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INVERSE METHODS FOR IDENTIFICATION OF CABLES PARAMETERS WITH APPLICATIONS

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Abstract

In the paper is presented a method for estimation of bending stiffness EI and viscous damping β of the cables. Such the cables are overhead line conductors modeled as the beams that respect the author condition which detaches the cable (conductor) model of the beam model. We propose the objective function specified in displacements for identification of the cable parameters. The possibilities of using the finite element method for identification of the parameters are also described. The analytical expressions of the kinetic energy of the cable with viscous or hysteretic damping are analyzed and energy methods are studied for control of vibration of overhead line conductors. The applications are referred in this area.

INTRODUCTION

For identification of the parameters of the cables by inverse methods, we define the objective function using the dimensionless analytical and experimental values of displacements.

The analytical expression of the free vibration modes and the resonance frequencies equation for the cable with some précised conditions on the extremities have produced, using our mathematical model of cables, deduced from Euler-Bernoulli beam model. The finite element method is also used in the paper for identification of cable parameters that compound the Stockbridge dampers, mounted for mitigation of the wind transverse induced vibrations (Karman effect) of overhead line conductors.

We calculate the analytical expression of the kinetic energy of the cable with viscous damping and we analyse the possibilities of using the viscous or hysteretic damping hypothesis.

MATHEMATICAL MODEL OF CABLE

The behavior of the cable (conductor) excited by the force $q(x,t)$, applied transversal on the cable, which action in the point of abscissa x, at the time t, is specified by the equation of motion:

$$\rho A\frac{\partial^2 w(x,t)}{\partial t^2}=-\beta\frac{\partial\, w(x,t)}{\partial\, t}+T\frac{\partial^2 w(x,t)}{\partial x^2}-EI\;\frac{\partial^4 w(x,t)}{\partial x^4}+q(x,t) \tag{1}$$

We note by ρA the mass unit length of cable, EI the bending rigidity of the cable, T the tension in the cable, β the viscous damping of the cable and $y(x,t)$ the corresponding displacement of the cable.

First we search the free transverse vibrations [1]-[5] of the cable without damping and for clamped extremities. The free vibrations are of standing waves form:

$$w_r(x,t)=w_r(x)\sin(\omega_r t+\varphi) \tag{2}$$

In (2) $\varpi_r=2\pi f_r$ and f_r is resonance frequency of the cable. We use:

$$\alpha^2=\frac{TL^2}{EI},\ \beta_r^4=\frac{4\pi^2\rho\, A f_r^2\, L^4}{EI},\ \delta_r=\left\{\frac{\alpha^2}{2}+\left(\frac{\alpha^4}{4}+\beta_r^4\right)^{1/2}\right\}^{1/2},\ \ \varepsilon_r=\left\{-\frac{\alpha^2}{2}+\left(\frac{\alpha^4}{4}+\beta_r^4\right)^{1/2}\right\}^{1/2},\ \xi=\frac{x}{L}$$

The expressions α , β_r , δ_r , ε_r verify the relationships:

$$\delta_r^2-\varepsilon_r^2=\alpha^2,\ \delta_r\ \varepsilon_r=\beta_r^2,\ \delta_r^4-\alpha^2\delta_r^2-\beta_r^4=0,\ \varepsilon_r^4+\alpha^2\varepsilon_r^2-\beta_r^4=0 \tag{3}$$

For detach the cable model of the beam model, we search the solution of (1) using our condition [4], performed by the cable wire:

$$\mathrm{e}^{-\delta_r}\approx\ 0 \tag{4}$$

The condition (4) is our condition of definition for the cable.

Any can verify that $\delta_r\xrightarrow[r]{}\infty$ and implicit $e^{-\delta_r}\xrightarrow[r]{}0$ because $f_r\xrightarrow[r]{}\infty$ for beam model and thus appear the conclusion that the beam model is substituted by the cable model for sufficient large integer r and thus for sufficient high frequency.

For the clamped cable and for the cable with clamped extremity at any end of the cable and the free extremity at another, the equation of resonance frequencies is:

$$\alpha^2\sin\ \varepsilon_r-2\ \beta_r^2\cos\ \varepsilon_r=0 \tag{5}$$

The characteristic equation of the cable with simply supported extremities (the third case analyzed) is now:

$$\sin \varepsilon_r = 0 \tag{6}$$

The analytical expressions of the free vibration modes for the cable, in three mentioned cases of boundary conditions are $w_{1r}(x)$, $w_{2r}(x)$, $w_{3r}(x)$ respectively:

$$\begin{aligned}
w_{1r}(x) &= C_r\left(e^{-\delta_r \xi} + \frac{\delta_r}{\varepsilon_r}\sin\ \varepsilon_r \xi - \cos\ \varepsilon_r \xi - \frac{\delta_r}{\varepsilon_r}\ e^{\delta_r(\xi-1)}\sin\ \varepsilon_r + e^{\delta_r(\xi-1)}\cos\ \varepsilon_r \right) \\
w_{2r}(x) &= C_r\left(e^{-\delta_r \xi} + \frac{\delta_r}{\varepsilon_r}\sin \varepsilon_r \xi - \cos \varepsilon_r \xi + \frac{\varepsilon_r}{\delta_r}\ e^{\delta_r(\xi-1)}\sin \varepsilon_r - \frac{\varepsilon_r^{\ 2}}{\delta_r^{\ 2}}\ e^{\delta_r(\xi-1)}\cos \varepsilon_r \right) \\
w_{3r}(x) &= C_r(\sin(\varepsilon_r \xi) - e^{\delta_r(\xi-1)}\sin \varepsilon_r),\ \xi = x/L
\end{aligned} \tag{7}$$

We specify the following particular solutions of the cable model for equation (1) with $q(x,t) = \beta = 0$, that are also particular solutions of the beam model:

$$w_{1r}(x) = e^{-\delta_r \xi},\ \ w_{2r}(x) = e^{\delta_r \xi},\ \ w_{3r}(x) = \sin\ \varepsilon_r \xi,\ \ w_{4r}(x) = \cos\ \varepsilon_r \xi \tag{8}$$

The identities from (3) can be used for justify the particular solutions (8).

The expressions of $w_r(x)$ from (7) verify equation (1) with $q(x,t) = \beta = 0$ because they are linear expressions of particular solutions from (8) and also they verify the imposed boundary conditions.

The author have deduced such as for the undamped vibrations, the analytical solution of the damped free vibration modes of the cable that verify the equation (1) with $q(x,t) = 0$.

If the initial conditions for the searched solution $w_i(x,t) = X_i(x)\ T_i(t)$ (with i fixed) of the equation (1) with $q(x,t) = 0$ are chosen $w_i(x_o,t_o) = D_{oi}$, $\frac{\partial w_i}{\partial t}(x_o,t_o) = V_{oi}$, where $X_i(x)$ is a vibrating mode from (7), then we have the following expression of them [5]:

$$w_i(x,t) = \frac{X_i(x)}{X_i(x_o)} e^{-\frac{\beta}{2\rho A}(t-t_o)} \{ (\frac{\beta}{2\rho A}\frac{D_{oi}}{\omega_{i\beta}} + \frac{V_{oi}}{\omega_{i\beta}})\sin \omega_{i\beta}(t-t_0) + D_{oi}\cos\ \omega_{i\beta}(\ t-t_o)\} \tag{9}$$

$$\omega_{i\beta} = \left\{ \omega_i^2 - (\beta_A)^2 \right\}^{1/2},\ \beta_A = \frac{\beta}{2\rho A},\ \omega_i \ge \beta_A,\ i = 1,2,...$$

The formula (9) is used for performing the objective function referred to unknown parameters EI, β, deduced by theoretical and experimental values of cable displacements and using the principle of the weighted least square method.

DETERMINATION OF THE PARAMETERS

The bending rigidity EI and the viscous damping β of the cable may be calculated using the theoretical $w_i^c(x,t)$ and experimental $w_i^{\exp}(x,t)$ values for the damped free vibration modes. The objective function is considered of the form:

$$f(EI,\beta)=\sum_{i,x,t} w_i(\, w_i^c(x,t)-w_i^{\exp}(x,t))^2 \tag{10}$$

where the index "i" is in the range determined by the number of the resonance frequencies supposed in the model, the index "x" represents a number of the points taken into account belong of the cable, the index "t" represents a number of the time values taken into account and the coefficient w_i represents the weight taken equal with $1/(w_i^{\exp}(x,\ t))^2$. The objective function is thus dimensionless.

The experimental values are measured by use an experimental stand with single overhead cable. The utility length of the cable is 33 m, the ends of the cable are fixed, T =1333 N and ρA = 0.757 kg/m. A vibrations exciter situated at the middle of the span generates the resonance frequencies used in the model.

The analyzed frequencies, according to experimental values, have the values $f_9 = 11.83\ Hz$, $f_{15} = 19.5\ Hz$, $f_{19} = 24.98\ Hz$, the points considered are $x_1 = 0.089\ m$, $x_2 = 2.0\ m$, $x_3 = 16.18\ m$ at the moments that specify the main values of the displacements. The values of the bending rigidity (EI = 40 Nm^2) and of viscous damping ($\beta = 2.675$ Ns/m) are determined by minimization of the function (10).

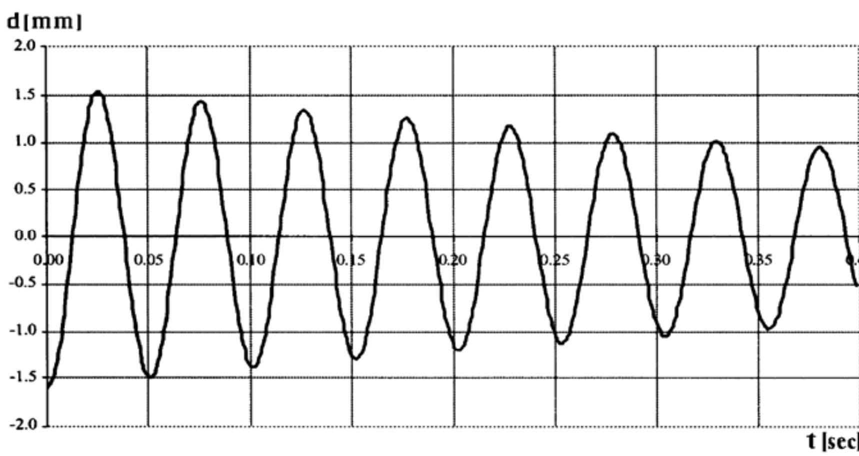

Figure 1 The diagram of damped displacement for resonance frequency

We present above (Fig.1) the diagram of damped displacement of cable for the resonance frequency of 19.5 Hz and for the point of abscissa $x = 16.18\ m$ of the span.

The bending rigidity of the cable that compound a damper and a damping of them was chosen by inverse method, using the experimental diagram of the dissipated energy versus frequency of the damper (Fig.2) and the theoretical diagram obtained by finite element method (FEM).

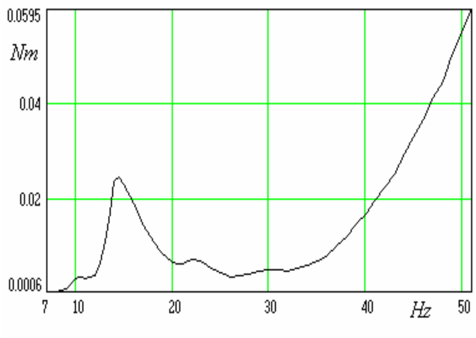

Figure 2 - Metered energy of damper Avb5

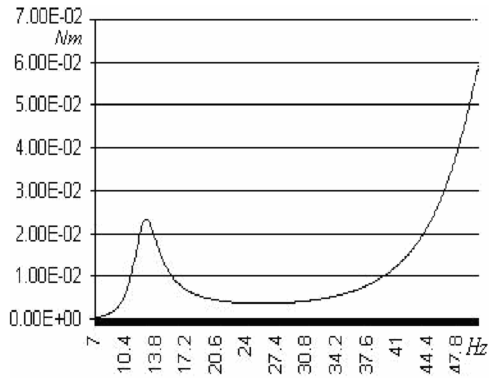

Figure 3 - Energy of damper Avb5 with FEM

The theoretical energy dissipated per cycle by the damper is performed, using the following formula [1]:

$$E(f) = \pi F(f) D(f) \sin(\varphi(f)), \tag{11}$$

where $E =$ energy dissipated per cycle by the damper for frequency f, $F =$ reacting force in the clamp for the same frequency f, $D =$ displacement of the damper in the clamp, $\varphi =$ phase angle.

The energy dissipated per cycle by the damper, in the clamp, is drawn in Fig. 3, where the parameters chosen for the cable that compound Stockbridge damper are $EI = 11\ [Nm]$ and the damping ratio 0.145%.

THE ENERGY BALANCE PRINCIPLE

We consider the analytical expression (9) of solution $w_i(x,t) = X_i(x)\ T_i(t)$ (with fixed i) of damped free vibrations for equation (1) where $q(x,t) = 0$.

This solution is for clamped cable to the extremities and with D_{0i}, V_{0i} initial conditions. Thus dissipated kinetic energy of the cable, per cycle, is:

$$E_{ic} = \frac{1}{2} m \iint_{(x,t)} \dot{T}_i^2(t) X_i^2(x) dx dt = \frac{1}{2} m \int_{t_0}^{t_0+\frac{2\pi}{\omega_i}} \dot{T}_i^2(t)\ (\int_0^L X_i^2(x) dx)\ dt \tag{12}$$

where $m = \rho A$ is the mass unit length of the cable and A section area of the cable.

First, we calculate the integral referred to variable x. We deduce as below:

$$\int_0^L X_i^2(x)dx = \frac{L}{\delta_i} + \frac{L}{2}(\frac{\delta_i^2}{\varepsilon_i^2}+1) + L\sin^2\varepsilon_i(\frac{2\delta_i}{\delta_i^2+\varepsilon_i^2} - \frac{\delta_i}{2\varepsilon_i^2} - 2\frac{\delta_i^2}{\varepsilon_i^2}\frac{\delta_i}{\delta_i^2+\varepsilon_i^2}) +$$
$$+L\cos^2\varepsilon_i\ (\frac{1}{2\delta_i} - \frac{4\delta_i}{\delta_i^2+\varepsilon_i^2}) + L\frac{\sin 2\varepsilon_i}{2\varepsilon_i}(2\frac{\delta_i^3}{\varepsilon_i\,(\delta_i^2+\varepsilon_i^2)} + 4\frac{\delta_i^2}{(\delta_i^2+\varepsilon_i^2)} - 2\frac{\varepsilon_i^2}{\delta_i^2+\varepsilon_i^2} - \frac{\alpha^2}{2\varepsilon_i^2}) \quad (13)$$

We discover that $\int_0^L X_i^2(x)dx \to L$ for $i \to \infty$ and therefore $\int_0^L X_i^2(x)dx \approx L$ for sufficient high frequencies. Also we calculate:

$$\int_{t_0}^{t_0+\frac{2\pi}{\omega_{i\beta}}} \dot{T}_i^2(t)dt = \frac{1}{X_i^2(x_0)}\int_{t_0}^{t_0+\frac{2\pi}{\omega_{i\beta}}} e^{-2\beta_A(t-t_0)}(V_{0i}\cos\omega_{i\beta}(t-t_0) -$$
$$-(\omega_i^2\frac{D_{oi}}{\omega_{i\beta}} + \beta_A\frac{V_{oi}}{\omega_{i\beta}}\ )\sin\omega_{i\beta}(t-t_0))^2\,dt \quad (14)$$

We get:

$$\int_{t_0}^{t_0+\frac{2\pi}{\omega_{i\beta}}} \dot{T}_i^2(t)dt = \frac{1-e^{-\frac{4\pi\beta_A}{\omega_{i\beta}}}}{4X_i^2(x_0)\beta_A}\ (V_{0i}^2+\omega_i^2 D_{0i}^2) \approx \frac{\pi}{X_i^2(x_0)\omega_{i\beta}}(V_{0i}^2+\omega_i^2 D_{0i}^2) \quad (15)$$

where the last equality is verified for sufficient high frequencies ($\frac{4\pi\beta_A}{\omega_{i\beta}} \to 0$).

In the initial conditions for the damped free vibration $w_i(x,t)$ can suppose that $V_{0i}=0$ when D_{0i} is the amplitude of the vibration. In this case we get, for sufficient high frequencies, when also $\int_0^L X_i^2(x)dx \approx L$:

$$E_{ic} = \frac{1}{2}m\int_{t_0}^{t_0+\frac{2\pi}{\omega_{i\beta}}} \dot{T}_i^2(t)\ (\int_0^L X_i^2(x)dx)\ dt \approx \frac{\pi\rho AL}{2X_i^2(x_0)\omega_{i\beta}}\omega_i^2 D_{0i}^2 \quad (16)$$

It is known [1] that the energy dissipated per cycle by cable, vibrating in the mode "i", in hysteretic damping hypothesis, is given by:

$$E_{ic} = \pi\frac{h_i L}{2}D_{0i}^2 \quad (17)$$

In (17) D_{0i} is the amplitude of the vibration, $V_{0i}=0$, L is the span length and h_i is the hysteretic damping coefficient for mode "i", $h_i = H\ i^3/L^3/8$, H constant.

From the relations (16) and (17) we find a correspondence between hysteretic and viscous damping coefficient such that to be dissipated the same energy, namely:

$$h_i = \rho A\ \omega_i^2 / \omega_{i\beta} / X_i^2(x_0),\ \omega_{i\beta}^2 = \omega_i^2 - \beta_A^2 \tag{18}$$

Concerning the wind energy there are several workers [3] which have been studied the wind energy imparted to the conductor. We use the relations:

$$\frac{P^w}{f^3 D^4} = \frac{E^w}{f^2 D^4} = \text{fnc}\,(\frac{y_o}{D}) = 10^z \tag{19}$$

where P^w is the wind power of a unit length of conductor, E^w is the wind energy of a unit length of conductor, f is frequency, D is the conductor diameter, y_o is the antinodes amplitude, y_o / D is defined as dimensionless amplitude, $\text{fnc}\,(\frac{y_o}{D})$ is named a reduced power (or reduced energy) of the wind. For the experimental diagram (fig.4) of Diana (Italy) about the wind energy, we propose [4] the following analytical expression of the value $z = \sum_{n=o}^{9} a_n X^n$, $X = \log_{10}(2\,y_o / D)$, in formula (19):

$$a_0 = 1.26575,\ a_1 = 1.69387,\ a_2 = -1.08622,\ a_3 = -12.7859,\ a_4 = -34.1620,$$
$$a_5 = -43.7526,\ a_6 = -31.583,\ a_7 = -13.190,\ a_8 = -2.96931,\ a_9 = -0.27798$$

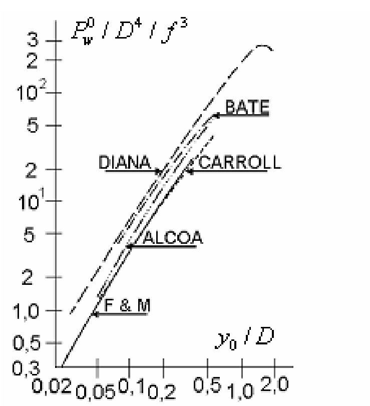

Figure 4 – The empirical Power (Energy) reduced of the wind

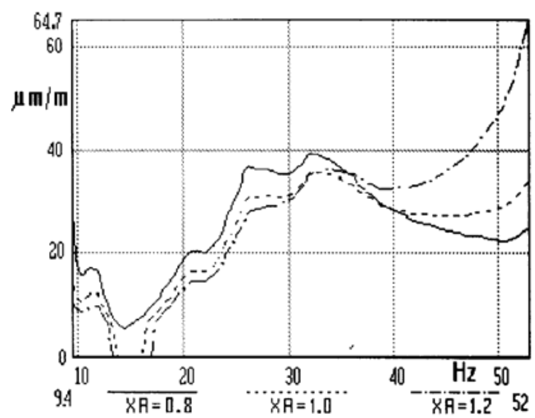

Figure 5 - DBS for cable with Avb5, L= 400m

The dynamic bending strain ε_r of the cable, at the rigidly clamped extremities, can be expressed by the following empirical relationship:

$$\varepsilon_r = k_\varepsilon d\ y_r(x_b) \tag{20}$$

Is denoted above $x_b =\ 0.089m$, d the diameter of outer strands of the cable and $y_r(x_b)$ the bending amplitude for the mode "r" of cable vibration.

The energy balance principle is expressed by the following equation:

$$E^w = E^c + E^d \tag{21}$$

with E^w the energy induced by the wind, E^c and E^d the energy dissipated by the cable for any mode of vibration, respectively by the dampers mounted on the span of transmission line. The energy balance permits us to determine amplitude of the vibration mode analyzed and the dynamic bending strain of the cable.

To illustrate the method, it was selected a span of a ACSR 54/19 cable with $D = 3.15\ 10^{-2}[m]$, $d = 3.5\ 10^{-3}[m]$, $H = 10791\ [Nm]$, $m = 1.982\ [Kg/m]$, $k_\varepsilon = 390\ [m^{-2}]$, $EI = 40\ [Nm^2]$, $\rho = 1.25\ [Kg/m^3]$, $T = 40050\ [N]$

In Fig. 5 are drawn the diagrams of dynamic bending strain versus frequency of the cable with span of $400m$ and with damper Avb5 mounted at the extremities of the span in the points XA specified.

CONCLUSIONS

The original analytical considerations about the definition of the cable and evaluation of the kinetic energy of the cable in viscous damping hypothesis, permits us to discuses a possible equivalence between the viscous or hysteretic damping hypothesis for the cable. The inverse methods and the energy balance principle presented here, can be used to establish whether or not is necessary to equip the cable of transmission line by Stockbridge dampers, their number and optimum spacing on the cable to assure the enlargement of the life time of the cable.

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