

# A RECEPTION PLATE METHOD OF MEASUREMENT OF THE SOURCE MOBILITY OF MACHINES IN BUILDINGS

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# Abstract

The major obstacle to a practical structure-borne source characterization is the difficulty in acquiring and assembling the requisite source data, of which there are two components. The first is the vibration activity of the source, conventionally measured as free velocity or a blocked force spectra, at each contact and usually for more than one component of excitation. The second is the mobility (or impedance) spectra, again at each contact and for each degree of freedom. Therefore, large data sets must be acquired and processed and the acquisition of mobility data is often difficult. A simplified method of obtaining the source mobility is described where the machine or component under test is attached to a low-mobility (thick) plate. The reception plate power, along with previously measured source free velocity, gives an approximate estimate of the average real part of the effective source mobility. This quantity can be simply expressed as single frequency banded value. A description is given of numerical models of reception plates systems, to achieve optimum structural dynamic performance. In particular, the reception plate is required to have a low mobility but with the high modal overlap factor. Reception plate estimates of source mobility are compared with directly measured values and the simplifying assumptions of the method are discussed.

### **INTRODUCTION**

A practical structure-borne sound source characterisation ideally should be a quantity indicative of source strength and involve source factors only. At the same time, it should be in a form appropriate for combining with receiver factors in predicting sound transmission in installed conditions.

The real part of the power *P* transferred from a source into a receiver can be expressed in terms of the free velocity  $v_{sf}$  of the source and the source and receiver mobilities,

$$P = \frac{1}{2} \left| v_{sf} \right|^2 \frac{\text{Re}(Y_R)}{\left| Y_s + Y_R \right|^2}$$
(1)

 $Y_s$  is the complex mobility at the contact of the source and  $Y_R$  is the complex mobility at the contact of the receiver. Therefore, three independent quantities are required to predict the power in the installed condition.

The measurement of the mobility spectra, at each contact and for each degree of freedom, is often difficult and complicated. The required effort is too great for a practical laboratory method.

This paper considers a simplified method of obtaining the source mobility by using an indirect measurement approach, where the machine or component under test is attached to a low-mobility (thick) reception plate.

### STRUCTURE-BORNE SOUND TRANSMISSION

The power transmitted to a receiving structure due to a single component of excitation at a contact point is given by Equation (1) in terms of the free velocity  $v_{sf}$  of the source and the source and receiver mobilities. In this study, forces perpendicular to the receiver plate only are considered since there is increasing evidence that although other components of excitation, particularly moments, cannot be neglected a priori, perpendicular forces generally dominate the structure-borne transmission into floors [1, 2].

For a source connected through multiple (N) contacts, the power through the ith contact is obtained from Equation (1) but where point mobility terms are replaced with effective mobilities,  $Y^{\Sigma}$ .

$$P_{SRi} = \frac{1}{2} |v_{sfi}|^2 \frac{\text{Re}(Y^{\Sigma}_{Ri})}{|Y^{\Sigma}_{Si} + Y^{\Sigma}_{Ri}|^2}$$
(2)

where

$$Y_i^{\Sigma} = Y_i + \sum \frac{F_j}{F_i} Y_{i,j}$$
(3)

 $Y_i$  is the point mobility at the i<sup>th</sup> contact,  $Y_{i,j}$  is the transfer mobility between the i<sup>th</sup> and j<sup>th</sup> contacts and  $\frac{F_j}{F_i}$  is the ratio of the forces at the j<sup>th</sup> and i<sup>th</sup> contact, respectively. Since the forces at the contacts cannot be known priori to the installation of the source, some assumption, concerning the force distribution, is required. For this study, a unit force distribution was assumed with zero phase difference between forces.

When the source of mobility  $Y_s^{\Sigma}$  is installed on a reception plate of mobility  $Y_R^{\Sigma}$ , if  $|Y_{Si}^{\Sigma}| \gg |Y_{Ri}^{\Sigma}|$  for all contacts i, then the total structure-borne power is

$$P_{SR}^{Total} \approx \frac{1}{2} \sum_{i}^{N} \frac{\left| v_{sfi} \right|^{2}}{\left| Y^{\Sigma}_{Si} \right|^{2}} \operatorname{Re}(Y_{Ri}^{\Sigma})$$
(4)

If it is assumed that  $\operatorname{Re}(Y_R^{\Sigma}) = \frac{1}{N} \sum_{i}^{N} \operatorname{Re}(Y_{Ri}^{\Sigma})$ , and  $\left|Y_S^{\Sigma}\right| = \frac{1}{N} \sum_{i}^{N} \left|Y_{si}^{\Sigma}\right|$ 

then equation (4) becomes

$$P_{SR}^{Total} \approx \frac{1}{2} \operatorname{Re}(Y_{R}^{\Sigma}) \frac{\sum_{i}^{N} |v_{sfi}|^{2}}{|Y_{s}^{\Sigma}|^{2}}$$
(5)

Total power  $P_{SR}^{Total}$ , from the source into a reception plate, can be obtained from the spatial average of the squared velocity,  $\tilde{v}^2$ ,

$$P = \tilde{\tilde{v}}^2 \eta \ \omega \ m \tag{6}$$

 $\eta$  is the loss factor of the plate of mass m.

Since  $\operatorname{Re}(Y_R^{\Sigma})$  is known, and  $v_{sfi}$  can be obtained from direct or indirect measurement, then the average value of  $|Y_S^{\Sigma}|$  can be estimated.

### NUMERICAL MODEL

A numerical model of a reception plate was assembled in order to conduct a preliminary study of the indirect method of obtaining source mobility. The reception plate was modeled as a homogeneous plate, according to the compendium of Gardonio and Brennan [3]. This allowed consideration of excitation by forces perpendicular to the plane of the plate and through moments about axes in the plane of the plate (see Figure 1 for dimensions, coordinate system and source-receiver contact positions). The reception plate was assumed to be 100 mm concrete with free edges. The assigned damping loss factors were obtained from laboratory measurements [2].

In Figure 2 shown is the predicted point mobility at a central position from [3] and the measured value from [4].



Figure 1: Dimensions, coordinate system and contact positions for idealised reception plate.



Figure 2: Predicted point mobility and measure value from laboratory reception plate.

#### Effective mobility of the reception plate

The transfer and point mobilities of reception plate can be calculated by using the homogeneous thin plate model at source contact positions. Then the receiver or reception plate effective mobilities,  $Y_{Ri}^{\Sigma}$ , can be obtained by using Equation (3).

#### Powers into the plate

When the machine of interest is attached to a simple reception plate, the structureborne sound transmission, through all contacts and components of excitation, is rendered down to a single value, equal to the reception plate power obtained from Equation (6).

In Figure 3 is shown the reception plate power for a single contact source at a central location using Equation (6), from an average of 5 velocities and also from an average of 10 velocities. Also shown is the exact power obtained from Equation (1). Results indicate that a small sample of 5 accelerometer positions will give a satisfactory approximation to the power  $P_{SR}^{Total}$  [4].



*Figure 3: Structure-borne power into the reception plate according to equation (6) for five and ten sample points, compared with exact value according to equation (1).* 

# SOURCE EFFECTIVE MOBILITY

#### **Case study 1: Whirlpool Bath**

The first source considered in this study was a whirlpool bath. In Figure 4 is shown the point mobility at one foot of the whirlpool bath. Also shown is the characteristic (i.e. infinite plate) mobility of the 100 mm concrete reception plate. It shows that the mobility of the reception plate is significantly lower than that of the source, so that Equation (5) applies.

The whirlpool bath was located at a central position, in contact with the reception plate through four mounts, Figure 1. The reception power,  $P_{SR}^{Total}$ , which is assumed to contribute to the bending field on the plate only, is obtained from Equation (6) from the mean square plate velocity. The average value of  $|Y_S^{\Sigma}|$  then can be estimated through Equation (5). In Figure 5 is shown the estimated average of  $|Y_S^{\Sigma}|$  as one third octave values. Also shown is the value of  $|Y_S^{\Sigma}|$  from directly measured source data.



Figure 4: Point mobility,  $Y_s$ , at one foot of a whirlpool bath and the characteristic mobility of the reception plate,  $Y_c$ .



Figure 5: The Magnitude of average effective mobility of a whirlpool bath

The agreement is within 5 dB over the frequency range of interest.

#### **Case study 2: Fan Unit**

The second structure-borne source considered was a fan unit, again connected through four mounts. In Figure 6 is shown the point mobility at one contact point. Again, the characteristic mobility of the reception plate also is shown, indicating that Equation (5) can be applied.



Figure 6: The point mobility,  $Y_s$ , at one contact point of the fan unit and the characteristic mobility of the reception plate,  $Y_c$ .

The average value of  $|Y_s^{\Sigma}|$  again can be estimated through Equation (5) as shown in Figure 7. Also shown is the value of  $|Y_s^{\Sigma}|$  from directly measured source data.



Figure 7: The Magnitude of average effective mobility of a fan unit

The agreement is within 5 dB over most of the frequency range with a difference of 10 dB at an anti-resonance of the source at 100 Hz. This is likely to be the result of the modal behaviour of the reception plate at those frequencies when the mobility of the reception plate is not significantly lower than that of the source.

# CONCLUSIONS

A reception plate method is used to obtain a source mobility representation, which corresponds to an average of the effective mobility at the contacts. This mobility quantity can be obtained without the requirement of direct measurement. Instead, the effective mobility of thick reception plates is required, which is relatively easy to measure.

The estimated source effective mobility results give promising agreement with measured source data over the frequency range of interest. The complicated source mobility measurement process can be simplified as a practical laboratory test that still gives a good accuracy.

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