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THE $\overline{\overline{G}}^{i}$ MATRIX : A TOOL TO MEASURE MODAL DENSITY AND APPROXIMATE VIBRATION RESPONSE AT MEDIUM FREQUENCY

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Abstract

This paper deals with an approach to measure the number of modes dominating the vibration response of structures excited in a frequency band : for system with low modal overlap the method gives a tool for measurement of modal density and for system of high modal overlap it permits to build reduced models from measured data.

INTRODUCTION

Vibration modes are commonly used to represent behaviour of vibrating systems, however in medium and high frequency this representation is not well adapted because of the huge number of modes participating to the response and of modal overlap . In this frequency range methods have been proposed to fit with phenomena , in particular Statistical Energy Analysis is widely used , Lyon [1]. A key point of these approaches is the use of modal density instead of keeping all modal information . Several papers have presented theoretical calculation of modal density ,that can also be measured by counting resonance frequencies appearing in frequency response functions of vibrating systems , however this is not easy specially for inhomogeneous industrial structures . In addition this method fails when modal overlap is achieved and a method based on input impedance measurement were proposed by Bourgine [2] and Clarkson [3] however it is not applicable to inhomogeneous structures. Using these and techniques , measurement of modal density have been reported for plates and cylinders (Clarkson and Pope [4]) or sandwich panels (Renji [5]) .

In this paper a different approach is presented to estimate experimentally or theoretically the number of modes dominating the vibration response of structures excited in a frequency band , namely , the effective modal density describing the response can be estimated with dominating eigen values of the $\overline{\overline{G}}^i$ Matrix .
When modal overlap is achieved the response is no more dominated by isolated modes but by a smaller number of groups of modes responding together . In this case it is not possible to estimate modal density with the present approach , however a small number of eigen vectors of the $\overline{\overline{G}}^i$ Matrix can be used to directly calculate the response of the structure instead of using standard mode decomposition . Experimental application of the method appears as an extension for medium and high frequency problems, of experimental modal analysis used at low frequency .

DEFINITION OF THE $\overline{\overline{G}}^i$ MATRIX

A linear system is driven at point X_i by an harmonic excitation of angular frequency ω_k and unit amplitude . The response at point X_j has the form (1) :

$$W^i(X_j,\omega_k)exp(j\omega_k t),\ \omega_k=\Omega_{min}+k\Delta \quad ,\ k=0,N-1 \tag{1}$$

The $\overline{\overline{G}}^i$ Matrix defined in equation (2) characterizes the response of the system at points X_j and angular frequency ω_k :

$$\overline{\overline{G}}^i=[g^i(j,k)]=[W^i(X_j,\omega_k)] \tag{2}$$

The response at point X_j when the excitation has the form (4) can be calculated with the $\overline{\overline{G}}^i$ Matrix as indicated in (5) or (6) .

$$F^i(t)=\sum_{k=1}^{N} A^i_k exp(j\omega_k t) \tag{3}$$

$$\left\{W^i\left(X_j,t\right)\right\}=\overline{W}^i\left(t\right)=\sum_{k=1}^{N} g^i\left(j,k\right)A^i_k \exp\left(j\omega_k t\right)=\overline{\overline{G}}^i\overline{F}^i\left(t\right) \tag{4}$$

where the driving force vector is introduced as :

$$\overline{F}^i(t)=\{A^i_k exp(j\omega_k t)\} \tag{5}$$

In these expressions $\{\ \}$ or a bar on a capital letter indicate a vector and a double bar on a capital letter indicates a matrix .

REDUCTION OF INFORMATION TO ESTIMATE THE RESPONSE

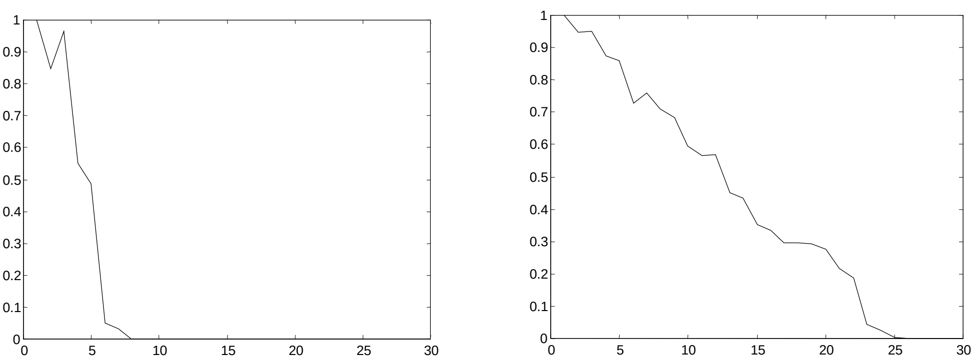
When medium or high frequency is concerned a lot modes are responding meaning that a lot of modal information is necessary for describing exactly vibrations of the system . However, simplified model can be established by reducing the modal information contained in the $\overline{\overline{G}}^i$ Matrix from eigen values and eigen vectors decomposition. One can neglect the second order contributions just ignoring the effects of small eigen values . In a standard procedure one can write the response in the form :

$$\overline{W}^i(t)=\sum_{\alpha=1}^{N} a^i_\alpha(t)\overline{V}^i_\alpha \cong \sum_{\alpha=1}^{M} a^i_\alpha(t)\overline{V}^i_\alpha \qquad (6)$$

with $\{a^i_\alpha(t)\}=\left[\langle \overline{V}^i_\alpha . \overline{V}^i_\beta \rangle\right]^{-1}\left\{\langle \overline{V}^i_\beta \overline{W}^i(t) \rangle\right\}$, and $[\ ]^{-1}$ indicates the inverse matrix (7)

EXAMPLE OF BEAM LONGITUDINAL VIBRATION

A beam of length L, cross section S, made of homogeneous material , excited by an harmonic force of unit amplitude at point X_i is considered. The response at point X_j can be calculated with forced waves decomposition (see [6], chapter 10) permitting the $\overline{\overline{G}}^i$ matrix calculation. Influence of resonant modes number, on $\overline{\overline{G}}^i$ matrix eigen values is presented figure 1, in the case of well separated modes (low modal overlap) .The major conclusion that can be drawn, is that the number of significant eigen values indicates the number of modes dominating the response , corresponding generally to resonant modes in the excited frequency band , however, when the number of resonant modes is large the number of significant eigen values is smaller, for example 30 modes and 25 significant eigen values in case 2 .



6 resonant modes in the excited band,. *30 resonant modes in the excited band*

Figure 1 Modulus of Normalized Eigen values of the $\overline{\overline{G}}^i$ matrix in the case of different frequency band of excitation .

This tendency was expected in the sense that some resonant modes can have a small response because excitation point located close to a node . Thus, dominant eigen values of the $\overline{\overline{G}}^{i}$ matrix permits estimation of modal density , however, this modal density is associated to a particular excitation point and appears as an effective modal density for a given excitation . Eigen vectors are presented in the following by plotting a continuous shape function . Figure 3 presents imaginary and real parts plots of the first eigenvector . The curves represent displacements along the beam that can be used as elementary motion to decompose beam vibration when excited in the frequency band . Shapes oscillate with an average wavelength slowly variable with eigenvalue order . The average wavelength of the first eigen vector shape is approximately equal to 2L/24, those of the eleventh eigen vector shape to 2L/29 , they lie in between maximum and minimum natural wave length in the excited band (2L/20 to 2L/35) .

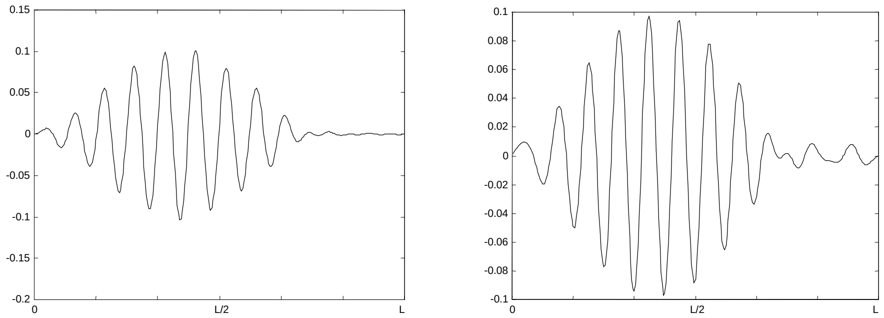
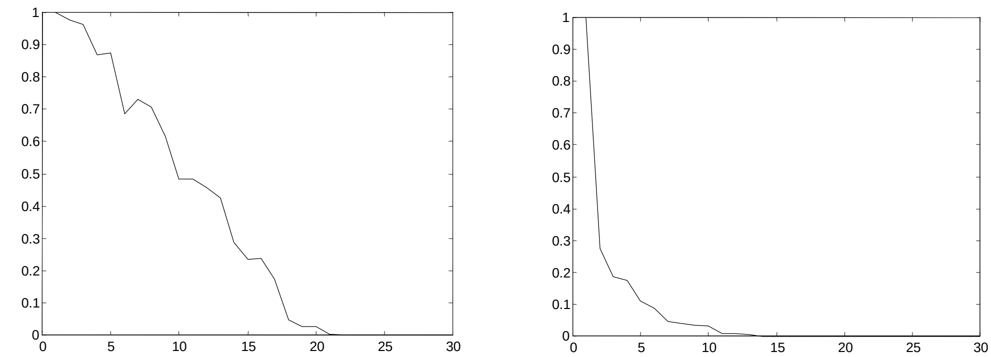


Figure 3) Real and imaginary part of the first eigenvector. Excitation at point 0.066L, modal overlap varying from 0.2 to 0.35, resonant modes : 20 to 35.

In a second time the influence of modal overlap is studied , it is important because in medium and high frequency problems, modes are often overlapping . This is done using the same beam as Figure 1, with increased damping loss factor.



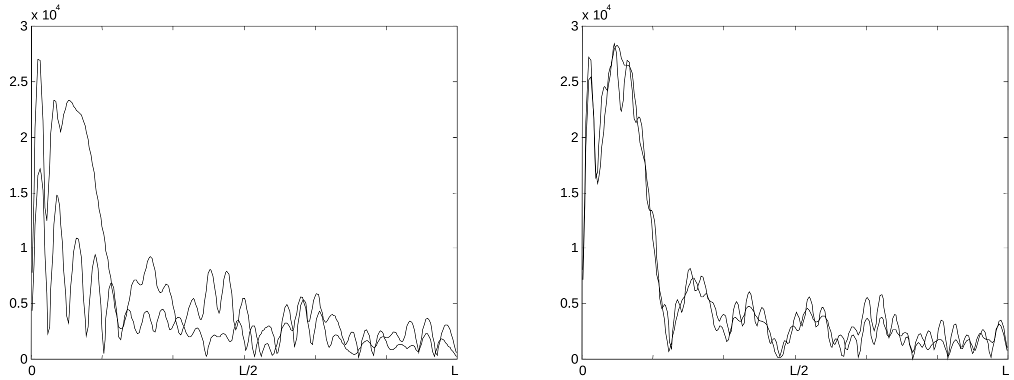
Case 1) modal overlap varying from 0.035 to 0.059. *Case 2) modal overlap varying from 1.120 to 1.880*

Figure 4. Normalized Eigen values of the $\overline{\overline{G}}^{i}$ matrix in the case of different modal overlap conditions . .24 resonant modes in the excited band.

The important point that can be seen in figure 4 is the decreasing influence of high order eigen values when modal overlap increases . In this case the method to estimate the modal density fails , but it describes reality in the sense that few elementary displacement shapes due to modes accumulation are sufficient to represent vibration field .

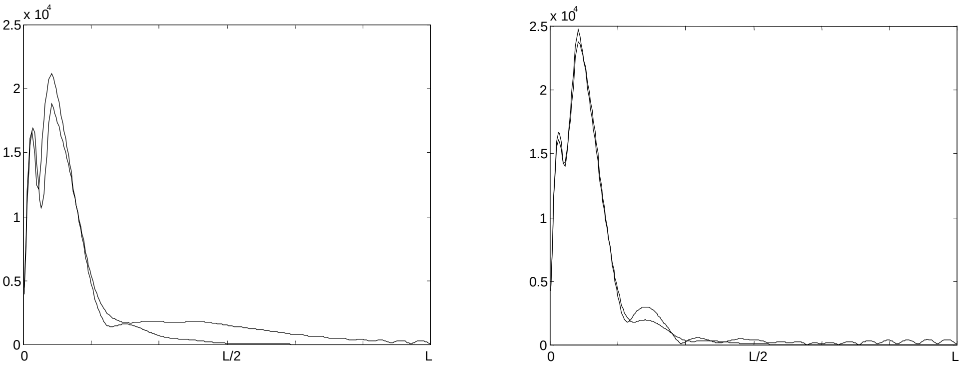
CALCULATION OF VIBRATION FIELD BY EXPANSION ON EIGEN VECTORS

For calculating the beam response at instant t the excitation vector must be defined , of course when time is varying the vector is changing . For sake of simplicity, a single excitation vector (representing the state of excitation at a given time) is used to calculate the beam response, it is constructed randomly in order to have a global tendency. In figure 5, the beam response is reconstructed for a variable number of eigen vectors , the case of 16 resonant modes with low modal overlap is considered . The exact solution is slightly different with studied case because of the random choice of the force vector .The reconstruction is improved when the number of eigen vectors increases . When it reaches the number of resonant modes the reconstruction is very good however some differences still exist and can be associated to non resonant modes contribution .
When modal overlap is achieved (figure 6), using two eigen vectors to reconstruct the displacement field is sufficient to have a good approximation even if 16 resonant modes lies in the excited band .



a) 5 eigen vectors *b) 16 eigen vectors*

Figure 5 . Comparison of exact and reconstructed modulus of vibration displacements along the beam for different number of $\overline{\overline{G}}^{i}$ *matrix eigen vectors. Low modal overlap case.*



a) 2 eigen vectors   *b) 5 eigen vectors*

Figure 6 .Comparison of exact and reconstructed vibration field along the beam for different number of eigen vectors. Modal overlap varying from 2 to 3.5 in the frequency band .

MEASUREMENT OF MODAL DENSITY ON LIGHTLY DAMPED PLATE

A steel rectangular plate (0.6 length, 0.4 width and 0.0007 thickness) was mounted in a rigid frame and transfer functions measured from one excited point to several receiving points located all over the plate . A laser vibrometer was used in order to scan the vibration field with a regular mesh. $\overline{\overline{G}}^{i}$ matrices were measured for different frequency band and corresponding eigen values calculated . Each time 380 measurement points all over the plate, using a regular mesh and 380 frequencies in a band, were used . One example is presented in figure 7, corresponding to the frequency band [0 Hz, 190 Hz]. The theoretical number of resonant modes in the band is 21.3 modes . Twenty one significant $\overline{\overline{G}}^{i}$ matrix squared normalized eigen values appears in accordance with expected value .

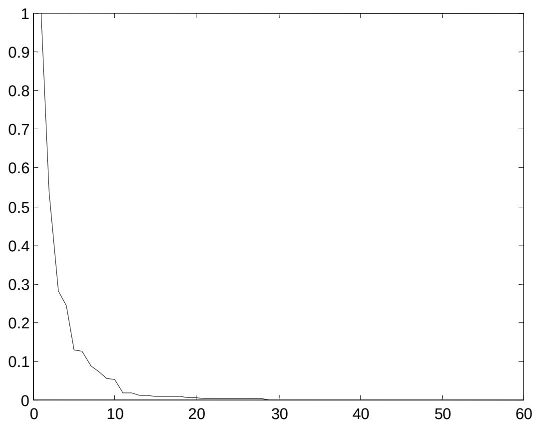


Figure 7. Measured $\overline{\overline{G}}^{i}$ *matrix normalized eigen values square moduli, frequency band [0 Hz, 190 Hz].*

RESPONSE OF A HIGHLY DAMPED PLATES FROM MEASURED $\overline{\overline{G}}^{i}$ MATRIX EIGEN VECTORS

Same measurements were done on a highly damped sandwich panel in order to construct the $\overline{\overline{G}}^{i}$ matrix . Eigen values are no more used to measure modal density because of modal overlap, but eigen vectors can be used to expand the plate response expecting that very few of them are sufficient to obtain a good approximation of displacement field.

The plate response is presented in figure 8 , the exact result is compared to the approximate displacement calculated with one and two eigen vectors . Displacements are plotted following the laser scanning order , namely points located on the first line then those of the second line , etc .This representation explains low displacement values appearing at a repeated period corresponding to the first and last points on a line, that are close to boundaries.

The expected result is clearly demonstrated : one eigen vector expansion is sufficient to obtain a quite good prediction , only small differences appear at low vibration level

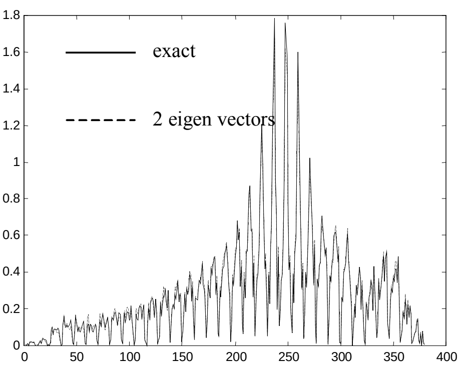


Figure 18 . Comparison of highly damped plate responses calculated directly with measured $\overline{\overline{G}}^{i}$ *matrix and approximated with : left : 1 eigen vector, left : 2 eigen vectors, frequency band [380 Hz, 570 Hz] .*

This result validates the possibility , when modal overlap is achieved , of representing vibration behaviour of structures excited by broad band forces, by expansion on few eigen vectors of the $\overline{\overline{G}}^{i}$ matrix . This is of high interest in medium and high frequency problems where the use of standard mode decomposition in not adapted .

Experimentally the method appears as an extension of experimental modal analysis techniques for medium and high frequency problems .

CONCLUSION

In this paper an approach based on the so called $\overline{\overline{G}}^i$ Matrix has been presented to estimate the number of modes dominating the vibration response of structures excited in a frequency band. It has been demonstrated that the effective modal density describing the response can be estimated with dominating eigen values of the $\overline{\overline{G}}^i$ Matrix . When modal overlap is achieved , that is often the case in medium and high frequency acoustic problems , the number of dominant eigen values of the $\overline{\overline{G}}^i$ Matrix decreases drastically indicating that response is no more dominated by isolated modes but by a smaller number of groups of modes responding together . In this case it is not possible to estimate modal density with the present approach , however the small number of eigen vectors of the $\overline{\overline{G}}^i$ Matrix can be used like a reduced model to directly calculate the response of the structure instead of using standard mode decomposition . The approach has been used experimentally and the possibility of modal density measurement demonstrated for plates having low damping . For highly damped sandwich panel , a reduced model built with only two eigen vectors of the experimentally build $\overline{\overline{G}}^i$ Matrix was sufficient to approximate accurately the plate response . The method appears finally as an extension of standard modal analysis for experimental identification of modes at low frequencies .

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REFERENCES

[1|R . H. Lyon, Statistical energy analysis of dynamical systems: Theory and Applications, M.I.T. press , Cambridge (1975) .

[2]A. Bourgine, Sur une approche statistique de la dynamique vibratoire des structures , (On a statistical approach of the structural vibrations). Thèse de Doctorat es sciences , Paris , (1973) .

[3] B. L. Clarkson , The derivation of modal densities from point impedances , Journal of Sound and Vibration 77 (4) (1981) 583-584.

[4] B. L. Clarkson and R. J. Pope , Experimental determination of modal densities and loss factors of flat plates and cylinders , Journal of Sound and Vibration 77 (4) (1981) 535-549.

[5] K. Renji , Experimental modal densities of honeycomb sandwich panels at high frequencies , Journal of Sound and Vibration 237 (2000) 67-79 .

[6] J.-L. Guyader, Vibrations des Milieux Continus , Hermes Science publication , Paris , (2002) , 445 p.