ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

HIGHER-ORDER SPECTRA FOR ON-LINE IDENTIFICATION OF ANC PLANTS

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Abstract

In the paper a new approach to the *on-line* identification of electro-acoustic plants models for active noise control systems, based on the higher-order spectra aided by signal averaging is presented. Two solutions are proposed: first employing the higher-order spectra only and second employing the both higher and second-order spectra. Proposed identification methods allow to reduce an influence of disturbances on identification results and obtain strongly consistent estimates in contrast to classical second-order identification algorithms. Signal averaging enhances signal-to-noise ratio without deteriorating noise attenuation. Results obtained with the proposed identification methods, computed on the basis of data acquired in the laboratory experiments, are provided and compared with results of classical second-order identification methods.

INTRODUCTION

Adaptive active noise control (ANC) systems are parameterized with the models of the secondary and feedback paths. If these paths are time-varying, the models have to be updated during ANC system operation [1]. Several problems like inherent feedback loops, low signal-to-noise ratio, and correlation of the identified path input and disturbance make *on-line* identification very difficult [2], [3]. In this case classical identification algorithms using second-order moments and spectra often give biased and inconsistent estimates. The proposed solutions employ higher-order spectra (HOS) or together higher- and second-order spectra aided by signal averaging. HOS contain some additional information about processed signals that is not available to extract using second-order spectrum (SOS) i.e. power spectral density. HOS are identically zero for some processes, including Gaussian processes, what is useful property from identification point of view. So if the disturbance signal is Gaussian, it theoretically doesn't influence the identification results, and in practice this influence is significantly reduced [8], [9], [13]. The proposed approach requires a special, non-Gaussian, excitation signal. In the laboratory experiments the excitation sequence is repeated several times and the measured data are averaged. Averaging enhances signal-to-noise ratio without deteriorating noise attenuation, and improves Gaussian properties of the disturbance. This allows to obtain strongly consistent estimates [13]. The results of real-world experiments show the effectiveness of proposed approach to electro-acoustic plants models identification in comparison to classical second-order identification methods.

ANC SYSTEM AND ON-LINE IDENTIFICATION PROBLEM

A feedforward active noise control system of interest, described in details in [1], [2], [3], creates a local spatial zone of quiet surrounding an error microphone in an enclosure. The laboratory room plan and block diagram of the ANC system is shown in *Figure 1*.

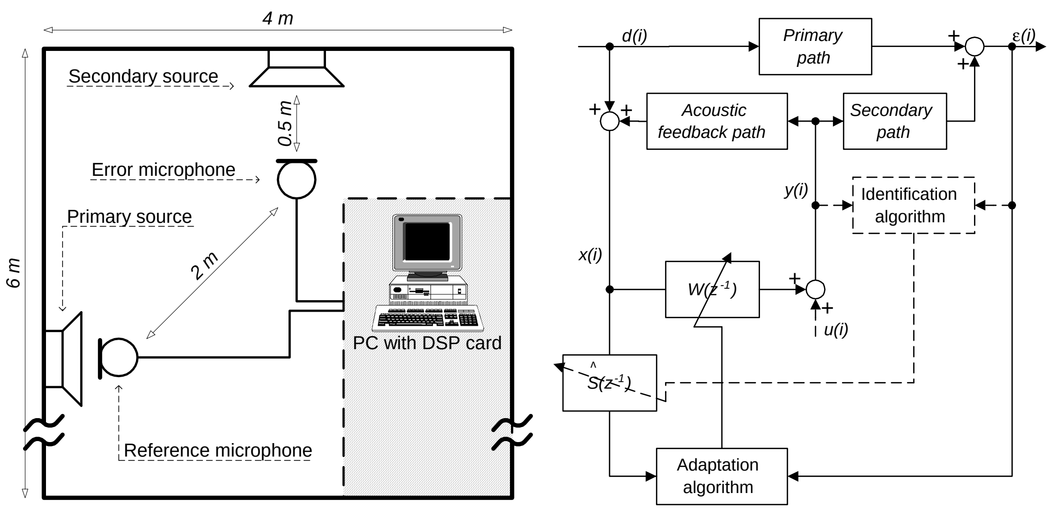

Figure 1: Laboratory room plan (left) and ANC system block diagram (right).

The ANC laboratory enclosure is placed at the Institute of Automatic Control, Silesian University of Technology, Gliwice, Poland [5]. The *primary path* represents the acoustic space between the source of an undesirable noise (disturbance) and the error microphone, and the *secondary path* corresponds with all electro-acoustic elements between the output of a control algorithm and the error microphone. The control signal $y(i)$, being a result of filtration of the disturbance (measured by the reference microphone, placed near the primary source) through a digital filter $W(z^{-1})$, is used to attenuate the disturbance. The coefficients of $W(z^{-1})$ are updated using adaptation algorithm that minimizes the mean square value of the signal $\varepsilon(i)$ from the error microphone. The reference signal $x(i)$ has to be filtered through the secondary path model $\hat{S}(z^{-1})$ before it is used in the adaptation algorithm. Since the sound generated by the secondary source affects also the reference microphone, there exists an *acoustic feedback path*, composed analogously to the secondary path. This influence may be compensated by the suitable length of filter $W(z^{-1})$. Filter $\hat{S}(z^{-1})$ has to be identified *off-line* (before activating the ANC system), and in many cases it should be also updated *on-line* (while ANC system is operating). The need of *on-line* identification follows from possible changes of geometry (e.g. movements of microphones, loud-speakers, people and objects) and conditions (e.g. temperature, humidity) in the enclosure. There are three main difficulties connected with *on-line* identification in the ANC system:

- It is a closed-loop system identification problem and it implies that the input and output of the identified path are correlated. Moreover, if the system works well, the identified ANC plant is not excited sufficiently. Therefore the external excitation $u(i)$ has to be introduced into the system and added to the control signal.
- The disturbance $d(i)$ affects simultaneously the input and output of the ANC plant. So there exists a "hidden variable" that influences both control signal $y(i)$ and error signal $\varepsilon(i)$. To avoid this undesirable influence a HOS-based identification method is proposed.
- The higher the ratio of the external excitation to the disturbance variances is, the better the identification results are. On the other hand, the external excitation reduces the noise attenuation, so increasing its variance is against the purpose of ANC system. Therefore the signal averaging procedure is proposed.

IDENTIFICATION METHODS

Data Acquisition - Signal Averaging

The signal averaging procedure is well known in system identification [4], [10]. In this procedure the periodic excitation sequence $u(i)$, $i = 1, 2, \ldots, N * M$, is composed of M repetitions of N-samples long primitive sequence $u_1(i)$, $i = 1, 2, \ldots, N$. Acquired secondary path input data $y(i)$ data are divided into M blocks and averaged according to the equation

$$\overline{y}(i) = \frac{1}{M} \sum_{m=0}^{M-1} y(i + m * N). \tag{1}$$

Averaged output signal $\overline{\varepsilon}(i)$ is calculated in the same way. The secondary path model is identified on the basis of the averaged data. This technique let us to improve signal-to-noise ratio without deterioration of the noise attenuation, because the variance of averaged disturbances is reduced proportionally to the number of repetitions M. Another useful property of averaging reveals when identification based on HOS is applied. According to the *Central Limit Theorem*, the averaged disturbance asymptotically approaches the Gaussian distribution [11], [12]. So the averaging procedure let us successfully use HOS-based methods, even if the disturbance does not satisfy the Gaussianity assumption.

Second-Order Estimator

The most known nonparametric identification method is the classical spectral analysis, employing second-order spectra to obtain the frequency response estimator $\hat{S}_{SO}(\mathrm{j}\Omega n)$

$$\hat{S}_{SO}(\mathrm{j}\Omega n) = \frac{\Phi_{\overline{y}\overline{\varepsilon}}(\mathrm{j}\Omega n)}{\Phi_{\overline{y}\overline{y}}(\Omega n)}, \tag{2}$$

for frequencies $\Omega = 2\pi/N$, $n = 0, 1, \ldots, N-1$ [7]. The estimates of input power spectral density $\Phi_{\overline{y}\overline{y}}(\Omega n)$ and input-output cross-power spectral density $\Phi_{\overline{y}\overline{\varepsilon}}(\mathrm{j}\Omega n)$ in the direct approach (periodogram) can be calculated as a multiplication of the proper *N*-point discrete Fourier transforms of $\overline{y}(i)$, and $\overline{\varepsilon}(i)$

$$\overline{Y}(\mathrm{j}\Omega n) = \sum_{i=0}^{N} \overline{y}(i)\mathrm{e}^{-\mathrm{j}\Omega ni}, \quad \overline{E}(\mathrm{j}\Omega n) = \sum_{i=0}^{N} \overline{\varepsilon}(i)\mathrm{e}^{-\mathrm{j}\Omega ni}, \tag{3}$$

according to the equations

$$\Phi_{\overline{y}\overline{y}}(\Omega n) = \frac{1}{N}\overline{Y}(\mathrm{j}\Omega n)\overline{Y}^*(\mathrm{j}\Omega n), \quad \Phi_{\overline{y}\overline{\varepsilon}}(\mathrm{j}\Omega n) = \frac{1}{N}\overline{Y}(\mathrm{j}\Omega n)\overline{E}^*(\mathrm{j}\Omega n), \tag{4}$$

where superscript * denotes complex conjugation.

Higher-Order Estimator

The proposed identification method is based on direct estimators of integrated bispectra (IB) [9], [13]. In the IB method the frequency response estimator $\hat{S}_{HO}(\mathrm{j}\Omega n)$ is a ratio of integrated cross-bispectrum of input and output signals to the integrated bispectrum of input signal, according to the equation

$$\hat{S}_{HO}(\mathrm{j}\Omega n) = \left(\frac{IB_{\overline{y}\overline{\varepsilon}}(\mathrm{j}\Omega n)}{IB_{\overline{y}\overline{y}}(\mathrm{j}\Omega n)}\right)^*, \tag{5}$$

where

$$IB_{\overline{y}\overline{y}}(\mathrm{j}\Omega n) = \frac{1}{N}R_{2\overline{y}}(\mathrm{j}\Omega n)\overline{Y}^*(\mathrm{j}\Omega n), \quad IB_{\overline{y}\overline{\varepsilon}}(\mathrm{j}\Omega n) = \frac{1}{N}R_{2\overline{y}}(\mathrm{j}\Omega n)\overline{E}^*(\mathrm{j}\Omega n), \tag{6}$$

and the proper *N*-point discrete Fourier transforms of $\overline{y}(i)$, $\overline{\varepsilon}(i)$ and $r_{2\overline{y}}(i)$

$$\overline{Y}(\mathrm{j}\Omega n) = \sum_{i=0}^{N} \overline{y}(i)\mathrm{e}^{-\mathrm{j}\Omega ni}, \;\; \overline{E}(\mathrm{j}\Omega n) = \sum_{i=0}^{N} \overline{\varepsilon}(i)\mathrm{e}^{-\mathrm{j}\Omega ni}, \;\; R_{2\overline{y}}(\mathrm{j}\Omega n) = \sum_{i=0}^{N} r_{2\overline{y}}(i)\mathrm{e}^{-\mathrm{j}\Omega ni}. \tag{7}$$

Signal $r_{2\overline{y}}(i)$ is defined as

$$r_{2\overline{y}}(i) = \overline{y}^2(i) - \frac{1}{N}\sum_{k=1}^{N}\overline{y}^2(k). \tag{8}$$

The IB estimates obtained in the direct way should be smoothed in the frequency domain, otherwise we receive the empirical transfer function estimator [7], and the identification method reduces to second-order.

Mixed-Order Estimator

Unlike the classical second-order identification methods, the IB-based method allows to obtain strongly consistent estimates in the presence of disturbance [13]. On the other hand the variance of the classical second-order estimates, outside the dominant frequency band of disturbance, is usually smaller than the variance of the higher-order ones [6]. A new solution is proposed that join the unbias of higher-order and low variance of second-order estimates. In this solution the frequency response estimator $\hat{S}_{MO}(\mathrm{j}\Omega n)$ is a linear combination of classical and higher-order estimators, weighted by the coherence function and one minus coherence function, respectively

$$\hat{S}_{MO}(\mathrm{j}\Omega n) = \kappa^2(\Omega n)\hat{S}_{SO}(\mathrm{j}\Omega n) + (1-\kappa^2(\Omega n))\hat{S}_{HO}(\mathrm{j}\Omega n), \tag{9}$$

where $\kappa^2(\Omega n)$ is the coherence function of $\overline{y}(i)$ and $\overline{\varepsilon}(i)$

$$\kappa^2(\Omega n) = \frac{|\Phi_{\overline{y\varepsilon}}(\mathrm{j}\Omega n)|^2}{\Phi_{\overline{yy}}(\Omega n)\Phi_{\overline{\varepsilon\varepsilon}}(\Omega n)}. \tag{10}$$

The frequency response estimate can be easily transformed to parametric model using e.g. the least squares approximation of frequency response for fixed model structure described in [13].

LABORATORY EXPERIMENTS

During the laboratory experiments, the ANC system was working with the sampling frequency 1 kHz. The attenuation frequency band was limited from below up to 20 Hz (by dynamical properties of the control loud-speaker) and from above down to 350 Hz (by anti-aliasing and forming analog filters). The enclosure was disturbed by a computer-simulated colored Gaussian noise or a real pump noise. Both disturbances have similar dominant frequency band, see power spectral densities in *Figure 2*.

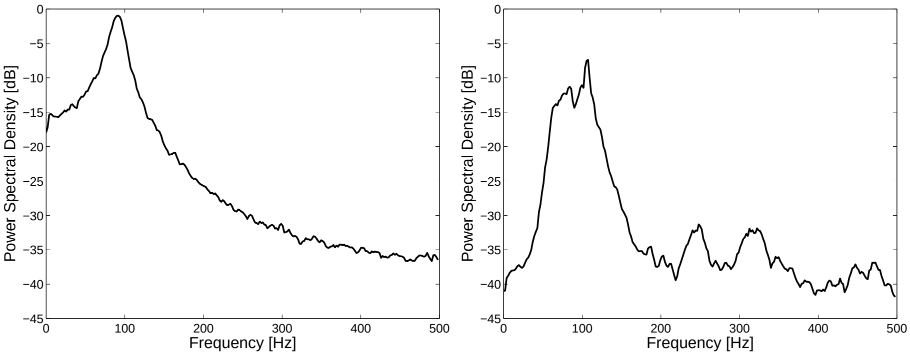

Figure 2: Power spectral density of colored Gaussian (left) and pump (right) noises.

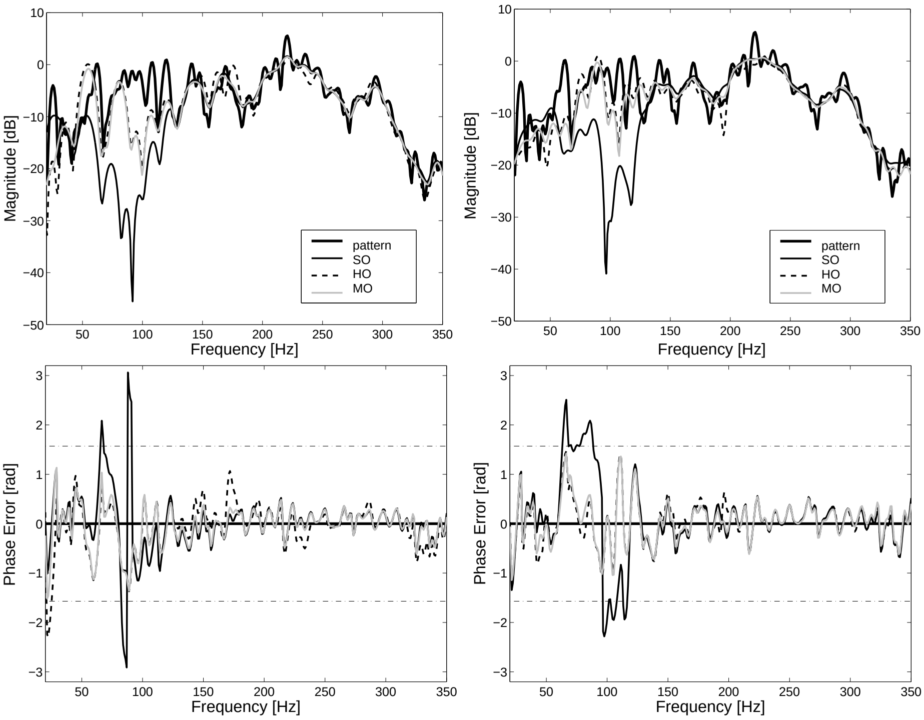

Figure 3: Frequency response magnitude and frequency response phase error of obtained estimates for colored Gaussian, $M = 5$ (left) and pump, $M = 20$ (right) disturbances.

Using each of the discussed identification methods a parametric model (a FIR filter with 100 coefficients) of the secondary path was found. An exemplary identification results are shown in *Figure 3* for different number of repetitions M, what follows from different powers of excitation signal (one-sided exponential white noise) in these experiments. Result of an additional *off-line* experiment in the undisturbed ANC system is treated as a pattern to which *on-line* identification results are compared. In the figure the phase is illustrated as the phase error, equal to the obtained frequency response phase minus the phase of the pattern. The phase errors should be in the range $(-\pi/2, \pi/2)$ to assure stability of the ANC system. We can observe that classical second-order estimates give greater errors than higher-order and mixed-order what influences satisfaction of the phase error condition. However, outside the dominant frequency band of disturbances, the second-order estimates are a little better than higher-order, which defect of higher-order estimator can be eliminated by using of mixed-order estimator.

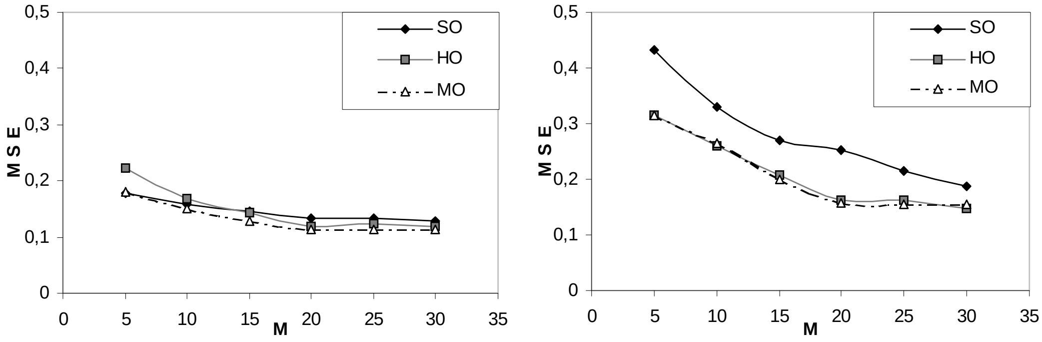

Figure 4: Mean square error of estimates in function of number of repetitions M in frequency bands from 40 to 350 Hz (left) and from 70 to 110 Hz (right)

Mean square error (MSE), representing a mean (square) distance between the obtained estimates and the pattern in a fixed frequency band, was calculated for two frequency bands: from 40 to 350 Hz and from 70 to 110 Hz. The first frequency band corresponds to the whole attenuation band, the second frequency band corresponds to the dominant frequency band of disturbances. Averaged MSE of 10 Monte Carlo experiments is shown in *Figure 4*. We can observe that in the first frequency band results of all methods are comparable, but in the second frequency band higher-order and mixed-order methods produce lower MSE than classical second-order methods.

CONCLUSIONS

In the paper two identification methods based on the higher-order spectra and signal averaging were proposed to solve a problem of the *on-line* identification of electro-acoustic plant models for active noise control systems. The proposed methods allow to reduce an influence of disturbances on identification results what was illustrated by laboratory experiments results.

Acknowledgements

The partial financial support of this research by The Polish Ministry of Education and Science (grant 3 T11A 012 29) is gratefully acknowledged.

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