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AN EXPERIMENTAL APPLICATION OF THE EQUIVALENT SOURCE METHOD TO SOUND RADIATION

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Abstract

The paper treats the acoustic radiation of a finite cylinder using the equivalent source method. The cylinder is mechanically excited by a white noise. The velocity distribution on the cylinder surface is found by measurements. The surface velocity reconstruction with minimum errors is made by optimizing the positions of equivalent simple sources (monopoles) and using different methods to solve the associated matrix equation. A comparison between the measured and synthesized acoustic pressure is incorporated into the procedure. The computing time is optimized by minimizing the number of sources without decrease of the accuracy. Using a suitable number of measurement points the error in computing the radiated acoustic power can be minimized.

INTRODUCTION

To solve the problem of acoustic radiation by a complex vibrating structure, different methods can be used. The most used method for solving acoustic radiation of vibrating bodies is the Boundary Element Method (BEM) [1]. It knows that exist many problems with the numerical implementation of Helmholtz integral in BEM and in the last decades many authors improved BEM (CHIEF method [2], boundary element multigrid method, etc).

The equivalent source method is moderately used in acoustics. In this paper the method is used for the computation of radiation of a vibrating body. The method consists in replacing the vibrating body with an acoustically equivalent set by simple sources located inside the body. This method is also called "wave superposition method [3]", "substitute source method" and when the equivalent sources are multipoles the method is sometimes called "method of multipole synthesis" [4].

The equivalent source method appears like an alternative to BEM. Some authors consider it to be superior to the usual boundary integral formulation of the exterior acoustic problem for two reasons. First, because the source and the collocation points never coincide, which avoids the singularity of the Green function and second because the non-existence and no uniqueness problems associated with the classical boundary integral formulation do not exist.

The advantage of this method is that the accuracy of solution for a radiator with a given surface boundary condition can be directly quantified.

The major disadvantage is the lack of indication of suitable positions of equivalent sources. The underlying principle of the equivalent source method in no way anticipates such positions.

Two tasks are associated with the equivalent source method. One is to find the (complex) source amplitudes at some given source positions which minimize the velocity error function. This task can be considered as little challenging. On the contrary, the other task, that to find suitable source positions, is a much more difficult one. An automatic search procedure for finding suitable source positions could be therefore of major help. The paper proposes such a procedure applicable to cylindrical surfaces.

The surface velocity field is obtained by measurements on a cylinder mechanically excited by white noise. In the rest of the study we shall focus on resonant frequencies only in view of their major contribution to the global velocity level.

Sinusoidal time variations will be assumed with a frequency ω. The time factor $e^{j\omega t}$ will be omitted throughout the paper.

THE EQUIVALENT SOURCE METHOD (ESM)

For the equivalent sources method the principal issue is optimal positioning of these sources as well as the optimal number of sources in view of reducing computation time.

Monopole Sources

To simplify the task, the low-order sources are used (monopoles or dipoles). The acoustic pressure generated by simple sources is given in (1).

$$p(M,\omega) = \iint_S j\omega\rho_0 \mu(M_0) g(M / M_0) \mathrm{d}s \tag{1}$$

where M_0 denotes the positions of equivalent sources and $\mu(M_0)$ the source distribution density. The requirement that the synthetic velocity matches the prescribed one is given by Esq. (2).

$$\iint_S \mu(M_0) \nabla g(M_S / M_0) ds = \hat{v}_n \tag{2}$$

The equation (2) in fact is a Fredholm integral equation of first kind. To solve it numerically (2) can be represented by an equivalent matrix equation. Such equations are known to be ill-posed and require the use of a suitable regularization technique such as truncated singular value or Tikhonov regularization. Alternative techniques to solve the matrix equation are iterative methods. The paper uses the conjugate gradient method.

It will be assumed that the number of sources is N and the number of points on the surface (control points) M. Equation (2) can be then expressed in a matrix form:

$$[v_n] = [T][Q] \tag{3}$$

For the general case when $N \neq M$ the solution reads:

$$[Q] = \left[[T]^*_{mn}[T]_{mn}\right]^{-1}[T]^*_{mn}[v_n] \tag{4}$$

where T^* is conjugate transpose matrix of T.

The calculus of matrix pseudo-inverse is made in the sense of the minimization of the root mean square error. The transfer matrix *i,j* element represents the complex amplitude of the normal surface velocity at the *i* point due to unit-strength elementary source at the *j* point [5] :

$$T_{ij} = -\frac{1}{4\pi}\frac{jk|r_j - r_i| - 1}{|r_j - r_i|^2} e^{jk|r_j - r_i|}\cos\theta_{ij} \tag{5}$$

where $|r_j - r_i|$ the distance between the control point and the equivalent source and θ_{ij} the angle between the vector $r_j - r_i$ and the outer normal to the surface.

Once the source strengths have been found, the pressure field can be calculated from following the relation :

$$[p] = [M]\cdot[Q] \tag{6}$$

where the M coefficient is:

$$M(|r - r_i|) = -j\omega\rho_0 \frac{e^{jk|r - r_i|}}{4\pi|r - r_i|} \tag{7}$$

On the surface of the radiator the pressure can be evaluated by substituting r from the equation (5) with control surface points coordinates.

The "greedy search" algorithm and surface search algorithm

Some authors consider that the placement of the equivalent sources inside the radiator should be done over an interior surface with the same shape like the vibrating surface. In such a case it is possible to have problems with the eigen frequencies of this interior surface.

To avoid this disadvantage Pavic [5] proposed a ''greedy search'' algorithm which finds the positions for the sources which reduces effectively the velocity errors.

It has been found in [5, 6] that in the case of low frequencies the ''greedy search'' algorithm places the equivalent sources near boundary points, approximately matching the boundary shape. For high frequencies the sources are positioned near the body center.

This paper proposes a novel ''equivalent surface search'' algorithm of the interior surfaces. The equivalent sources distributed on the surface are of monopole type. The main steps of the new search procedure are given below:

a) A large number of interior surfaces similar to the body surface is defined.
b) Each interior surface is checked from the surface velocity reconstruction point of view and a single surface is found giving the smallest deviation between the original velocity and that created by the equivalent sources.

The surface found in this way is checked from the point of view of fictitious eigenvalues which may influence the synthesized pressure.

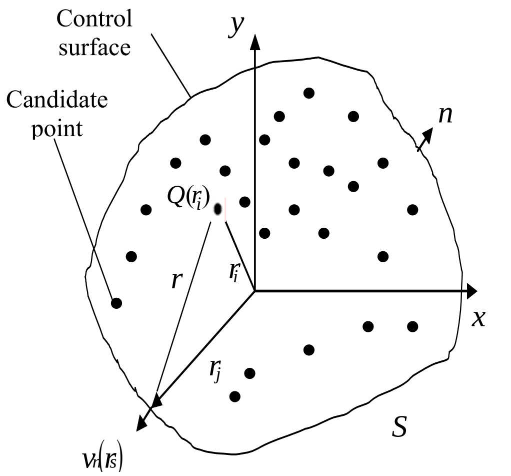

Figure 1 - Model for the implementation of the optimised equivalent sources.

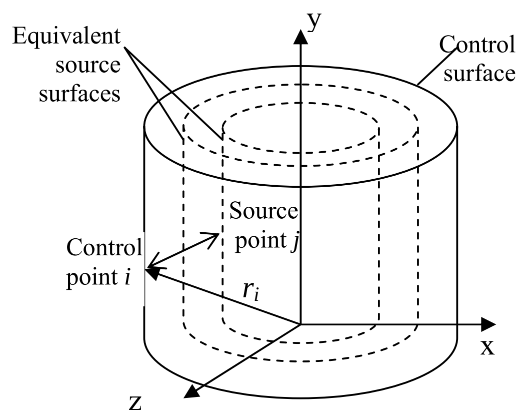

Figure 2 - The positioning of equivalent sources on interior surfaces.

In this paper two types of positioning source are used and compared:

- the positioning selected by the "greedy search" algorithm [5, 6];
- the positioning of sources over an interior surface of vibrating cylinder [7].

EXPERIMENTAL APPLICATION ON A CYLINDER

The 0.6 m long, 4 mm thick steel cylinder of 0.3 m diameter is mechanically excited by a shaker driven by white noise, Figure 3. The velocity distribution on the cylinder surface is measured by an accelerometer scanned along a uniform grid of 144 measurement points grouped in 12 lines. Due to the large number of measurement points the accelerometer was fastened to the cylinder by a small magnet.

Figure3 – The cylinder mechanically excited by a shaker.

Figure 4 – The measurement set-up.

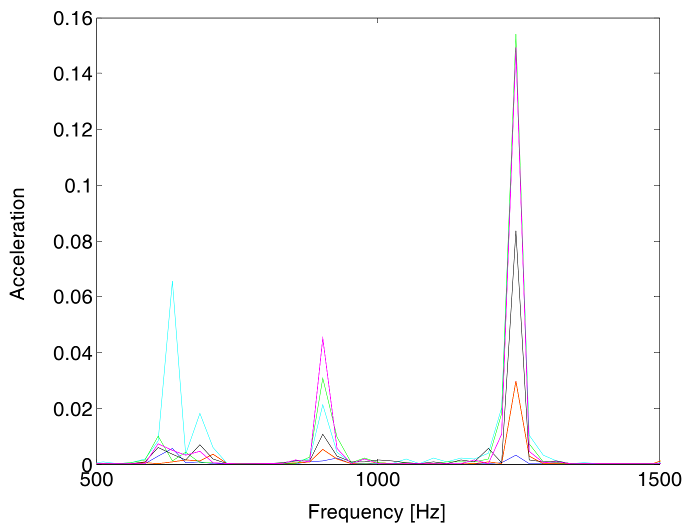

Figure 5 – The acceleration spectra from line1 of cylinder.

From Figure 5 it is seen that the cylinder resonant frequencies in the range 500-1500 Hz are 650 Hz, 950 Hz and 1250 Hz. The acoustic radiation of the cylinder is further studied at these resonant frequencies. Figure 6 shows the eigenmode of cylinder at 950 Hz computed by a finite element method.

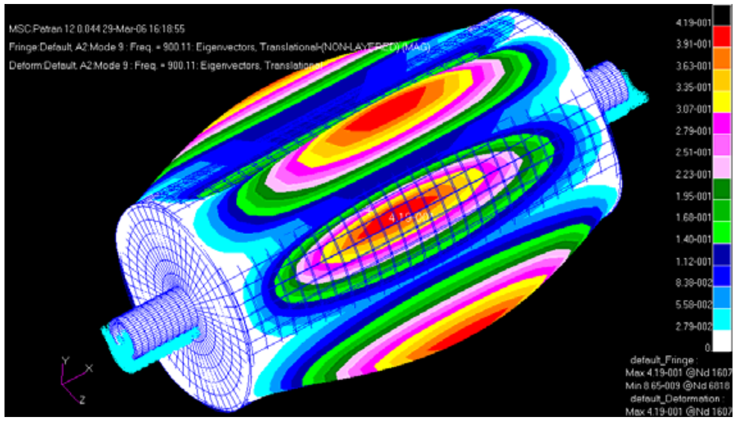

Figure 6 – The cylinder eigenmode at 1245 Hz.

THE ACOUSTIC RADIATION USING ESM

To compute the acoustic radiation of the cylinder with the equivalent source method it uses the model with 195 control points on boundary surface and 1900 candidate sources like in Figure 8.

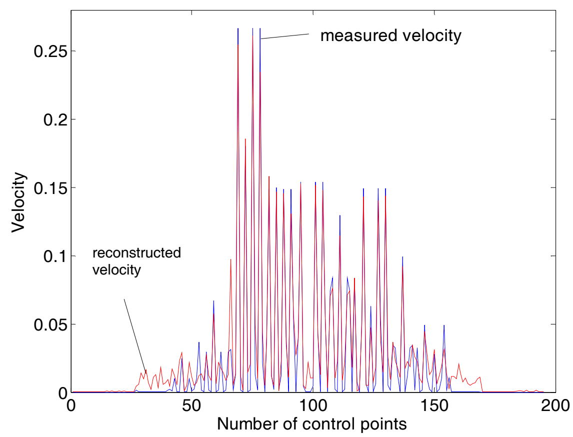

Figure 7 – Comparison between measured and reconstructed velocity at 1245 Hz.

In the Figure 7 is given a comparison between measured and reconstructed velocity field for the boundary of cylinder at the 1254 Hz. It has been assumed that the vibration of the cylinder flanges contribute little to sound radiation. Figure 9 presents the optimised 100 positions found by the greedy search approach. The surface of optimised sources tends to copy the body shape. Figure 10 presents the optimised surface position chosen with the new surface search algorithm. This algorithm is two time faster than the greedy search but it suffers from error point of view. More points are necessary to obtain the same level of error. The maximum pressure level is obtained in the zone around the excitation point (Figure 11). Comparison between measured and reconstructed velocity at 1245 Hz shows that it needs by an optimised equivalent source surface with 125 sources to obtain the same velocity errors like "greedy search "algorithm with 100 sources (Figure 12). Figure 13 presents the acoustic pressure on YZ plane situated at 2 m from cylinder axis.

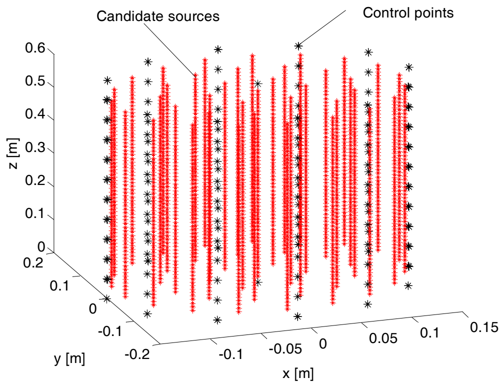

Figure 8 – The control points (black) and candidate sources (red).

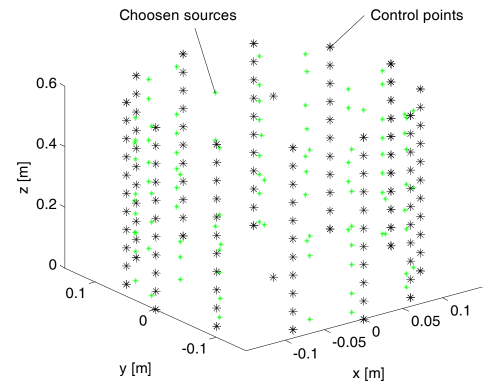

Figure 9 – The control points (black) and 100 optimised source positions (green).

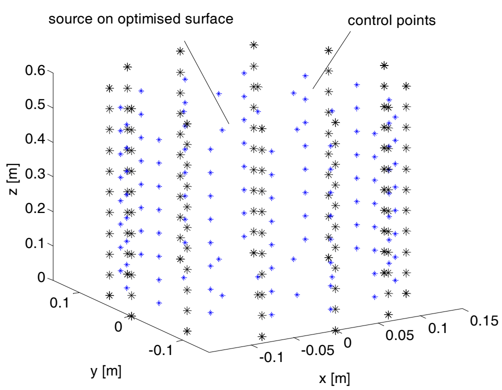

Figure 10 – The control points (black) and optimised surface(blue) – 1245 Hz.

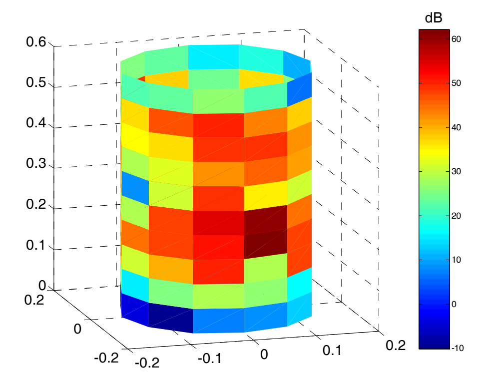

Figure 11 – The pressure on the control surface at 1245 Hz.

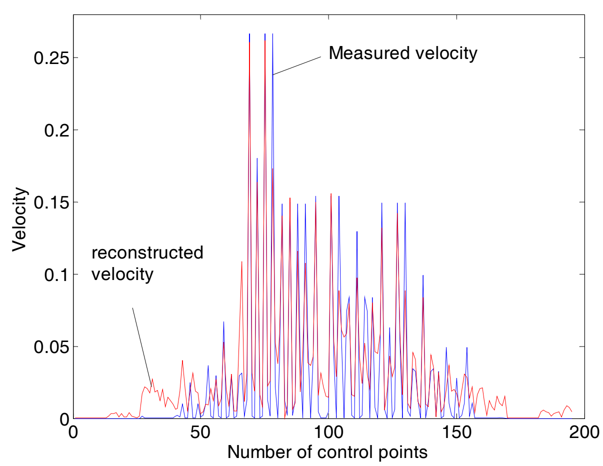

Figure 12 – Comparison between measured and reconstructed velocity at 1245 Hz with an optimised equivalent surface with 125 sources.

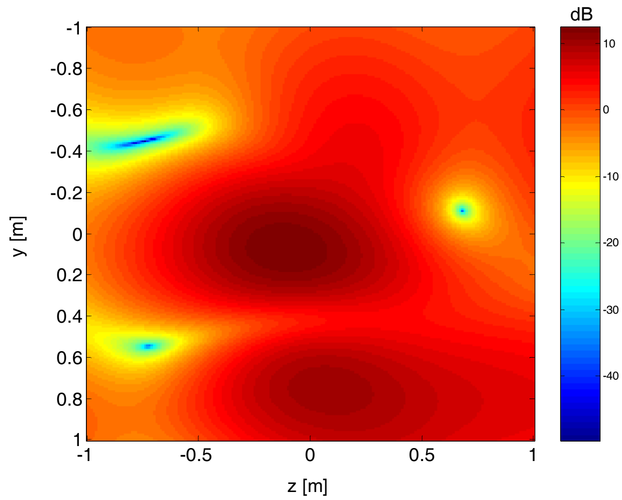

Figure 13 - The acoustic pressure on YZ plane situated at 2 m from cylinder axis.

CONCLUSIONS

The modelling of the cylinder radiation with the help of the equivalent sources method gives good results if the positioning of the sources are optimised. The new surface search algorithm was shown to halve the computation time. The study has demonstrated the inadequacy of using single accelerometers for surface vibration measurement. Advanced methods like scanning laser Doppler velocimetry will certainly be better suited for the measurement task.

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