ACOUSTICAL MEASUREMENT ACCOMPANYING TENSILE TEST: NEW MODALITY FOR NONDESTRUCTIVE TESTING AND CHARACTERIZATION OF SHEET MATERIALS

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Abstract

A series of uniaxial tensile tests was performed for sheet materials like paperboard (PPR) and Low Density Polyethylene(LDPE). In parallel with a tensile test, the natural frequency was measured through an acoustical excitation of these sheets considered as membranes.

Firstly, it was shown both theoretically and experimentally that, at a given load, the frequency is sensitive to the local deviation in the standard thickness or to the presence of cracks inside the material. It means that this acoustic measurement can be used as one of the methods of damage assessment, or nondestructive testing in general.

Secondly, the resonance frequency shift was continuously monitored for increasing strain on polyethylene and paperboard, and the curves obtained were compared to the stress-strain curves for material characterization. They were not the same and showed a non-monotonic stiffness variation for the polyethylene. It was shown that the resonance frequency shift correlates with the stress-strain curve for material characterization under tensile test.

INTRODUCTION

Materials characterization is important to perform failure analyses on samples. Samples can be analyzed to determine the failure mode, and solutions to prevent further problems. Meanwhile, Fracture Mechanics methodology has strongly indicated the importance of nondestructive testing (NDT) as a decision-supporting information tool [1].

A previous work [2] has considered vibration-based technique by providing a firm mathematical and physical foundation to the issue related to defect detection in sheet materials. It was shown that the presence of a scaterrer in form of crack shifts the resonance frequency of the material. As a matter of fact, it seemed important to investigate the quantitative aspect of this observation, by studying the variation of the resonant frequency with an increase in defect severity, which is included within the frame of this paper.

Different nondestructive methods are used for material characterization [3]. The dynamic response through a non-contact vibration-based technique is used in this study to investigate some phenomena accompanying elastic and plastic deformations on paperboard and LDPE, such as the resonant frequency shift. Beforehand, the mechanical properties of materials are determined in tensile testing, using a destructive method. The specimen under investigation, held by clamps, is subject to mechanical loading until rupture. However, there are relationships between mechanical properties and modal parameters, specifically the resonance frequency. It is well known that for a specimen under tension, the frequency is proportional to the speed of the transversal wave propagating along the material, which in turn is related to the remote stress,thus to the rigidity of the material. The changes in the material structural integrity may then be detected by a non-contact vibration-based technique. Hence, this method may be used to determine the physical and mechanical properties of materials.

DAMAGE SEVERITY ASSESSMENT ON PAPERBOARD

Theory

Consider the sample of paperboard in Figure 1 as a membrane subjected to tension (i) and transversal acoustical excitation (ii). The equation of motion of the membrane without account
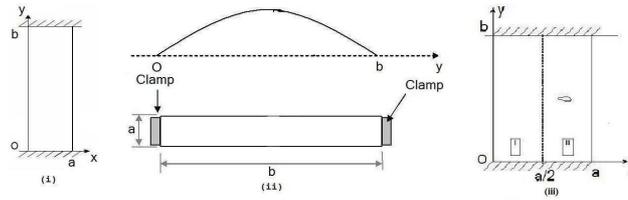
Figure 1: Specimen configuration.

of external pressure, has the form [4]:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} \right) = 0 \tag{1}$$

where ξ is the displacement of membrane along the z - axis from its equilibrium position z=0, $c = \sqrt{T/(\rho h)}$ is the velocity of propagation of bending wave which is determined by the tensile force T per unit length of boundary of membrane, ρ is the density and h is the thickness of the membrane. The free vibration of a rectangular membrane can then be represented as a series of natural modes:

$$\xi = \sum_{m,n=0}^{\infty} \xi_{mn} = \sum_{m,n=0}^{\infty} A_{mn} \cos\left(m\pi\frac{x}{a}\right) \sin\left(n\pi\frac{y}{b}\right) \sin\left(\omega_{mn} t + \varphi_{mn}\right). \tag{2}$$

where the constants A_{mn} and φ_{mn} are amplitude and phase, ω_{mn} is the natural frequency of

the mode,

$$\omega_{mn} = c\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}, \quad m = 0, 1, 2, 3, ... \tag{3}$$

and a, b are the dimensions of the membrane. Each mode in the solution (2) satisfies the boundary conditions

$$\xi_{mn}(x, y = 0) = 0, \quad \xi_{mn}(x, y = b) = 0, \quad \frac{d\xi}{dx}(x = 0, y) = 0, \quad \frac{d\xi}{dx}(x = a, y) = 0. \tag{4}$$

The conditions (4) correspond to immovable boundaries at $y = 0, y = b$ and to free boundaries at $x = 0, x = a$.

$$\frac{\partial^2 \xi^{(1,2)}}{\partial t^2} - c_{1,2}^2 \left(\frac{\partial^2 \xi^{(1,2)}}{\partial x^2} + \frac{\partial^2 \xi^{(1,2)}}{\partial y^2}\right) = 0 \tag{5}$$

Index 1 here corresponds to the left hand section, and 2 to the right-hand section. Let the membranes vibrate in the ξ_{m1} mode. If the two sections of the membrane are joined, they vibrate with the frequency ω, which lies between ω_2 and ω_1:

$$\omega_2 < \omega < \omega_1 \tag{6}$$

and the solutions to (5) are in the form:

$$\xi^{(1,2)} = A^{(1,2)}(x) \sin\left(\pi \frac{y}{b}\right) \cos(\omega t) \tag{7}$$

where ω is an unknown frequency.

From the previous equations, it is shown [Mfoumou et. al.] that the fundamental frequency of the membrane consisting of two different sections of equal size is given by

$$\omega_2 = \omega_1 \left(1 - \frac{c_1 - c_2}{2c_1}\right) \tag{8}$$

And if $c_2 < c_1$, the frequency shifts down, and the absolute value of the relative frequency shift is:

$$\frac{\omega - \omega_1}{\omega_1} = -\frac{c_1 - c_2}{2c_1} \tag{9}$$

Experiment

The experimental setup is shown in Figure 2. A cracked panel as illustrated in Figure 1-(iii) is considered. The 2 mm long crack is cut manually using a razor blade. The width and length of the specimen are w=30 mm and L=550 mm. The paperboard used has a thickness h=100 μm.
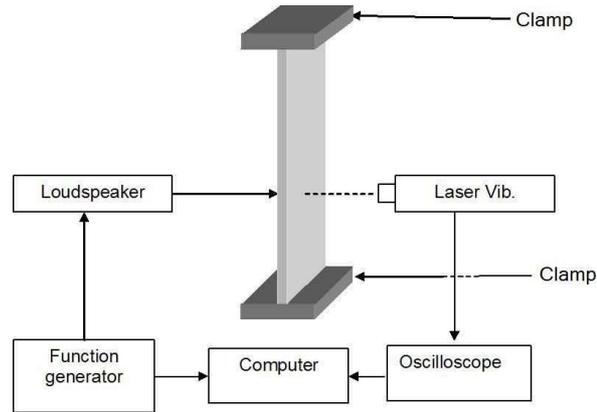
Figure 2: Experimental configuration.

The test is made on a MTS Universal Testing Machine with a 100N loadcell. The specimen is vertically positioned between the upper and lower clamps and is increasingly loaded until a load of 16N is reached. The MTS software TextWorks is used to monitor the loading process. The test is performed at a room temperature of 23^oC and 40% humidity, at a cross-head speed of 5mm/min.

The specimen is transversaly excited using a 8Vpp sine signal through a loudspeaker and the third bending mode (on the length direction) is monitored for increasing crack length. A VS-100 OMETRON laser vibrometer is used for sensing of vibrations from the material. In parallel, the updated load resulting from the material softning due to the increase in crack length is continuously recorded from the TestWorks display board.

Results and correlation

It was shown in a previous work [Mfoumou et. al.] that the resonance frequency shifts down by introduction of a defect in form of crack. As a matter of fact, the frequency continuously decreases due to the increase in crack length, and monitoring of the third bending mode shows a good agreement between experimental and analytical results, seen in Figure 3.

ACOUSTICAL MEASUREMENT FOR MATERIAL CHARACTERIZATION

Standard mechanical characterization

The tensile test is widely used to provide basic information on the strength of materials and as an acceptance test for the specification of materials. In the tensile test the specimen is subjected to a continually increasing uniaxial tensile force while simultaneous observations are made of the elongation of the specimen.

Two rectangular strips - one of paperboard and one of polyethylene - are investigated experimentally. The densities and sizes of the materials are specified on table 1 below. The
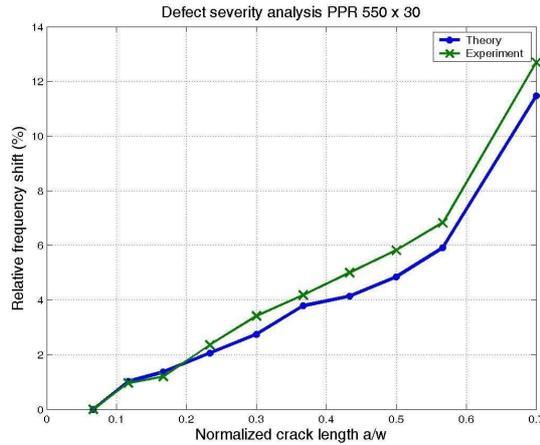
Figure 3: Third mode relative frequency shift (equation (9)) with defect severity.

Material	Density (g/cm^3)	Length (cm)	Width (cm)	Thickness (μm)
PPR	0.684	55	3	100
LDPE	0.91	10	1.5	27

Table 1: Materials under investigation.

same equipment as in the previous case is used. The specimen is increasingly loaded by moving the crosshead up. The test is performed under displacement control at a cross-head speed of 0.5 mm/min for the plastic and 0.07 mm/min for paperboard. An engineering stress-strain curve is then constructed from the load/elongation measurements and is illustrated in Figures 4a-b both for paperboard and LDPE.
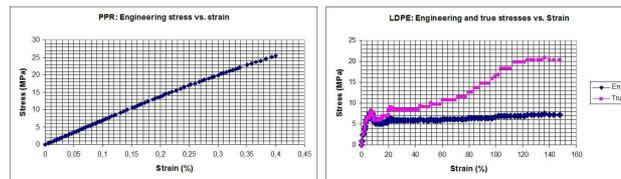
Figure 4: The stress-strain curves from standard test.

Acoustic measurement

In parallel with the tensile test, the specimen is transversaly excited with a loudspeaker. A reference frequency corresponding to one resonance frequency of the material is considered within the elastic region of the stress-strain curve. That resonance frequency is shifting with increase in strain, and the characteristic curves are plotted in Figure 5a-b.

The general shapes of the frequency-strain curves in Figure 5 require further explanation. For both materials, the second mode resonance frequency is monitored. In the elastic region, the load increases, and the resonance frequency is proportional to the square roots of strain and elastic modulus. When the load exceeds a value corresponding to the yield strength, the specimen undergoes strong plastic deformation. Further details are given below.

The observations made so far lead to the conclusion that the stress-strain curve correlates with the frequency-strain curve for material characterization.

Analysis

Consider the general expression of resonance frequencies given by equation (3). As the mode numbers m and n are constant for a given resonance frequency, it follows that the resonance frequency is proportional to the transversal wave velocity:

$$\omega_{mn} = \alpha c = \alpha \sqrt{\frac{F}{\rho a h}} \tag{10}$$

where F is the applied load and $\alpha = \sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}$.

It can also be demonstrated that the previous equation can be rewritten as follows:

$$\omega_{mn} = \alpha \sqrt{\frac{\sigma}{\rho}} \tag{11}$$

where σ represents the remote stress that elongates the specimen.

For the second bending mode, $m = 0$, $n = 2$, we have $\alpha = \frac{2\pi}{b}$. It follows that

$$\omega_{mn} = \frac{2\pi}{b} \sqrt{\frac{\sigma}{\rho}} = \alpha \sqrt{\frac{E\epsilon}{\rho}} \tag{12}$$

Figures 5a-b show the plots of analytical (equation 12) and experimental results. An acceptable agreement is obtained, and besides, the shapes of the curves are more illustrative than those of tensile tests. In equation 12, the resonance frequency is linearly proportional to the
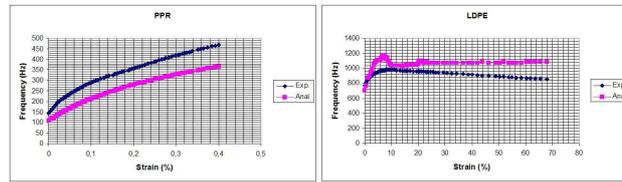
Figure 5: Analytical and experimental frequency-strain.

square root of the strain and thus, shifts up with increase in strain. When the material enters the plastic region, the Young's modulus starts (in principle) to decrease. However, the stress

to produce continued plastic deformation increases with increasing strain, i.e., the materials strain-harden. The volume of the specimen remains constant during plastic deformation, and as the specimen elongates, it decreases uniformly along the gauge length in the cross-sectional area.

Initially the strain hardening more than compensates for this reduction in Young's modulus, and the frequency (proportional to square root of strain) continues to rise with increasing strain. Eventually a point is reached where the decrease in stiffness is greater than the strain hardening. Because the stiffness now is decreasing far more rapidly than strain hardening, the actual load required to deform the specimen falls off and the resonance frequency likewise decreases until fracture occurs.

CONCLUSIONS

The issue related to low frequency acoustic measurement for damage severity assessment is proven in this work with a good accurary. A previous work [Mfoumou et. al] has shown that the resonance frequency shifts down with introduction of a crack in the material. As a continuation, this work correlates the analytical and experimental variation of the resonance frequency with an increase in the defect severity, with a good agreement. Therefore, the low frequency investigation of the vibration-based technique is sensitive to changes in standard thickness or crack inside the material.

More importantly, the possible connectivity of the standard mechanical testing (Fracture Mechanics) to acoustical measurement (NDT) is successfully proven for material characterization in this work. It is successfully shown that the resonance frequency shift measurement correlates with the stress-strain curve under tensile test. However, a non-monotonic stiffness variation is observed using the frequency measurement from the polymer behaviour, which eventually highlights a new investigation trend for fracture behaviour of materials.

References

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