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EXPERIMENTAL STUDY ON ACTIVE VIBRATION CONTROL VIA INVERSE MODELING

Liu Dongyue[1]， Zhiyi Zhang[2], Hongxing Hua[2]
(1 Naval research Center, 100073 Beijing, Box1303-14)
(2 Vibration Shock Noise Laborotory, Shanghai Jiao Tong University, 200030 Shanghai)
Email: hhx@sjtu.edu.cn, chychangcn@yahoo.com

Abstract
For lightly damped structures under periodic excitation, vibration responses are generally composed of forced vibration and the natural vibration. Lightly damped modes and periodic vibration render the synthesis of active controllers very difficult and high order controllers are usually generated with less feasibility in practical implementation. To obviate this difficulty, two active control strategies, active damping and harmonic cancellation, are considered to control vibration over a broad frequency band. In the experiment of active vibration suppression, harmonic cancellation and active damping are implemented simultaneously, where harmonic cancellation is based on an inverse modeling technique and no observer is needed. The inverse of the dynamic model of a test rig with two elastic plates is guaranteed by the collocated placement of transducer and actuator, which theoretically results in a minimum phase system. The inverse model was realized on a digital controller, and the experimental results show that vibration suppression can be achieved if the transport delay of control signal is sufficiently small.

INTRODUCTION

Passive vibration isolation systems are widely used in engineering since they are reliable and effective in most cases. However, performance compromise should be considered in passive vibration isolation systems due to the inherent conflict that suppression of mount resonance will deteriorate high frequency performance. In contrast, active isolation systems can achieve good performance simultaneously at resonance and higher frequencies, which is the essential feature that makes active systems attractive. Generally, an active system works in combination with a passive mount so that the advantages of both passive and active isolation systems are retained. In recent years, a great deal of progress has been achieved on the development and application of active isolation systems that can be found mainly in automotive suspension, disturbance isolation of experimental platforms, etc [1-2].

In the area of vibration control of marine vessels, vibration isolation of power machinery is of great importance. The reduced vibration transmission to hull structures will decrease noise radiation of structures, which generally improves the acoustic environment for crew and passengers and in particular the underwater acoustic signature of vessels. Recently, a great deal of investigation has been conducted regarding the active vibration isolation of power machinery and the involved vibration control of flexible structures [3-5]. Vibration isolation of power machinery is usually connected with flexible structures. Due to the strong dynamic coupling between machines and their flexible supporting structures, vibration of hull structures is composed of natural and forced vibration induced by power machinery. As a result, the active isolation and control of vibration needs to deal with harmonic and random vibration to which active vibration attenuation strategies may be different dramatically.

Active control strategies have been investigated widely and deeply during the past two decades, ranging from local control strategies to global control strategies using feedback of displacement, velocity, acceleration, force or the combination of them [6-10]. Since velocity feedback is effective in attenuating not only the resonance of an isolation system but also resonances of flexible structures, it has been applied largely in active vehicle suspensions and active damping [11-12]. However, force feedback is theoretically more stable than velocity feedback even in the case of flexible support of isolated machinery, which might have been a basis for the development of some active isolation units that use force gauges and force feedback.

Velocity feedback is very effective in suppressing random vibration, but it is not so effective in reducing harmonic vibration as usually expected. Harmonic vibration induced by rotating machinery often composes a major part of the total vibration of structures. To control harmonic vibration, cancellation techniques are frequently utilized, which are constructed on adaptive strategies, observer based gain scheduling, or inverse modeling with intelligent networks, etc [13-15]. Adaptation of controllers is attractive since controllers can track any changes of a disturbance and react to it. However, adaptive controllers may be tardy in response to transient disturbances and sometimes unsatisfactory in control. If an observer is used, the shortcomings of adaptive controllers will not exist, but the order of the observer may be very high when there are plenty of harmonic components in measured responses. Therefore, a device of fast computing capability cannot be excluded. Inverse modeling is to embed an inverse model of the plant to the closed loop so that the plant input can be recovered from its output. Inverse modeling can be realized online, which makes control strategies adaptive to plant uncertainties. For nonlinear systems, intelligent networks are powerful in inverse modeling, on which many researches have been carried out. However, this paper has dealt with the inverse modeling in a static way, using a fixed inverse model in the closed loop to avoid any time-consuming online updating. By identifying the system model which is a minimum phase model and exchanging its the poles and zeros, an inverse model is obtained. With this inverse model, harmonic disturbances injecting to the input are recovered and a cancellation signal is added to the input to counteract the effect of disturbances. In the experiment, a vibration isolation system is used to investigate the efficacy of velocity feedback as well as inverse modeling in active vibration control. The discussion includes three sections. In Section 2 the theoretical background of active damping and harmonic cancellation based on inverse model is given briefly. Section 3 gives the experimental results of active isolation, and conclusions are made in Section 4.

ACTIVE DAMPING AND HARMONIC CANCELLATION

Active damping

Active damping is realized by inserting active units between two systems connected through passive mounts, and active forces are proportional to the velocities of those points under control, as shown in Fig. 1. The vibration equation of this system is governed by equation (1), where $f_1 = -g_1\dot{x}_1$, $f_2 = -g_2\dot{x}_2$ and $g_1, g_2 \geq 0$. According to this equation, velocity feedback may introduce off-diagonal elements into the damping matrix, which results in an unsymmetric damping matrix and consequently system instability for large feedback gain. The stability of flexible systems with velocity feedback is conditional. However, in the special case where the supporting base is rigid and of infinite mass ($y_1 = y_2 = 0, Y = 0$), the velocity feedback as given in equation (1) is absolutely stable.

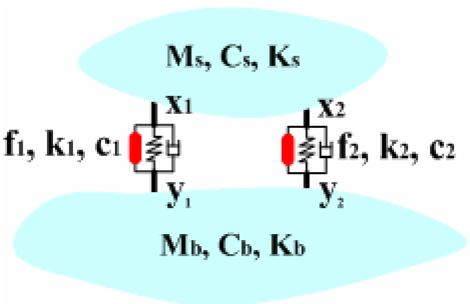


Fig.1 Two elastic bodies connected through passive mounts and active units

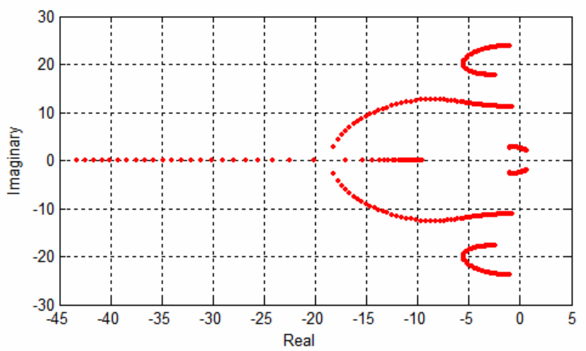


Fig.2 Root loci of the active damping system

$$\begin{bmatrix} M_s & * & * & & & \\ * & m_{s1} & * & & & \\ * & * & m_{s2} & & & \\ & & & m_{b1} & * & * \\ & & & * & m_{b2} & * \\ & & & * & * & M_b \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{Y} \end{Bmatrix} + \begin{bmatrix} C_s & * & * & & & \\ * & c_{s1}+c_1 & * & -c_1 & & \\ * & * & c_{s2}+c_2 & & -c_2 & \\ & -c_1 & & c_{b1}+c_1 & * & * \\ & & -c_2 & * & c_{b2}+c_2 & * \\ & & & * & * & C_b \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{Y} \end{Bmatrix} +$$

$$\begin{bmatrix} K_s & * & * & & & \\ * & k_{s1}+k_1 & * & -k_1 & & \\ * & * & k_{s2}+k_2 & & -k_2 & \\ & -k_1 & & k_{b1}+k_1 & * & * \\ & & -k_2 & * & k_{b2}+k_2 & * \\ & & & * & * & K_b \end{bmatrix} \begin{Bmatrix} X \\ x_1 \\ x_2 \\ y_1 \\ y_2 \\ Y \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_1 \\ f_2 \\ -f_1 \\ -f_2 \\ F \end{Bmatrix}, \quad f_1 = -g_1\dot{x}_1,\ f_2 = -g_2\dot{x}_2 \qquad (1)$$

Generally, for the suspended structure (Ms, Cs, Ks) as shown in Fig. 1, velocity feedback is equivalent to the role of a linear damper and the natural vibration of this structure is actively damped. If the feedback gain is properly given, the damping effect also suppresses vibration of the supporting base. In order to give a straight demonstration, consider a vibration system of three degrees of freedom, the mass, damping and stiffness matrices of which are as follows:

$$\begin{bmatrix} 100 & & \\ & 100 & \\ & & 100 \end{bmatrix}, \begin{bmatrix} 900+g & & \\ -g & 1600 & \\ & & 900 \end{bmatrix}, \begin{bmatrix} 8.21 & -7.86 & 3.21 \\ -7.86 & 11.43 & -7.86 \\ 3.21 & -7.86 & 8.21 \end{bmatrix} \times 10^5,$$

where g is the feedback gain.

If the velocity feedback gain varies from 0 to a sufficient large value, the root loci of this active system will cross the imaginary axis and enter into the right half plane, as shown in Fig.2. As the feedback gain increases, damping ratios increase as well except that of the first mode, which is clearly illustrated in Fig. 3. Therefore, there exists an optimal gain that ensures the stability and performance of active vibration control or isolation of elastic structures.

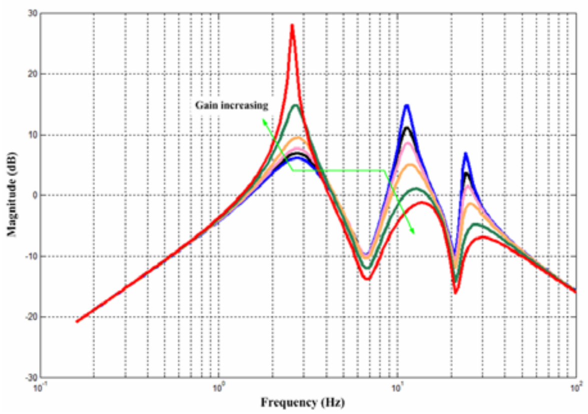


Fig.3 Variation of damping ratios to feedback gain

Harmonic cancellation

As shown in Fig.3, active damping only changes the damping ratios of natural modes and attempts to suppress the natural vibration. If the disturbance F in equation (1) is harmonic, the effect of active damping depends on the extent to which the frequencies of disturbances are close to the natural frequencies. Since active damping may deteriorate certain natural vibration as indicated in Fig.3, isolation of harmonic disturbances whose frequencies are near the amplification region cannot be achieved. So far, lots of harmonic cancellation techniques have been presented to tackle this problem. The mechanism of harmonic cancellation is given in Fig.4, where the measured output is the sum of responses induced by the control signal and harmonic disturbances, and the transfer path II includes the dynamics of active mounts. The amplitude and phase of the counteracting signal are adjusted by the controller, which can be either adaptive or fixed, to minimize the output.

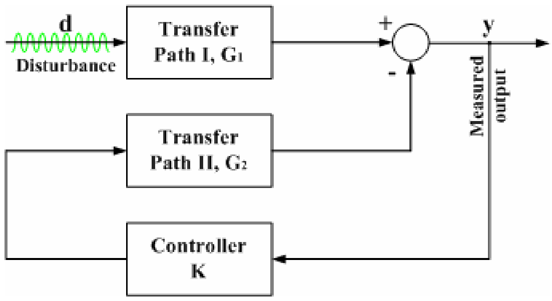


Fig.4 Mechanism of harmonic cancellation

In the block diagram shown in Fig.4, G_1 is usually unknown and G_2 can be obtained by identification. Suppose the controller is of fixed structure, the transfer function from the disturbance d to the output y is as follows:

$$y = (G_2 K + I)^{-1} G_1 d \tag{2}$$

The sensitivity $(G_2 K + I)^{-1}$ determines the attenuation of $G_1 d$ at particular frequencies. In light of equation (2), the controller K plays an important role in shaping the sensitivity. Therefore, to achieve a maximum attenuation of harmonic vibration, the controller K is expected to have as high gain as possible at those discrete frequencies. However, the accompanying problem is the order of K is proportional to the number of sinusoidal components and consequently the controller may be difficult to implement in a real system. To obviate this circumstance, the controller can be chosen as the inverse of G_2 provided that G_2 is invertible. Suppose K is the inverse of G_2, equation (2) then reduces to

$$y = (\mu G_2 G_2^{-1} + I)^{-1} G_1 d \tag{3}$$

where $K = \mu G_2^{-1}$, $\mu > 0$. The cancelled output y now becomes

$$y = G_1 d / (1 + \mu) \tag{4}$$

In light of equation (4), increasing the feedback gain μ will decrease the measured output proportionally. But the gain cannot be arbitrarily given when taking into account unmodeled high frequency dynamics of the system. A structural system comprises actually an infinite number of modes, and an identified model always involves merely the lower order modes. As a result, equations (3) and (4) are valid only within a limited frequency band. To guarantee the invertibility of G_2, the measurement point should be collocated with the active mount since the transfer function from the acting force to the collocated response corresponds to a minimum phase system in this special case. The transfer function can be expressed with a finite number of modes:

$$h(s)=\sum_{r=1}^{N}\frac{\alpha_r}{s^2+2\xi_r\omega_r s+\omega_r^2},\quad \alpha_r\geq 0 \tag{5}$$

where ω_r is the natural frequency of the system. This transfer function is stable and its phase is within $0\sim 180°$.

EXPERIMENTAL STUDY

The experimental model

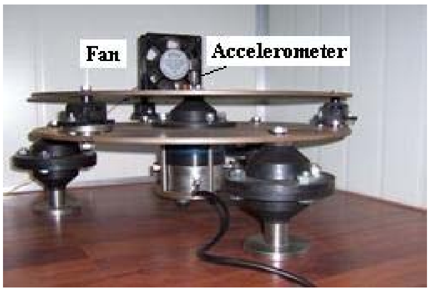


Fig.5 The experimental model

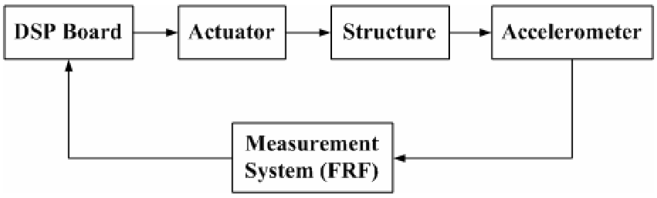
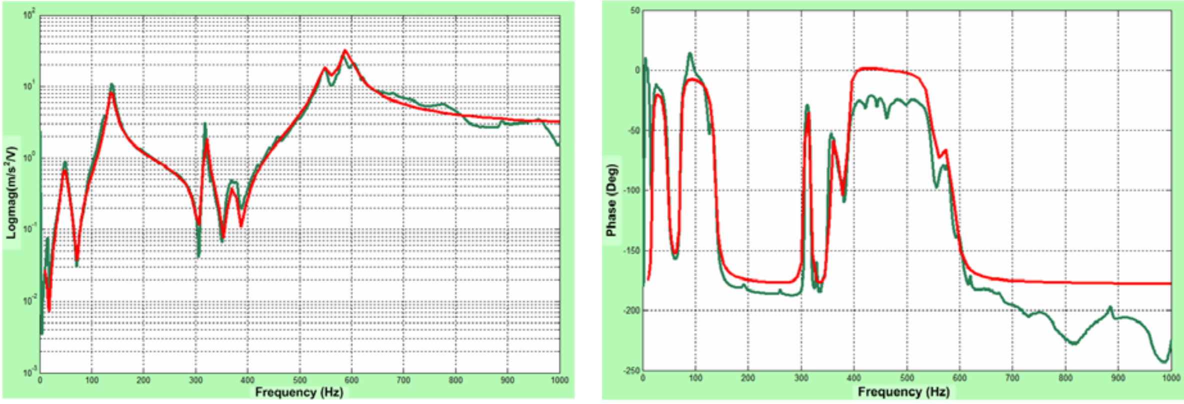


Fig.6 FRF measurement

The experimental model consists of two elastic plates which are suspended on rubber isolators, as shown in Fig.5. There is an active unit (an electromagnetic actuator) installed between the two plates, and an accelerometer is placed in collocation with this active unit. The model is disturbed by a fan mounted on the top of the upper plate. Running of the fan will force the upper plate to vibrate. The active unit is used to counteract the induced vibration in response to acceleration sensed by the accelerometer. The fan can also be installed at the lower plate, but for the sake of easy installation, it is at the present position, which only changes the transfer path I illustrated in Fig.4.

Identification of the transfer path II

The transfer path II is the transfer function G_2 from the signal input to the actuator to the measured acceleration. To obtain G_2, the frequency response function (FRF) was measured at first with a digital measurement system. As illustrated in Fig.6, the measured FRF includes dynamics of the digital signal processing (DSP) board, the actuator, the structure and the accelerometer. Fig.7 gives the measured results (non-smooth curves), which exhibits some time delay in the loop. In fact, time delay is mainly caused by the DSP board, which needs



(a) Magnitude *(b) Phase*

Fig.7 The measured FRF (non-smooth) and the identified FRF (smooth)

time to complete signal converting and computation. To derive an analytical model from the measured FRF, identification was conducted in the time domain. First, the impulse response was obtained from the measured FRF, and then the data were input to the algorithm, ERA, to generate a state space model directly. Equation (6) is the algorithm:

$$H(0)=\begin{bmatrix} h(0) & h(1) & h(2) & \cdots \\ h(1) & h(2) & h(3) & \cdots \\ h(2) & h(3) & h(4) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, H(0)=UV^T, A=U^+H(1)(V^T)^+, B=V(1,:)^T, C=U(1,:), D=h(0) \quad (6)$$

where the Hankel matrix $H(1)$ is a matrix having one row shift from $H(0)$, U^+ is the generalized inverse of U derived from SVD, h is the discrete impulse response, and $U(1,:)$ represents the fist row of U. However, the order of (A, B, C, D) is usually very high and renders the identified (A, B, C, D) difficult to use in a real digital system. Therefore, model reduction is necessary to deriving a small scale model retaining the dominant modes. Fig.7 also gives the FRF of the reduced model, which appears to approximate well to the exact one. Given (A, B, C, D), the inverse model of (A, B, C, D) can be obtained by exchanging the poles and zeros of $D+C(sI-A)^{-1}B$ or by direct manipulating matrices (A, B, C, D), as is given in equation (7).

$$a=A-BD^{-1}C, b=BD^{-1}, c=-D^{-1}C, d=D^{-1} \quad (7)$$

Experiment of active damping and harmonic cancellation

Although the simulation results are satisfactory, performance of these active control strategies needs to be examined on a real system. Fig.8 illustrates the experimental system that consists of all hardware units, in which the digital controller is a TI DSP and its time delay is about 0.04ms consumed to complete minimum computation.

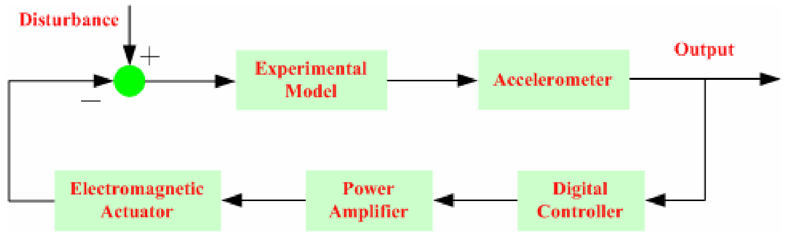
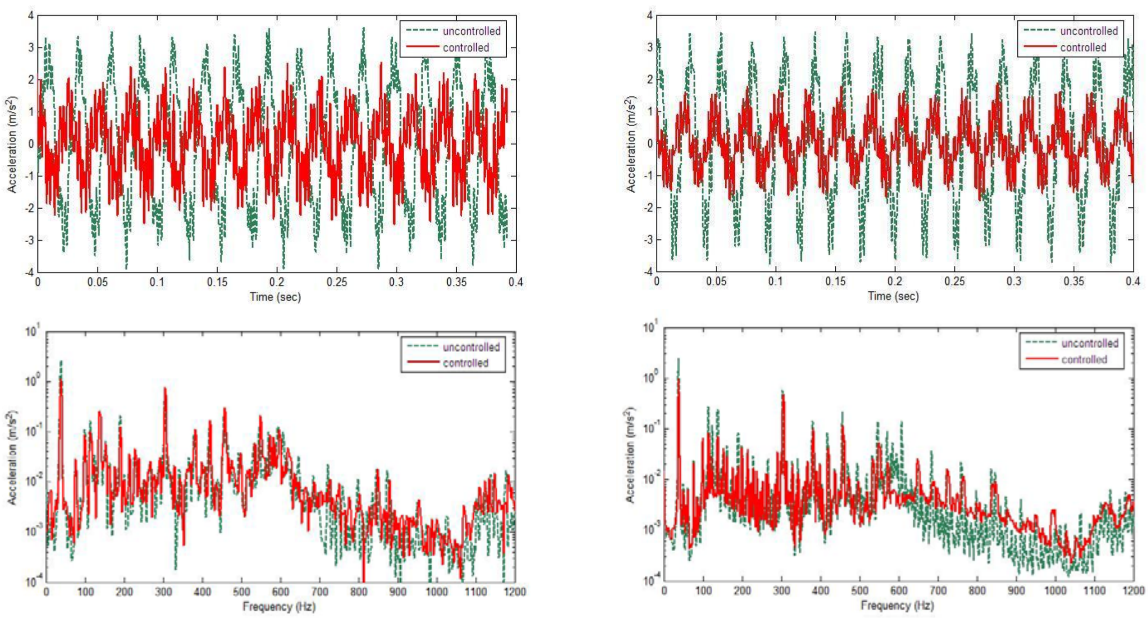


Fig.8 Block diagram of the experimental system

To examine the actual cancellation effect of inverse modeling, the inverse model and a LPF were coded into the DSP board. Disturbance was simulated by the fan (Fig.5), whose fundamental frequency is about 38Hz. Since the disturbance corresponding to this frequency was dominant, the upper plate vibrated nearly at a single frequency. Signal transfer through the digital controller caused time delay of the feedback loop to increase almost to 0.08ms, leading to a reduced gain margin. Therefore, the feedback gain was limited in this experiment.

Fig.9 (a) is the result of vibration control with harmonic cancellation. The cancellation effect is shown to be mainly at the fundamental frequency with a reduction in amplitude by over 50%. However, the vibration of natural modes is not suppressed, which is different from the simulation results. Fig.9 (b) is the result of vibration control with velocity feedback and harmonic cancellation, where the vibration of most natural modes is shown reduced except that the natural vibration at about 300Hz has indiscernible changes. From the time series of responses one also can see the effect of active control, suppressing vibration of the fundamental frequency prominently.

However, the experimental results are not so impressive as in simulation, which can be attributed to time delay in the feedback loop and the flexible support on which the active unit was installed. Time delay reduces gain margin of the closed loop and the flexible support results in complicated modal characteristics at high frequencies. Both can cause instability of the closed loop if unmodeled dynamics become noticeable as feedback gain grows up.



(a) Harmonic cancellation *(b) Velocity feedback + Harmonic cancellation*

Fig.9 Performance of active control strategies

CONCLUSIONS

Active damping and harmonic cancellation based on inverse modeling have been discussed with simulation and experiment. According to the obtained results, active damping is effective to natural vibration suppression and harmonic cancellation is suitable for counteracting harmonic disturbances. However, feedback gain is constrained by time delay as well as support flexibility. In other words, the closed loop is conditionally stable for vibration control/isolation with flexibly mounted actuators. The presented discussion has only dealt with one active mount, which can not give any results about coupling among active mounts. Further work needs to be focused on this issue as well as the improvement of harmonic cancellation based on inverse modeling.

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