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ANALYSIS OF NONLINEAR FORCED VIBRATION USING COMPONENT MODE SYNTHESIS METHOD

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Abstract

In this paper, an analytical approach for nonlinear forced vibration of a multi-degree-of-freedom system is proposed using the component mode synthesis method. The whole system is divided into some components and a nonlinear modal equation of each component is derived using the unconstrained vibration modes. The modal equations of all components and the conjunction conditions are solved simultaneously, and then the modal responses of components are derived. Finally, the dynamic responses of the whole system can be obtained. The degrees of freedom of modal equations can be reduced when the lower vibration modes are only adopted in each component. As a numerical example, a simple five-degree-of-freedom system is considered, in which all spring have cubic type nonlinearity. As a result, it is shown that even if the lower vibration modes of each component are only adopted, the accurate dynamic response near the first resonance can be obtained.

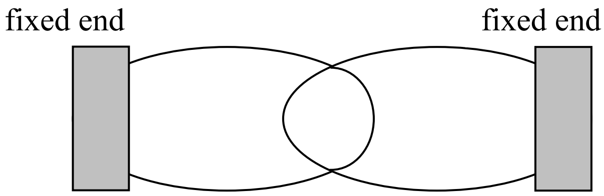
INTRODUCTION

Recently, the nonlinear vibration analysis becomes more important from the viewpoint of machine condition monitoring and diagnosis. Usually, the equation of motion with nonlinear properties is constructed, for example, using F.E.M., then the equation is transformed into modal equation with a few modal responses by the modal analysis technique. Then the nonlinear vibration is calculated by the direct integration, the method of multiple scales, harmonic balance method [1] and so on. However, when the system has huge degrees of freedom, the eigenvalue analysis needs large computer storage and calculation time. Because of the reason, the component mode synthesis (C.M.S.) method has been developed [2] for applying to the linear system only. In the previously paper, an analytical method using C.M.S. method was proposed for a nonlinear system [3]. But it is applicable only for the specific problem because there are many assumptions in the formulation.

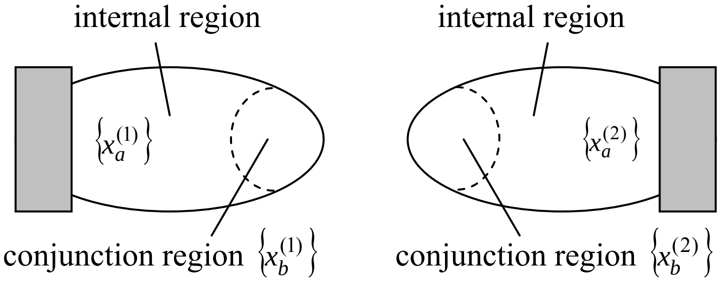
In this paper, a new analytical approach for nonlinear forced vibration of a multi-degree-of-freedom system is proposed using the C.M.S. method. The whole system is divided into some components and a nonlinear modal equation of each component is derived using the unconstrained vibration modes. The modal equations of all components and the conjunction conditions are solved simultaneously, and then the modal responses of components are derived. Finally, the dynamic responses of the whole system are obtained. The degrees of freedom of modal equations can be reduced when the lower vibration modes are only adopted in each component. As a numerical example, a simple five-degree-of-freedom system is considered, in which all spring have cubic type nonlinearity. The system is divided into two components with three degrees of freedom and the applicability of the proposed method is inspected.

FORMULATION BY C.M.S. METHOD

In this paper, the nonlinear forced vibration is analyzed when the amplitude of vibration is so large that the nonlinear property of the element cannot be ignored. And it is assumed that the nonlinearity can be expressed as the polynomials of displacement, there are no rigid modes when the whole system is divided and no external force acts in the conjunction region. For simplicity of the explanation, the damping property is ignored and the whole system is divided into two components as shown in Fig.1.



(a) whole system



(b) two components

Figure 1 – Analytical model

Equation of motion of component

The equation of motion for the component 1 can be obtained by F.E.M. as follows:

$$\left[M^{(1)}\right]\left\{\ddot{x}^{(1)}\right\}+\left[K^{(1)}\right]\left\{x^{(1)}\right\}+\left\{N^{(1)}\left(x^{(1)}\right)\right\}=\left\{f^{(1)}\right\}, \tag{1}$$

where $\left\{x^{(1)}\right\}=\left\{x_a^{(1)} \quad x_b^{(1)}\right\}^T$ is the displacement vector and the subscript 'a' and 'b' denote the internal and conjunction region, respectively. The term $\left\{N^{(1)}\left(x^{(1)}\right)\right\}$ means the nonlinear property composed of the polynomials of displacement. The term $\left\{f^{(1)}\right\}$ is composed of the external force $\left\{f_a^{(1)}\right\}$ and the internal force $\left\{f_r^{(1)}\right\}$.

The eigenvalue analysis for the linear system of Eq.(1) is carried out, and the natural frequencies squared and the natural vibration modes can be obtained as $\left[\omega^{(1)2}\right]$ and $\left[\Phi^{(1)}\right]$, respectively. Using the transformation of $\left\{x^{(1)}\right\}=\left[\Phi^{(1)}\right]\left\{\eta^{(1)}\right\}$, Eq.(1) can be rewritten as follows:

$$\left[I^{(1)}\right]\left\{\ddot{\eta}^{(1)}\right\}+\left[\omega^{(1)2}\right]\left\{\eta^{(1)}\right\}+\left[\Phi^{(1)}\right]^T\left\{N^{(1)}\left(\eta^{(1)}\right)\right\}=\left[\Phi^{(1)}\right]^T\left\{f^{(1)}\right\}. \tag{2}$$

Using the similar procedure, the equation of motion for the component 2 can be obtained in the modal coordinate system as follows:

$$\left[I^{(2)}\right]\left\{\ddot{\eta}^{(2)}\right\}+\left[\omega^{(2)2}\right]\left\{\eta^{(2)}\right\}+\left[\Phi^{(2)}\right]^T\left\{N^{(2)}\left(\eta^{(2)}\right)\right\}=\left[\Phi^{(2)}\right]^T\left\{f^{(2)}\right\}, \tag{3}$$

where $\left\{\eta^{(2)}\right\}$ is the modal displacement of the component 2.

Conjunction condition of two components

The physical displacement of the conjunction region has to be same such as $\left\{x_b^{(1)}\right\}=\left\{x_b^{(2)}\right\}$. This condition can be expressed as follows:

$$\left[\Phi_b^{(1)}\right]\left\{\eta^{(1)}\right\}=\left[\Phi_b^{(2)}\right]\left\{\eta^{(2)}\right\}. \tag{4}$$

The internal force acting on the component 1 must be the opposite direction of the one acting on the component 2, but the amplitude must be same as follows:

$$\left\{f_r^{(1)}\right\}=-\left\{f_r^{(2)}\right\}=-\left\{f_r\right\}. \tag{5}$$

Synthesizing of two components

Introducing Eq.(5) into Eqs.(2) and (3), premultiplying the first result by $\left[\Phi_b^{(1)}\right]$ and the second one by $\left[\Phi_b^{(2)}\right]$, and subtracting and using Eq.(4), the internal force $\{f_r\}$ can be obtained as follows:

$$\{f_r\} = [R_1]\left\{f_a^{(1)}\right\} - [R_2]\left\{f_a^{(2)}\right\} - [P_1]\left\{\eta^{(1)}\right\} + [P_2]\left\{\eta^{(2)}\right\} - [Q_1] + [Q_2]. \tag{6}$$

where

$$\begin{gathered}[R_i] = [\Phi_0]^{-1}\left[\Phi_b^{(i)}\right]\left[\Phi_a^{(i)}\right]^T, [P_i] = [\Phi_0]^{-1}\left[\Phi_b^{(i)}\right]\left[\omega^{(i)2}\right], \\ [Q_i] = [\Phi_0]^{-1}\left[\Phi_b^{(i)}\right]\left[\Phi^{(i)}\right]^T\left\{N^{(i)}\left(\eta^{(i)}\right)\right\}(i=1,2), [\Phi_0] = \left[\Phi_b^{(1)}\right]\left[\Phi_b^{(1)}\right]^T + \left[\Phi_b^{(2)}\right]\left[\Phi_b^{(2)}\right]^T\end{gathered}. \tag{7}$$

Introducing Eq.(6) into Eqs.(2) and (3) again, the next type of equation can be obtained.

$$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{\eta}^{(1)} \\ \ddot{\eta}^{(2)} \end{Bmatrix} + \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{21} & \tilde{K}_{22} \end{bmatrix}\begin{Bmatrix} \eta^{(1)} \\ \eta^{(2)} \end{Bmatrix} + \left\{N\left(\eta^{(1)}, \eta^{(2)}\right)\right\} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}\begin{Bmatrix} f_a^{(1)} \\ f_a^{(2)} \end{Bmatrix}. \tag{8}$$

The coefficient matrices and nonlinear terms can be concretely expressed when the subject is given.

The degree of freedom is checked here. When the degrees of freedom of the internal region of the component 1 and 2 are n_1 and n_2, and the one of the conjunction region is n_c, the total degree of freedom is $n_1 + n_2 + n_c$. In the method proposed in this paper, when all vibration modes are adopted in each component, the total degree of freedom is $n_1 + n_2 + 2n_c$, which is more than the original degree of freedom. But when a few vibration modes are only adopted, the total degree of freedom can be reduced. This point is an advantage of this method.

Equation (8) is a typical nonlinear differential equation. The forced vibration can be obtained by the direct integration, the method of multiple scales, harmonic balance method and so on.

NUMARICAL EXAMPLES

Numerical model and equation of motion of component

To check the applicability of the method, a simple five-degree-of-freedom system is considered as shown in Fig.2(a), and it is divided into two components as shown in Fig.2(b). In this example, no external force acts on mass m_3, the whole system is divided there. All springs have cubic type nonlinearity, $f_s = k\left(x + \beta x^3\right)$, where f_s and x are the restoring force and relative displacement, respectively.

For the component 1, the coefficient matrices in Eq.(1) and the nonlinearity vector is obtained as follow:

$$\left[M^{(1)}\right]=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3/2 \end{bmatrix},\left[K^{(1)}\right]=\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix},$$

$$\left\{N^{(1)}\right\}=\left\{\begin{matrix} k_1\beta_1 x_1^{(1)3}+k_2\beta_2\left(x_1^{(1)}-x_2^{(1)}\right)^3 \\ k_2\beta_2\left(x_2^{(1)}-x_1^{(1)}\right)^3+k_3\beta_3\left(x_2^{(1)}-x_3^{(1)}\right)^3 \\ k_3\beta_3\left(x_3^{(1)}-x_2^{(1)}\right)^3 \end{matrix}\right\},\left\{x^{(1)}\right\}=\left\{\begin{matrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{matrix}\right\},\left\{f^{(1)}\right\}=\left\{\begin{matrix} f_1^{(1)} \\ f_2^{(1)} \\ f_r^{(1)} \end{matrix}\right\}, \tag{9}$$

where $x_1^{(1)}$, $x_2^{(1)}$ and $x_3^{(1)}$ are x_1, x_2 and x_3 in Fig.2(a), respectively. For the component 2, they can be similarly obtained. The specifications are as follows:

$$\begin{aligned} &m_1=m_2=m_3=m_4=m_5=1.0\,kg,\ k_1=k_2=k_3=k_4=k_5=1.0\times10^5\ N/m, \\ &\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=0.3\,m^{-2} \end{aligned}.$$

Natural frequencies for the linear system

Primarily, for the linear system, the natural frequencies are calculated. The exact result is obtained from the full order model, which is a five-degree-of-freedom system. There are three vibration modes in each component, $n_1=n_2=3$, so when all of them are adopted, the total degrees of freedom become six. This case is indicated as ‘cms33’ in this study. Similarly, when the lower two or one modes are adopted, the total degrees of freedom become four or two, respectively, and these cases are indicated as ‘cms22’ or ‘cms11’.

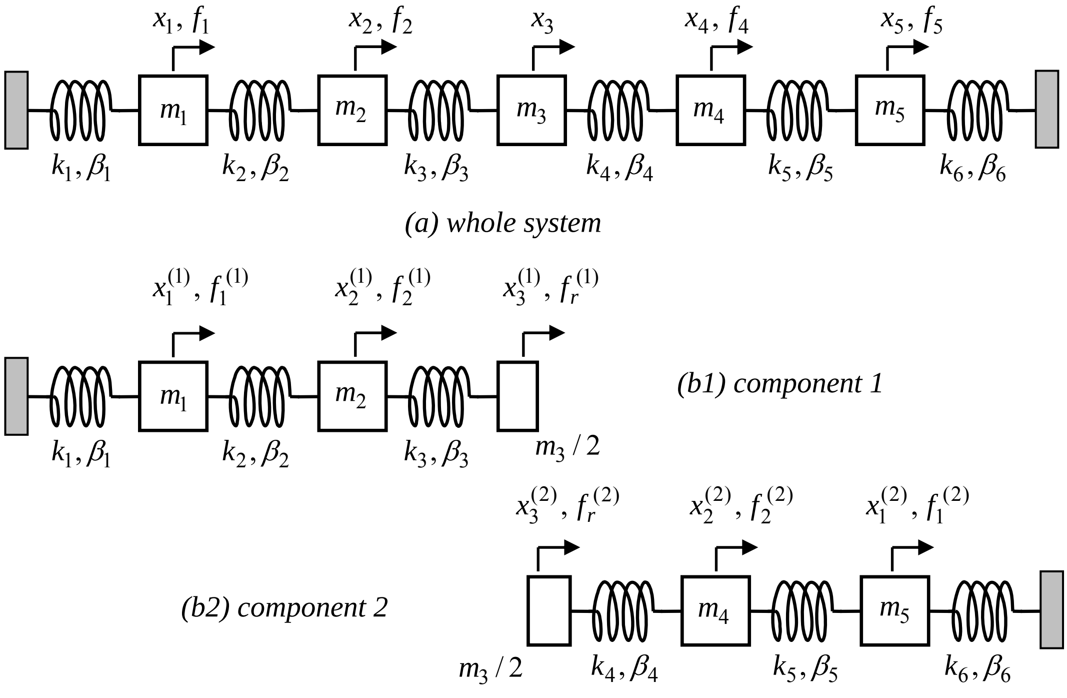


Figure 2 – A five-degree-of-freedom system

The results are shown in Table 1. From the table, it is recognized that the lower natural frequencies are determined, the degrees of freedom of components can be reduced, that is to say, the total degrees of freedom can be reduced. In the case of ‘cms22’, the result of ω_2 is not correct because the degrees of freedom of components are so few that the second vibration mode, that has a node at the center, cannot be correctly expressed.

Table 1 – Comparison of natural frequencies

Natural frequency [Hz]	exact	cms33	cms22	cms11
ω_1	26.05	26.05	26.05	26.05
ω_2	50.33	50.33	53.59	——
ω_3	71.18	71.18	71.18	——
ω_4	87.17	87.17	——	——
ω_5	97.23	97.23	——	——

Nonlinear forced response by harmonic balance method

It is assumed that the periodic external force with the frequency ω acts on the second mass; $f_2 = F_2 \cos\omega t$ and $F_2 = 1\times 10^3\ N$. The steady state responses are calculated by using the harmonic balance method. In the case of ‘exact’, the result is calculated directly from the full order model. The other hand, in the case of the proposed method, it is calculated from Eq.(8).

The resonance curves at x_3 near the first natural frequency are shown in Fig.3. The results of ‘cms33’ and ‘cms22’ agree very well with the result of ‘exact’ one. In the case of ‘cms11’, there is slight difference. It is recognized that an accurate result can be obtained for the first resonance in the case of few degrees of freedom.

Next, the steady state response are calculated when the vibration modes are truncated in the nonlinear modal equation for the full order model. When all vibration modes are adopted, that is indicated as ‘direct5’, the result is equal to the one of ‘exact’. When the lower four or three vibration modes are adopted, the cases are indicated as ‘direct4’ or ‘direct3’, respectively. The results of ‘exact’, ‘cms22’, ‘direct4’ and ‘direct3’ are shown in Fig.4. The result of ‘cms22’ agrees well with ‘exact’ than ‘direct4’ and ‘direct3’.

Finally, the calculation load is checked. The size of eigenvalue analysis, the number of variables and the number of nonlinear terms have much influence on the calculation load. That is shown in Table 2. In the case of ‘direct4’, for example, the eigenvalue analysis has to be done for the full order model once, that is shown ‘1 for (5x5) matrix’. The variables are $\eta_i\ (i = 1,\cdots,4)$ and the nonlinear terms are the polynomials of the third power of variables like $\eta_1^3, \eta_1^2\eta_2, \cdots$. In the case of 'cms22', the eigenvalue analysis has to be done for each component with three degrees of freedom, that is shown '2 for (3x3) matrix'. The variables are $\eta_i^{(j)}$ $(i=1,2; j=1,2)$ and the nonlinear terms are the polynomials of the third power of variables like $\eta_1^{(1)3}, \eta_1^{(1)2}\eta_2^{(1)}, \cdots$. In the case of 'cms22', though the eigenvalue analysis is done for a lower order matrix and the number of nonlinear terms is few, the acceptable result can be obtained. It is recognized that the method proposed in this study is effective for the actual application.

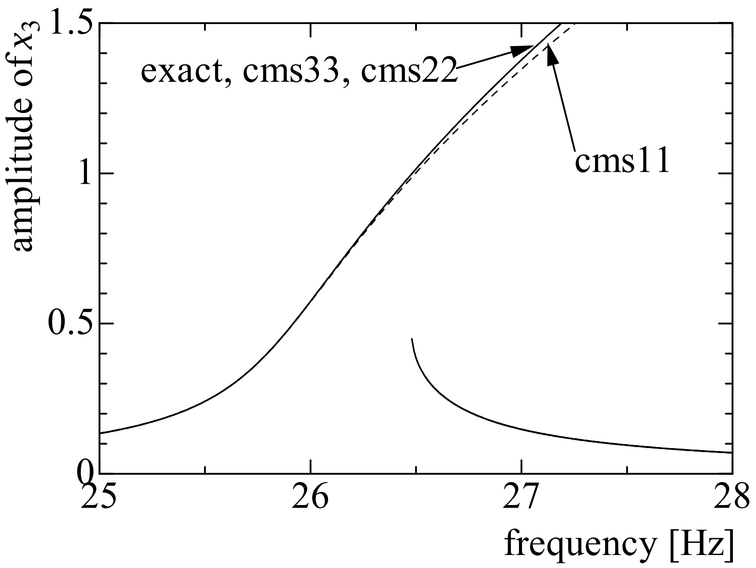


Figure 3 – Comparison of resonance curve among exact and C.M.S. method

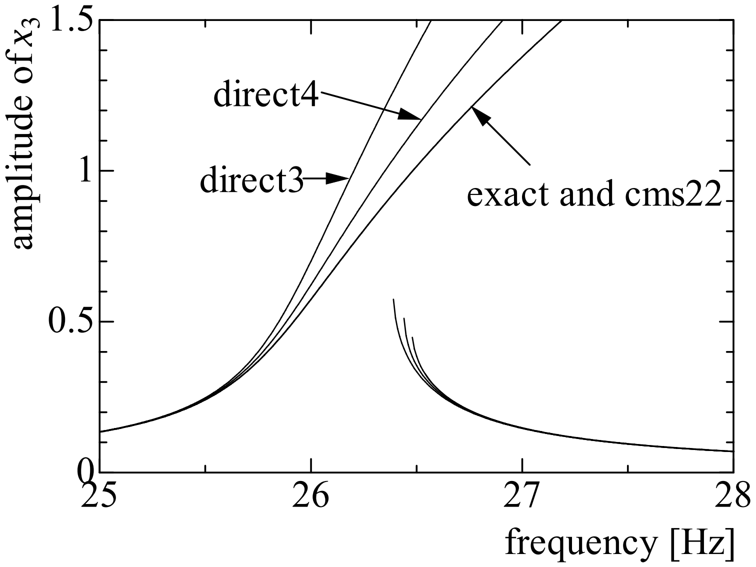


Figure 4 – Comparison of resonance curve among exact, modal truncation and C.M.S. method

Table 2 – Comparison of calculation load

	Eigenvalue analysis	Variables	Nonlinear terms
direct5	1 for (5 x 5) matrix	5	26
direct4	1 for (5 x 5) matrix	4	17
direct3	1 for (5 x 5) matrix	3	10
cms33	2 for (3 x 3) matrix	6	20
cms22	2 for (3 x 3) matrix	4	8
cms11	2 for (3 x 3) matrix	2	2

CONCLUSIONS

In this paper, an analytical approach for nonlinear forced vibration of a multi-degree-of-freedom system is proposed using the component mode synthesis method. By using the method, the degrees of freedom of modal equations can be reduced when the lower vibration modes are only adopted in each component. As a numerical example, a simple five-degree-of-freedom system is considered. As a result, it is shown that even if the lower vibration modes of components are only adopted, the accurate dynamic response near the first resonance can be obtained.

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