nodes, by

$$e_{BC} = \sum_{i=1}^{N} \left| \varepsilon_v(\mathbf{x}_i) \right|^2 \Big/ \sum_{i=1}^{N} \left| \bar{u}_n(\mathbf{x}_i) \right|^2 . \tag{3}$$

In each case, data relative to the best solution (over the 24 ones) is denoted by '*'. In a precedent investigation [1] it was shown that two indicators based on $\{A_m\}$ can provide valuable information on the solution quality: $\hat{A}$, the mean value of the source strength magnitudes and Q, which represents the magnitude of the monopole term in the multipole expansion of the equivalent source set, given respectively by

$$\hat{A} = \frac{1}{M} \sum_{i=1}^{M} \left| A_i \right|, \qquad Q = \left| \sum_{i=1}^{M} A_i \right| . \tag{4, 5}$$

These two quantities differ, essentially, in taking or not into account phase effects, which may lead to Q being significantly smaller than $\hat{A}$, due to the high degree of cancellation occurring, the equivalent sources being, in general, arranged like dipoles. It was shown that e_{BC} varies strongly with the number of sources and that $\hat{A}$ always increases with M, although in a manner that differs according to the type and size of the support used. Consequently, selecting the range of M corresponding to good solutions is equivalent to selecting a range of acceptable values for $\hat{A}$.

The effect of the frequency

In order to identify appropriate source sets, the following criterion is used: among the whole set of solutions, those with an e_{BC} value lower than 0.7 are selected. This limiting value has been chosen for corresponding, in the case $L = \lambda$ (i.e., $n = 1$), to a good field reconstitution on a circle with radius $R = 2\lambda$, with about 95% of control points presenting an error < 1 dB. Results show that the best solution obtained with the linear support is always better than the one provided by the circular one (Figure 2) and, that the higher the frequency, the more pronounced the discrepancy between the two supports efficiency (not shown). Regarding Q^*, it was found that, for each frequency, it is almost constant for $a \leq 0.5$; increasing for $a > 0.5$, what indicates a degradation in the solution quality. As for $\hat{A}^*$, it increases with frequency and decreases as the support size increases. It was also found that, while source amplitudes must not exceed a certain upper bound for $\hat{A}$, a minimum value exists below which a good quality solution cannot be obtained. It was verified that given two acceptable solutions, the one with lower e_{BC} presents, in general, the higher $\hat{A}$. Moreover, all the curves '$\hat{A}^*$ x a' can be interpolated by straight lines, their expressions being given in Table 1. This result allows, for a given frequency and support size, an estimation of $\hat{A}$ corresponding to an appropriate equivalent source set.

Table 1: equations for the "$\hat{A}^*$ x a" lines

$n = L/\lambda$	0.5	1	1.5	2
$\mathscr{L}$	$(-4.8a + 3.8).10^1$	$(-3.7a + 1.0).10^3$	$(-3.8a + 2.3).10^6$	$(-1.9^a + 1.3).10^7$
$\mathcal{C}$	$(-5.3a + 3.6).10^0$	$(-1.6a + 0.9).10^6$	$(-2.2a + 0.5).10^7$	$(-8.9a + 4.1).10^6$

For each frequency, support size and source number considered, the distance between two adjacent sources, d_s, can be expressed, in non-dimensional form, for the linear and circular case, respectively by

$$kd_s = 2\pi\, na/(M-1) \qquad\qquad kd_s = 2\pi\, na\, sin(\pi/M). \qquad (6a, 6b)$$

Results show that the values of kd_s corresponding to acceptable solutions are always small, with a mean value of about 0.6; the range explored varies from 0.01 to 10. Furthermore, if one considers only the 'best' solutions, taken as those with $e_{BC} \leq 0.6$, it is found that, for each frequency, the curves 'kd_s* x a' can be reasonably well approximated by lines with roughly the same angular coefficient (around 1.5). Insertion of this value in equations (6) yields a rule that relates, for a given support, the cube edge and the frequency to the 'optimal' number of monopoles. This quantity is given, in the linear and circular cases, respectively by

$$M^* \approx 1 + \frac{kL}{1.5} \qquad\qquad M^* \approx \frac{\pi}{1.5} kL. \qquad (7a, 7b)$$

It was verified that equations (7) provide, if not exactly the number of sources associated to the best solution, numbers that always correspond to a solution fulfilling the quality criterion adopted.

The aspect ratio effect

For parallelepiped bodies and normal incidence, since u_n is non-zero only on the two faces normal to **k**, the fields to be reconstructed are basically 'dipolar'. However, as η increases, the main difference between the two incidence cases considered is that, while for $\Phi_{inc} = 0$ the scattered field becomes more and more concentrated along the **k** direction, when $\Phi_{inc} = \pi/2$, the affected region increases with η. Figure 2 shows, for $\Phi_{inc} = 0$, the evolution of the boundary error (restricted to $e_{BC} \leq 1$) when, for a given support, the number of sources increases from 2 to 25.

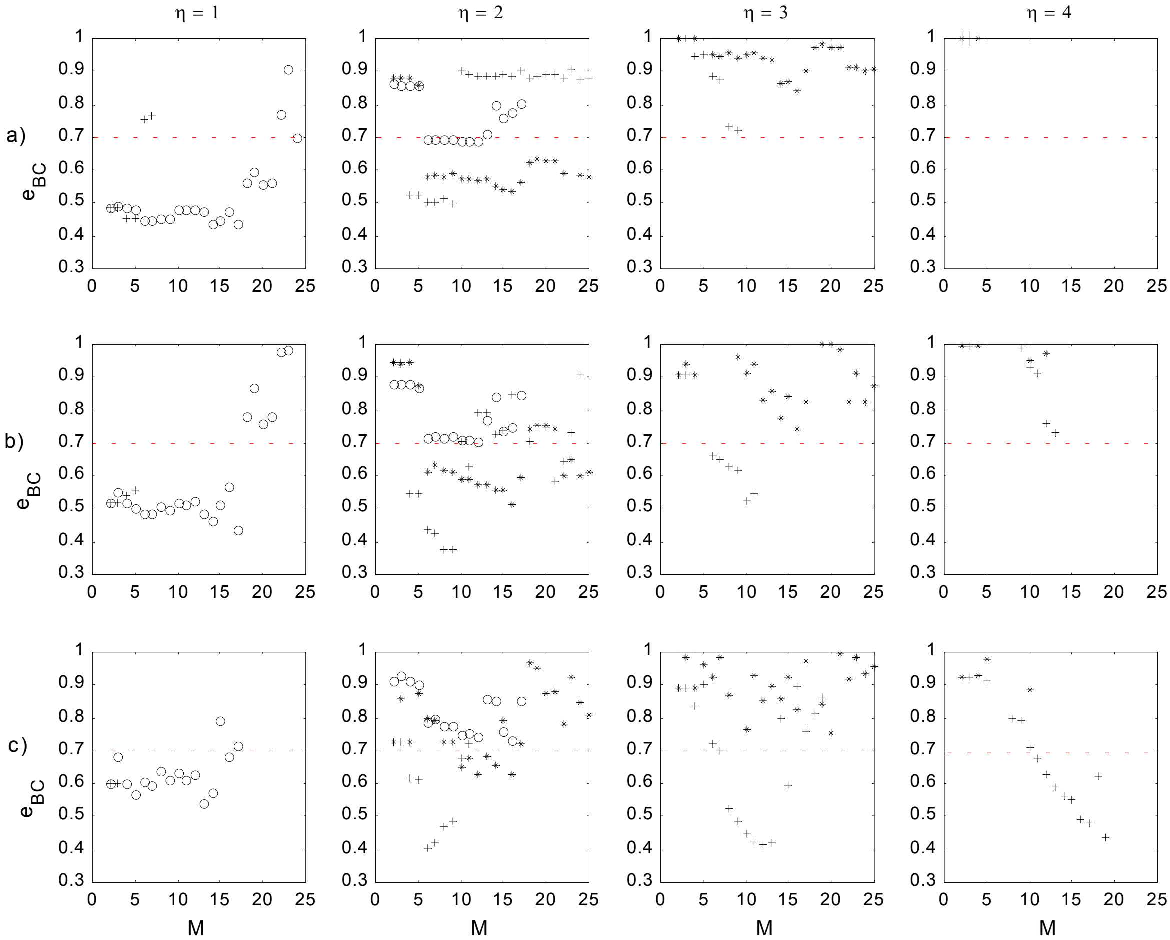

Figure 2:Evolution of the boundary error with the number of sources for η = 1, 2, 3 and 4, Φ_{inc} = 0, with $\mathscr{L}$(+), $\mathscr{C}$(○) and $\mathscr{E}$(♦); a = 0.25 (a), *0.5* (b) *and 0.75* (c).

While for $\eta = 1$, good quality solutions are obtained with the circular support, this support cannot provide such good solutions for $\eta = 2$ and is totally inappropriate for bodies with higher aspect ratio. For $\eta = 2$ the elliptical support with $a \leq 0.5$ yields good solutions for all M, even if the best solutions are reached with the linear one. When $\eta > 2$, the only appropriate support is the linear one and its size has to increase with η, what makes it able to deal with a higher number of monopoles and allows a better covering of the body boundary. Actually, the pressure field to be reconstructed is so concentrated on the x-axis that it only can be simulated efficiently with a relatively small number of monopoles when the sources are located on this axis. In this case, it was verified that the rule giving the optimal number of monopoles for the linear support is still valid, provided L is substituted by 'ηL' in equation (7a).

When **k** is normal to the largest side of the parallelepiped, i.e., for $\Phi_{inc} = \pi/2$, the results (Figure 3) shows that $\mathscr{L}$ is appropriate only for bodies with aspect ratio up to 2, $a < 0.5$ and a relatively low number of sources. Otherwise, excessive values for Q and $\hat{A}$ are generated (not shown), responsible for poor solution quality.

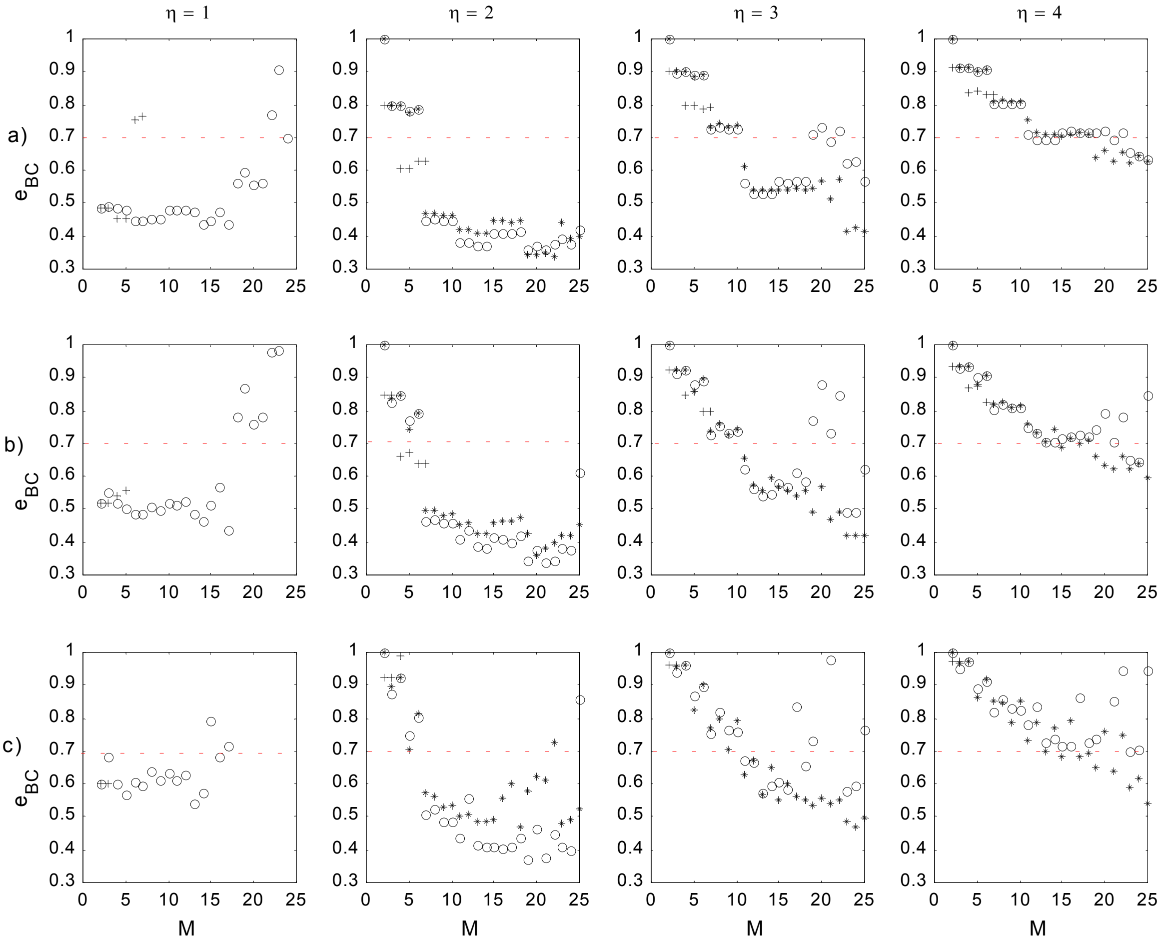

Figure 3: Evolution of the boundary error with the number of sources for η = 1, 2, 3 and 4, $\Phi_{inc} = \pi/2$, with $\mathscr{L}$(+), $\mathscr{C}$(○) and $\mathscr{E}$(♦); a = 0.25 (a), *0.5* (b) *and 0.75* (c).

For η > 2, the best solutions are obtained with the circular and elliptical supports, quality tending to increase with the number of monopoles used. However, it can be observed that, while a quality loss occurs with $\mathscr{C}$ for $M > 17$, this does not happen with $\mathscr{E}$. This is due to the fact that, since the elliptical support has a better match to the body geometry, the distances between the sources and the nodes on the boundary are more homogeneous, what makes the linear system to be solved more stable. Consequently, even for high values of M, the source amplitudes generated with $\mathscr{E}$ are kept below the acceptable upper limit (not shown). In fact, $\mathscr{E}$ is the only of the three supports which remains efficient for the higher values of η.

A natural extension of the elliptical support is the *double linear* one, made of two parallel lines normal to **k**, containing each $M/2$ monopoles regularly distributed (so that the sources are arranged in pairs, favoring the formation of dipoles parallel to **k**), and denoted by $\mathcal{LL}_M$. With the aim of investigating, for a given η, the influence of M and of the positioning of the sources on the solution quality, the solutions obtained with the source configurations $\mathcal{LL}_4$, $\mathcal{LL}_6$, $\mathcal{LL}_8$ and $\mathcal{LL}_{10}$ were computed for a between 0.01 and 0.99. The corresponding e_{BC} curves are shown in Figure 4.

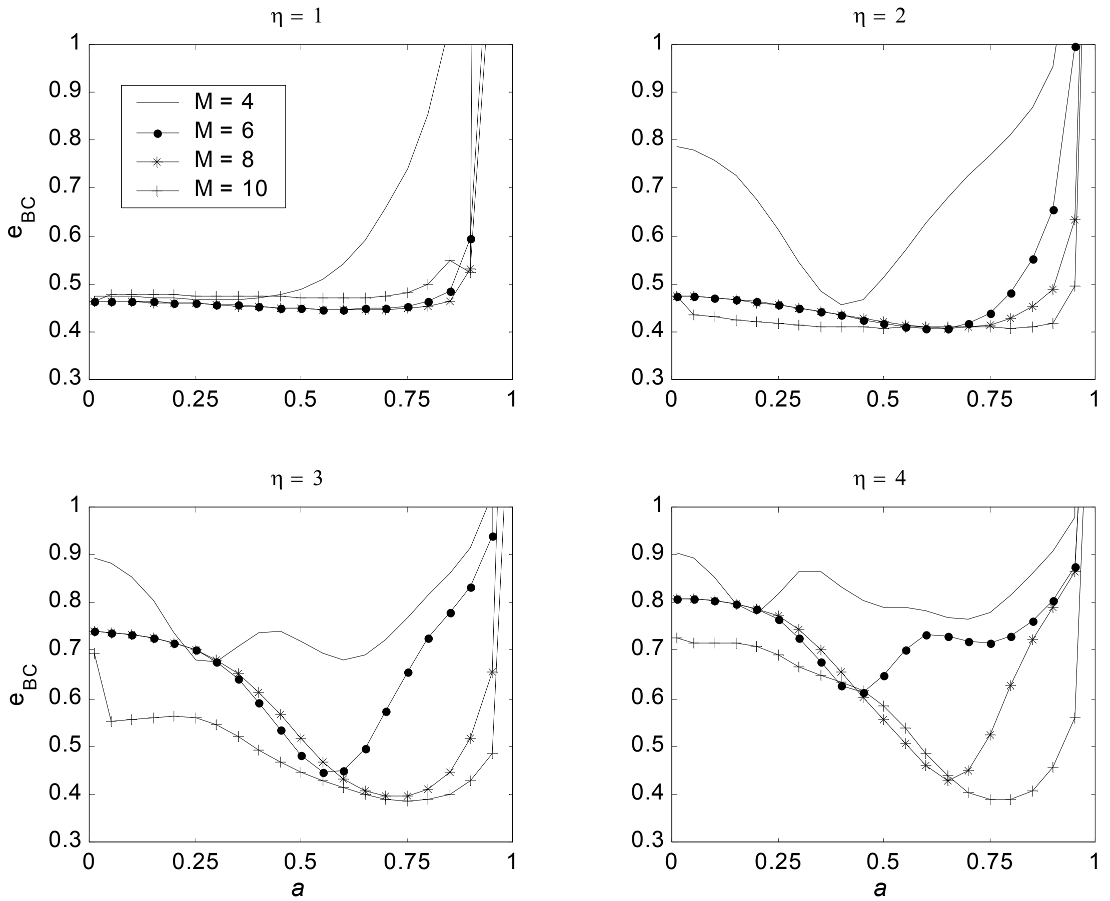

Figure 4:Evolution of the boundary error with the reduction factor for parallelepiped with η = 1, 2, 3 4 and $\Phi_{inc} = \pi/2$, obtained with $\mathcal{LL}_4$, $\mathcal{LL}_6$, $\mathcal{LL}_8$ and $\mathcal{LL}_{10}$.

Results show that the support $\mathcal{LL}$ is efficient in the majority of cases, notably for the highest η value used (η = 4), when the simple supports tested failed. While in this case e_{BC} values lower than 0.7 were attained only with the elliptical support using at least 20 monopoles, solutions with the same quality are obtained with the double linear one with only 6 sources. With 8 or 10 sources (i.e., 4 or 5 pairs of sources), solutions with e_{BC} of the order of 0.4 are generated. Although for each η, the lowest e_{BC} is generally obtained for the highest M, the data suggest that there is a minimum value for M that guarantees a good solution, this number being given by

$$M^{*} \sim 2(\eta + 1). \tag{8}$$

Distinct bands of a values corresponding to good solutions show up in Figure 4. Their width and positioning, centered on a^*, varies with M and η. The knowledge of a^* provides the value of d_s^*, the 'optimal' distance between 2 monopoles in the same line. For the 16 cases investigated it was found that, whereas kd_s^* varies significantly, the ratio kd_s^*/η is roughly constant, around 1.1. With equation (7a) this leads, for M up to 14, to

$$a^{*} \sim 0.17\left(\tfrac{1}{2}M - 1\right). \tag{9}$$

Equations (8) and (9) constitutes a simple rule for determining, given the body aspect ratio, the size of the double linear support and the optimal number of monopoles to be used.

CONCLUSIONS

It was shown that, for a cubic scatterer, the best solutions are obtained with a linear support parallel to the incident wave vector, the advantage over the circular support increasing with frequency. Two rules concerning identification of appropriate source configurations for the linear and circular cases have been found: one relating the side of the cube and the frequency to the 'optimal' number of monopoles; the other furnishing, for a given frequency and support size, an estimation of the mean amplitude value for an appropriate source set. For scatterers with aspect ration higher than unity, when **k** is normal to the largest side of the parallelepiped, the best solutions are also reached with the linear support, provided its size increases with η. When **k** is normal to the largest side of the parallelepiped, the linear support is inappropriate, the most efficient of the simple supports being the elliptical one. It was found, however, that with a relatively small number of sources, a double linear support leads always to good solutions, which are significantly better than the best ones obtained with the simple supports. In this case, it was found a simple rule that furnishes, for a given aspect ratio, the size of the support and the optimal number of monopoles.

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