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A STUDY OF THE DYNAMIC BENDING STIFFNESS OF SANDWICH BEAMS WITH FOAM-FILLED HONEYCOMB CORES

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Abstract

Sandwich composite materials have been widely used in recent years for the construction of spacecraft, aircraft, and ships, mainly because of their high stiffness-to-weight ratios. The dynamic bending stiffness of such materials is difficult to measure because it depends on frequency unlike ordinary non-composite materials. In this paper, Nilsson's sixth-order differential equation model is reviewed and the frequency response function method is used to characterize the dynamic bending stiffness of sandwich beams with foam filled honeycomb cores. The four-point bending method is used to characterize the static bending stiffness. Experiments with static and vibrating beams show that these methods can yield good estimates for the dynamic bending stiffness.

INTRODUCTION

A honeycomb panel is a thin lightweight plate with a honeycomb core with hexagonal cells. Layered faceplates are bonded to both sides of the core as shown in Fig. 1. Each component is by itself relatively weak and flexible. When combined into a sandwich panel, the elements form a stiff, strong and lightweight structure. The facesheets carry the panel bending loads and the core carries the shear loads. In general, the honeycomb core is strongly orthotropic. The dynamic characteristics should be expected to be different in each direction. The two main principal in-plane directions *x* and *y* are defined in Fig. 1.

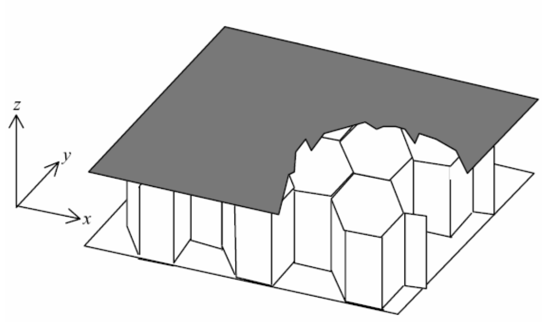


Figure 1 – Sandwich panel with a honeycomb core

A large number of papers have been published on the dynamic properties of sandwich structures. Among them, the Timoshenko [1] and Mindlin [2] models are frequently referenced to. One of the first fundamental studies on the bending and buckling of sandwich plates was published by Hoff [3]. In this paper, Hamilton's principle is used to derive the differential equations governing the bending and buckling of rectangular sandwich panels subjected to transverse loads and edgewise compression. Many of the basic ideas introduced by Hoff form the basis for many subsequent papers on the bending of sandwich plates.

Another classical paper was published by Kurtze and Waters [4] in 1959. The thick core is assumed to be isotropic and only shear effects are included. The bending stiffness of the plate is found to vary between two limits. The high frequency asymptote is determined by the bending stiffness of the faceplates. The model introduced by Kurtze and Waters was later improved by Dym and Lang [5].

Common for many of those references is that the governing differential equations derived are of the fourth order. For increasing frequencies, the results disagree strongly with measured vibrations.

A more general description of the bending of sandwich beams has been given by A. C. Nilsson [6]. In this model, the faceplates are again described as thin plates. However, the general wave equation is used to describe the displacement in the core. When the frequency is increased, Nilsson’s sixth-order differential equation model [6] gives more accurate results.

The aim of this paper is to describe a simple measurement technique for determining some of the material parameters of composite beams.

THEORY OF SANDWICH BEAMS

For a honeycomb beam, the lateral displacement w can be determined when determining the differential equations governing the motion of the structure.

From the study of Eva Nilsson [7], the equation governing w is

$$\begin{aligned} & -2D_2D_1\frac{\partial^6 w}{\partial x^6}+2D_2I_\rho\frac{\partial^6 w}{\partial x^4\partial t^2}-\left(D_1\mu+2D_2\mu+I_\rho G_e t_c\right)\frac{\partial^4 w}{\partial x^2\partial t^2} \\ & +G_e t_c\left(D_1\frac{\partial^4 w}{\partial x^4}+\mu\frac{\partial^2 w}{\partial t^2}\right)+I_\rho\mu\frac{\partial^4 w}{\partial t^4}=\left(D_1+2D_2\right)\frac{\partial^2 p}{\partial x^2}-G_e t_c p-I_\rho\frac{\partial^2 w}{\partial t^2}, \end{aligned} \tag{1}$$

where D_1 is the bending stiffness per unit width of the beam, D_2 is the bending stiffness of a single faceplate, I_ρ is the mass moment of inertia per unit width, μ is the mass per unit area, G_e is the effective shear modulus of the core and t_c is the thickness of the core, as shown in Fig. 2.

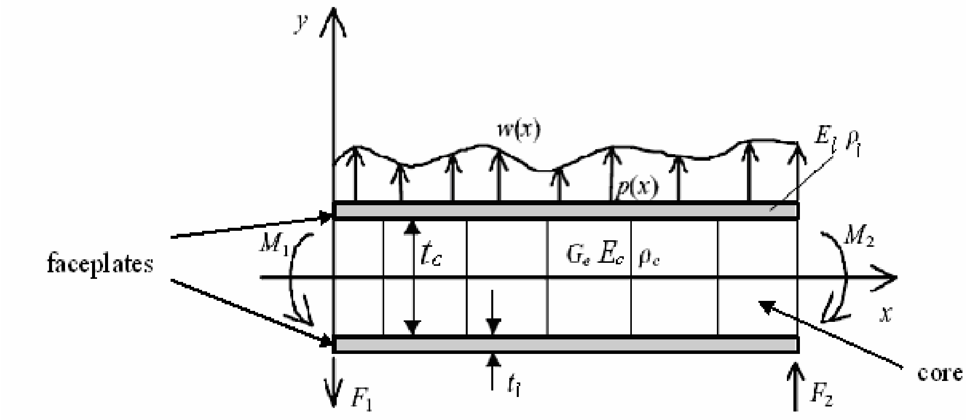


Figure 2 – Excitation of a beam and resulting forces and moments. Dimensions and material parameters for faceplates and core are shown

For free vibrations, the external pressures *p* is equal to zero and for honeycomb panels the moment of inertia can be assumed to be very small. Using these assumptions, the resulting equation for w Eq. (1), is reduced to

$$2D_2\frac{\partial^6 w}{\partial x^6}+\left(\frac{D_1+2D_2}{D_1}\right)\mu\frac{\partial^4 w}{\partial x^2\partial t^2}-G_e t_c\left(\frac{\partial^4 w}{\partial x^4}+\frac{\mu}{D_1}\frac{\partial^2 w}{\partial t^2}\right)=0, \tag{2}$$

which is the wave equation for the bending of beams neglecting the moment of inertia.

Dynamic bending stiffness

The corresponding bending stiffness at the frequency f_n is shown in the following equation according to the ref. [7]:

$$D_n=\frac{\mu 4\pi^2 f_n^2 L^4}{\alpha_n^4}, \tag{3}$$

where α_n is given in Table 1 for three boundary conditions.

Boundary conditions	n	1	2	3	4	5	n>5
Free-free Clamped-clamped	α_n	4.73	7.85	11.00	14.14	17.28	$n\pi+\pi/2$
Free-clamped	α_n	1.88	4.69	7.85	11.00	14.14	$n\pi-\pi/2$
Simply supported	α_n	3.14	6.28	9.42	12.56	15.7	$n\pi$

Table 1 – α_n for three boundary conditions

By means of Eq. (3) the bending stiffness can be determined at each natural frequency. The accuracy of the procedure depends on how well the boundary conditions can be simulated.

The bending stiffness for a honeycomb panels depend on the bending stiffness of each faceplate, the thickness of the faceplates and core and the shear stiffness of the core. The general principles governing the bending stiffness of a honeycomb panel are discussed in reference [7]. The bending stiffness must satisfy the equation:

$$\frac{A}{f}D^{3/2} - \frac{B}{f}D^{1/2} + D - C = 0, \tag{4}$$

where *A*, *B* and *C* are functions of bending and shear stiffness as follows:

$$A = \frac{G_e t_c}{\mu^{1/2} 2\pi D_1}; \quad B = \frac{G_e t_c}{\mu^{1/2} 2\pi}; \quad C = 2D_2 . \tag{5}$$

Using the measured data these parameters can be determined by means of the least square method. The quantity *Q* is defined by:

$$Q = \sum \left(A\frac{D_i^{3/2}}{f_i} - B\frac{D_i^{1/2}}{f_i} + D_i - C \right)^2 . \tag{6}$$

where D_i is the measured bending stiffness at the specific frequency f_i. The parameters *A*, *B* and *C* are to be chosen so that *Q* has a minimum. This means that the constants are solutions to a system of equations obtained from $\partial Q / \partial A = \partial Q / \partial B = \partial Q / \partial C$. By means of this regression analysis, the measured data are used to determine a number of parameters in the formula defining the total bending stiffness of the beam as function of frequency. In this way the bending stiffness can be estimated in the entire frequency range.

Once the constants *A*, *B* and *C* are determined, the static bending stiffness of the sandwich beam D_1 and the bending stiffness of the faceplate D_2 can be determined.

Boundary conditions

The beam must satisfy certain boundary conditions at each end. The three boundary conditions for a beam are summarized in Table 2. β is the angular displacement due to bending of the core.

Clamped ends	$w = 0$	$\beta = 0$	$\frac{\partial w}{\partial x} = 0$
Simply supported ends	$w = 0$	$\frac{\partial \beta}{\partial x} = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$
Free ends	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial \beta}{\partial x} = 0$	$D_1 \frac{\partial^2 \beta}{\partial x^2} = I_\rho \frac{\partial^2 \beta}{\partial t^2}$

Table 2 – Boundary conditions for end of beam

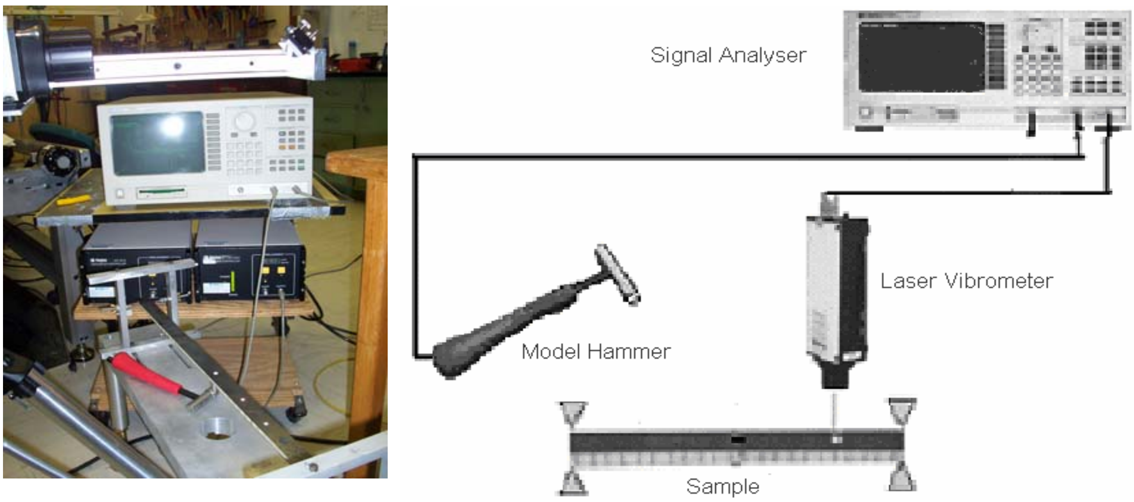
Vibration measurements

For lightweight structures, such as honeycomb beams, excitation by a shaker should be avoided when the frequency response function (FRF) is measured. The mounting of the shaker can change the vibrational modes of the structure considerably. Excitation by an impact hammer is preferred. Since mass is not added to the structure in such measurements. The structure is given an impact which starts the measurement period. Averages are made from 5-12 different excitation points. The drawback with impact hammer excitation is that it is difficult to excite the higher modes of the beam. For a good measurement result it is important to hit the structure with the hammer perpendicular to the surface of the faceplates.

EXPERIMENTS

Set up

In the experiments, the input channel 1 of the dynamic signal analyzer was connected to the modal hammer, which gave an impulse to the beam, and input channel 2 was connected to the laser vibrometer to detect the response of the beam. The dynamic signal analyzer gives the output of the frequency response function of the beam when it is excited by the impulse given by modal hammer. The samples were tested with the three special boundary conditions: free-free, simply supported-simply supported and clamped-clamped. The set up is shown in Fig. 3.



(a) *(b)*

Figure 3 - (a) Experimental setup; (b) Schematic of the experimental setup.

Samples

The sandwich composite structures with honeycomb cores, used in the experiments, are shown on Fig.4. The facesheets are made with woven cloth impregnated with a resin, and the core is a lightweight foam-filled honeycomb structure.

The geometry and density of the sample beam used in the experiments is shown in the Table 3.

Length	Total thickness of the structure	Thickness of one faceplate	Mass per unit area
0.6 m	0.00732 m	0.00038 m	2.176 kg/m^2

Table 3 - Geometry and density of the sandwich beam.

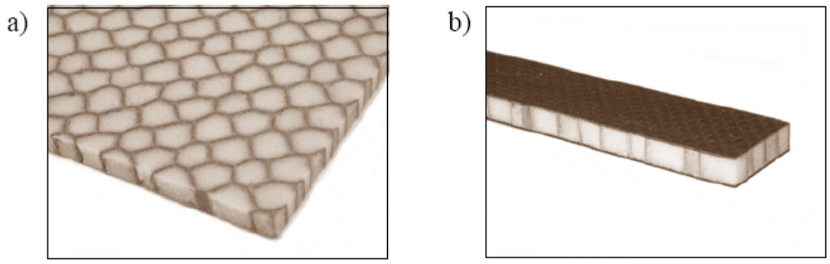


Figure 4 Honeycomb sandwich composite; (a) core, (b) composite beam.

RESULT ANALYSIS

Boundary conditions

In the measurements, the boundaries were found to have some effect. For a clamped beam, shear is induced at the boundaries, thus rendering the beam more flexible as compared to a beam with free ends. The natural frequencies of a clamped beam are consequently lower than the corresponding natural frequencies for the same beam with free ends. Hence, the frequency-dependent stiffness is affected by the boundaries. The natural frequencies for the three special boundary conditions are shown in the Table 4. This is due to the fact that shear in the beam is induced by the clamped boundaries to a larger extend as compared to the case with free boundaries. The apparent bending stiffness for the clamped beam is therefore somewhat lower than for the beam with free ends as shown in Fig. 5.

Boundary conditions	1	2	3	4	5	6	7	8	9	10
Free – free	119.2	317.5	582.9	883.0	1206.5	1579.2	1959.9	2345.8	2806.1	3247.3
Simply supported – simply supported	53.7	206.1	431.6	710.2	1022.4	1373.0	1742.4	2113.1	2542.2	2963.8
Clamped – clamped	118	306.2	557.5	860	1188.5	1557.3	1939.2	2331.2	2789.0	3228

Table 4 - Natural frequencies for the three different beam boundary conditions.

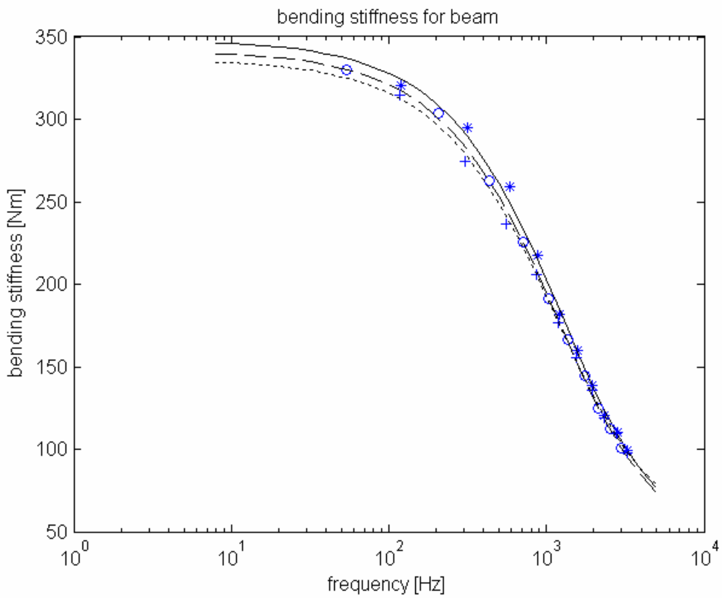


*Figure 5 - Bending stiffness for the sandwich beam for three different boundary conditions. Measurement points *, + and ∘ : bending stiffness for the natural frequencies of the beam with free, simply supported and clamped ends, respectively.*
Solid, dashed and dotted curves: calculated static bending stiffness of the beam with free, simply supported and clamped ends, respectively.

Dynamic bending stiffness

Since the sample used in the experiments is orthotropic, then in the two principle directions, there are two sets of stiffnesses as shown in Fig. 6. The bending stiffness is frequency dependent and decreases with increasing frequency. The bending stiffness at low frequency is dominated by the bending stiffness of the entire beam. At high frequency, the bending stiffness is dominated by the bending of the faceplates only.

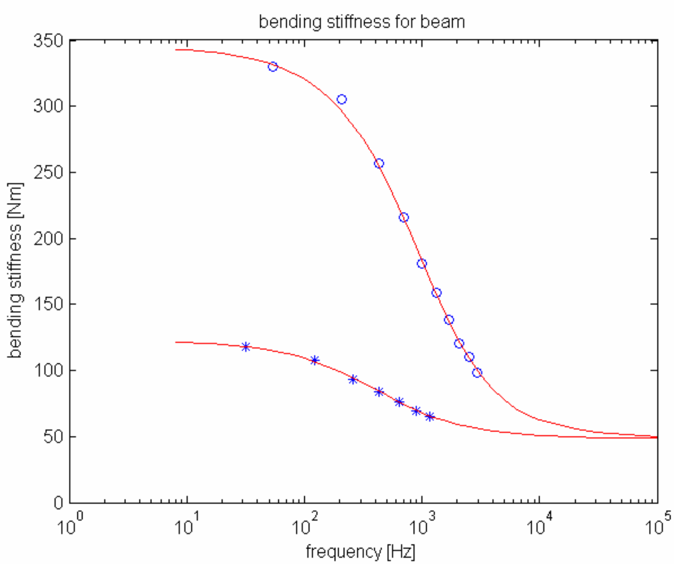


Figure 6 - Bending stiffness for the sandwich beam.
*Measurement points * and ◦ : bending stiffness for the natural frequencies in two principle directions. Curves –: calculated static bending stiffness, based on material parameters for faceplates and core.*

For sandwich beams with isotropic faceplates and orthotropic cores, the stiffness in the two principal directions is found to be equal as the frequency increases as shown in Fig. 7.

The static stiffness of the entire beam D_1 and the static stiffness of the faceplate D_2 are determined by us of the four-point bending method. In the low frequency range, when $\omega \to 0$, the first part of the equation is dominant, and then the apparent dynamic stiffness is close to D_1. The bending stiffness is consequently determined by pure bending of the beam. In the high frequency range, when $\omega \to \infty$, and then the apparent dynamic stiffness is close to $2D_2$. The faceplates are assumed to move in phase. In this frequency range, the bending stiffness for the entire beam is equal to the sum of the bending stiffness of the two faceplates. The comparison for the *x* direction is as shown in Table 5:

Static stiffness by four-point method		Two stiffness limits from dynamic characterization	
D_1 [Nm]	D_2 [Nm]	$\omega \to 0$ [Nm]	$\omega \to \infty$ [Nm]
120	24	122.7	48.6

Table 5 –Comparison of static stiffness measured by four-point bending method and two stiffness limits from dynamic characterization.

CONCLUSIONS

Several theoretical models for the determination of the dynamic bending stiffness of sandwich beams were reviewed. Nilsson's sixth-order differential equation model was introduced. A simple measurement technique for determining the material parameters of composite beams was used. The experimental results show that this technique can be used to determine the dynamic stiffness of composite sandwich beams.

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