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OPEN KINEMATIC CHAINS WITH MULTIPLE IMPACTS

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Abstract

The planar impact with friction of kinematic chains with an external surface is presented in this study. The links are rigid and the joints have no clearance. There are multiple impacts at the end points of the kinematic chain during the dynamical process. The force-deformation equation contains a damping term to reflect dissipation in the contact area. Planar collisions of a two-link chain with two contact points are numerically studied to compare the outcomes.

INTRODUCTION

The subject of impact is of great interest for scientists and engineers in the area of robotics. The goal is to develop methods that can predict the behavior of links after collisions. A collision between two bodies is a contact event that occurs at a common point of contact. Analytical solutions (obtaining post impact velocities in terms of pre-impact velocities) of rigid body collision problems in classical mechanics are formulated in terms of two principles: Newton's law of motion and Coulomb's law of friction. In addition, the solutions require the knowledge of two material constants: coefficient of friction and coefficient of restitution.

Kane and Levinson [6] showed that Newton's approach may predict erroneous energy results in rigid body problems when friction is present. Keller [7] formulated a three dimensional differential approach that resolved the energy paradox by using the kinetic definition of the coefficient of restitution. An important recent contribution to the area was the definition of a new coefficient of restitution (the energetic definition) by Stronge based on the internal dissipation hypothesis (Stronge [9]).

The collisions of kinematic chains with external surfaces were considered in Hurmuzlu and Chang [3]. They formulated an algebraic solution of impacts of planar multi-body system with two contact points based on the kinematic formulation of the coefficient restitution (with an energy correction scheme proposed in Brach [1]).

Hurmuzlu and Marghitu [4] studied rigid body collisions of planar kinematic chains with multiple contact points. Marghitu and Hurmuzlu [8] presented a three dimensional solution scheme based on the differential formulation of impact equations that incorporates the three definitions of the coefficient of restitution is presented.

In this article first the impacting system and the related coordinate frames are presented and then, the impulsive forces are presented. A simplified two link chain with two contact points is numerically studied. The focus is on the relation between the post and pre-impact energies, slippage and rebounds at the contact points.

IMPACTING SYSTEM

A general representation of the kinematic chains that are considered here is shown in Fig. 1. Consider the n interconnected rigid links $B_1, ..., B_n$. The kinematic chain has frictionless revolute joints. The end A_j, $j = 1, 2, \ldots, k$ of the chain collides with the surface S_j, $j = 1, 2, \ldots, k$. The collision at A_j leads to several outcomes depending on the initial conditions, the contact force and the coefficient of friction μ_j at the contact point A_j among the surface and the chain. The coordinate axes are aligned with surface S_0, whereas surfaces S_j are taken at angles θ_j with the horizontal. The normal and tangential directions at A_j are defined as shown. The contacting ends rebound as a result of the collision. The link B_1 has one rotational

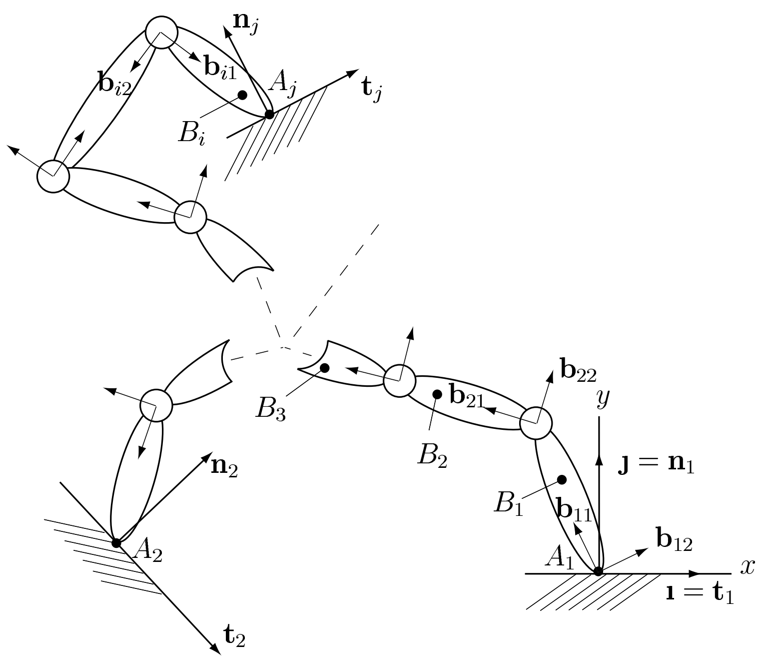

Figure 1: A general representation of an kinematic chain with frictionless revolute joints

and two translational degrees of freedom. Bodies $B_i,\ i = 2, 3, \ldots, n$ have rotational joints, and therefore have one relative degree of freedom each. Let $R[\boldsymbol{\imath},\ \boldsymbol{\jmath},\ \mathbf{k}]$ define a fixed inertial reference frame and R_i a set of reference frames attached to bodies B_i. The coordinate axes of R are aligned with the plane surface S_0. We express by $\mathbf{b}_{i1},\ \mathbf{b}_{i2}$ and $\mathbf{b}_{i3} = \mathbf{k}$, $(i = 1, 2, ..., n)$ the corresponding mutually perpendicular unit vectors along the axis of R_i. Let $\mathbf{n}_j$ be the unit vector of the normal to the surface S_j and directed from S_j into the contacting link. Let $\mathbf{t}_j$ be unit vector in the common tangent plane to the surface S_j and the contacting link satisfying: $\mathbf{n}_j = \mathbf{t}_j \times \mathbf{k}$.

The orientation of each body with respect to its lower adjacent body is defined through the angles ϕ_i where $i = 1, \ldots, n$. The relative translations of body B_1 is given by d_1 and d_2. The generalized coordinates for the system can be expressed as the vector

$$\mathbf{q} = \{d_1,\, d_2,\, \phi_1,\, \phi_2,\, \phi_3, \ldots, \phi_n\}^T \tag{1}$$

The unit vectors of an axis frame of a body can be expressed as a linear combination of the unit vectors of the axis frame of an adjacent body as:

$$R_i = \mathbf{S}_{i\;i-1}\, R_{i-1}, \tag{2}$$

where R_{i-1} is the rotational matrix. The angular velocity of B_i in R can be written as

$$\boldsymbol{\omega}_{i+2} = u_i \mathbf{k}, \tag{3}$$

and the vector of generalized speeds is given by

$$\mathbf{u} = \{u_1, u_2, u_3, u_4, u_5, \ldots, u_n, u_{n+1}, u_{n+2}\}^T \tag{4}$$

where, $u_1 = \dot{d}_1$ and $u_2 = \dot{d}_2$. The velocity of the A_j end is

$$\mathbf{v}_j = v_j^n \mathbf{n}_j + v_j^t \mathbf{t}_j \quad (j = 1, \ldots, k), \tag{5}$$

where, v_j^n is the normal and v_j^t is the tangential velocity. The vector of contact forces is given by

$$\mathbf{F} = \left\{F_1^t, F_1^n, \ldots, F_k^t, F_k^n\right\}^T \tag{6}$$

where, F_j^n is the normal contact force and F_j^t is the tangential contact force. The impulses at the contact points are obtained by integrating Eq. (6), which gives:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1^t \\ \tau_1^n \\ \vdots \\ \tau_k^t \\ \tau_k^n \end{bmatrix} = \begin{bmatrix} \int_0^t F_0^t dt \\ \int_0^t F_0^n dt \\ \vdots \\ \int_0^t F_k^t dt \\ \int_0^t F_k^n dt \end{bmatrix} \tag{7}$$

where,

$$\boldsymbol{\tau}_j = \tau_j^t \mathbf{t}_j + \tau_j^n \mathbf{n}_j \quad (j = 1, ..., k) \tag{8}$$

IMPACT FORCES

The normal impulsive forces, F_j^n, $j = 1, 2, \ldots, k$, are determined by combining the classical Hertzian contact theory (Goldsmith [2]) and elastic-plastic indentation theory (Johnson [5]). At a contact point there is a linear relationship between the plastic deformation q_{pj} and the normal contact force F_j^n, as follows

$$q_j^p = \eta(F_j^n - F_j^c). \tag{9}$$

In the above equation the coefficient η has the following expression

$$\eta = \frac{1}{2\pi R_j H}, \tag{10}$$

where H characterizes the plastic property of the material and can be approximate with the Brinell hardness, and R_j is the radius of the end impacting link. The critical value of the impact force F_j^c, can be expressed in terms of the yield stress σ_y

$$F_j^c = \frac{8\pi^3 R_j^3 \sigma_y^3}{k_1^2}, \tag{11}$$

where

$$k_1 = \frac{2}{3(1-\nu^2)} E\sqrt{R_j}. \tag{12}$$

The Poisson's ratio is ν and the Young's module is E.
The elastic deformation q_j^e as a function of the contact force is described by the Hertz's law

$$q_j^e = \left(\frac{F_j^n}{k_1}\right)^{2/3}, \tag{13}$$

and the total normal deformation for the elasto-plastic impact is the sum of elastic and plastic deformation

$$q_j = q_j^e + q_j^p = \left(\frac{F_j^n}{k_1}\right)^{2/3} + \eta(F_j^n - F_j^c). \tag{14}$$

The critical deformation q_j^c corresponds to the force F_j^c, and the maximum deformation q_m appears when the maximum force F_j^m is applied. The impact force at the impact point is

$$\mathbf{F}_j = F_j^n \mathrm{n} + F_j^t \mathrm{t}, \tag{15}$$

where F_j^t is the friction force.

The possible cases of motion at the contact point are

I. the end is slipping: $F_j^t = -\mu\,\mathrm{Sign}(v_j^t) F_j^n$. The experimental data show that a dry friction coefficient can be used to model the impact of the links.

II. the end is not slipping: $v_j^t = 0$ subject to $\left|F_j^t / F_j^n\right| \leq \mu_j$.

Contact force was determined and examined. When a link has a nonzero tangential velocity at the onset of the collision $v^t \neq 0$, there will be a phase of slip.

APPLICATION

In this section the impact problem of a two link chain, link 1 and link 2, with two contact points (Fig. 2) is presented. A cartesian reference frame xOy is chosen, a mobile reference frame x_1Oy_1 is attached to the link 1, at point A, and a mobile reference frame x_2Oy_2 is attached to the link 2, at point B. The lengths of the links 1 and 2 are $l_1 = l_2 = l = 153$ mm. The masses of the links 1, and 2 are $m_1 = m_2 = 0.03871$ kg. Also, for this application $\alpha = \beta$ and $\theta = \phi$. The mass moments of inertia for the links 1 and 2 are $I_{C1} = 0.19712$ and $I_{C2} = 76.53893$. The contact surface S is horizontal, and the chain includes slender members (steel bars with a density of 7801.0 kg/m^3, each with a diameter of 6.35 mm) that are connected with a rotational joint E. The velocity of the point A (impacting point on link 1) on the Ox and Oy axes is denoted by v_{Ax} and v_{Ay}. The velocity of the point B (impacting point on link 2) on Ox and Oy axes is denoted by v_{Bx} and v_{By}. The initial angular velocities of the link 1, and link 2 are $\omega_1 = \omega_2 = \omega_{in}$. The kinetic energy of the link 1 is T_1. The kinetic energy of the link 2 is T_2. The total kinetic energy for the two link chain is $T = T_1 + T_2$. Friction is negligible for the rotational joint M. The coefficient of static friction is $\mu_s = 0.7$ and the coefficient of dynamic friction $\mu_k = 0.57$ (for dry steel). The gravitational acceleration $g = 9.807$ m/s^2 is considered. Two sets of simulation shown in Fig. 3 and Fig. 4 were

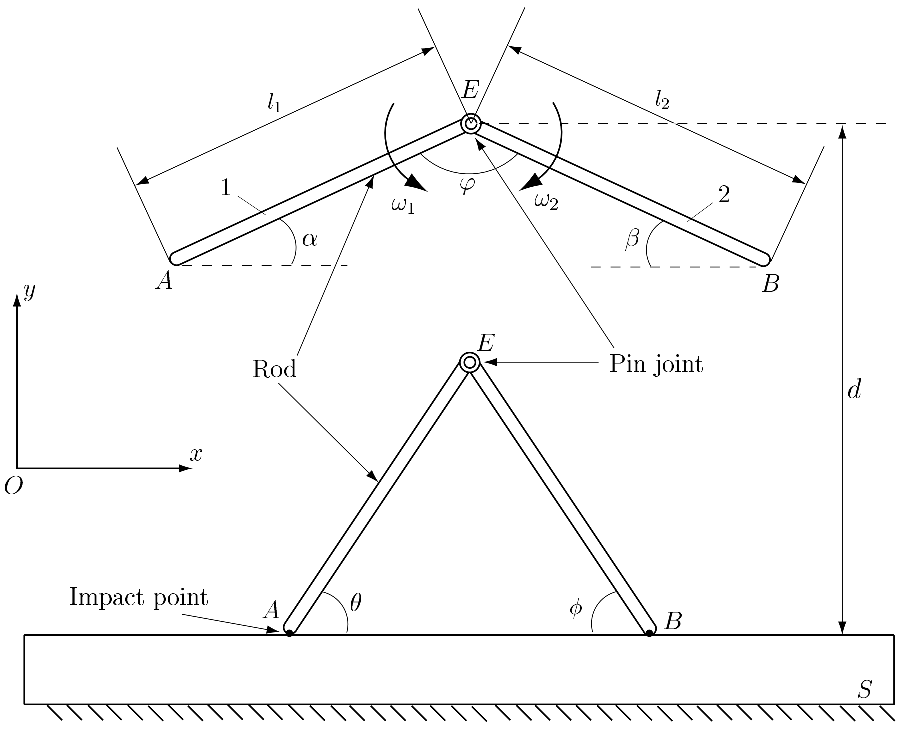

Figure 2: Impact of two link chain with two contact points

conducted to determine the kinetic energy denoted by T, angular velocity denoted by ω, and the Ox and Oy velocity of the point A denoted by v_{Ax} and v_{Ay}, for the two link chain before

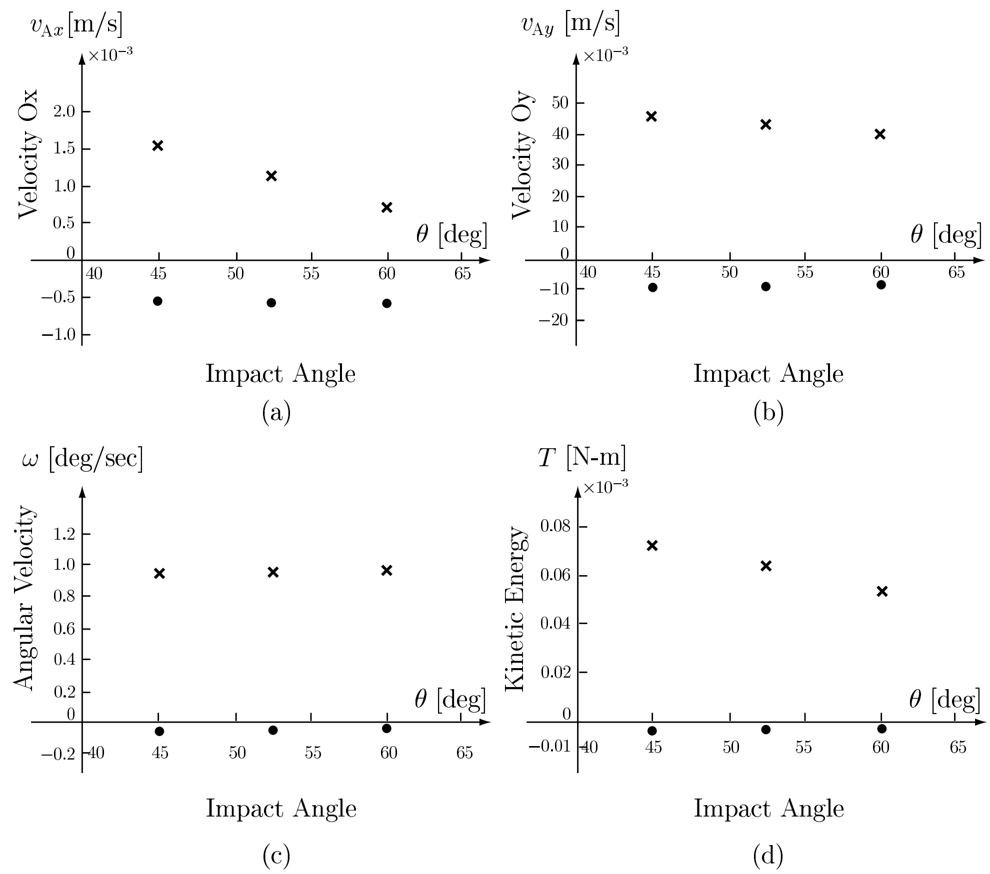

Figure 3: Impact of two link chain with an initial angular velocity $\omega_{in} = 1\ m/s$

and after the impact. For each simulation the two link chain is released such as the distance between the pin joint E and the surface S is d=constant.

The the distance d was considered $d = 200$ mm. When released, link 1 and link 2 have an initial angular velocity $\omega_{in} = 1$ deg/sec for the first simulation set, and an angular velocity $\omega_{in} = 5$ deg/sec for the second simulation set respectively. Three different values for the impact angle θ have been taken into consideration for each simulation set. The values of the impact angle θ are $\theta = \theta_1 = 45°$, $\theta = \theta_2 = 52.5°$, and $\theta = \theta_3 = 60°$.

Results of the simulation are depicted in Fig. 3 and Fig. 4. In the figures, the symbol "$\times$" was used to shown the values obtained immediately before the impact and the symbol "•" was used to shown the values obtained immediately after the impact.

For the first simulation set ($\omega_{in} = 1$ deg/sec, $\theta = \theta_1, \theta_2, \theta_3$) the Ox velocity v_{Ax} vs. the impact angle θ, Oy velocity v_{Ay} vs. the impact angle θ, the angular velocity ω vs. impact angle θ, and the kinetic energy T vs. impact angle θ, for post and pre-impact were computed and plotted in Fig. 3.a, Fig. 3.b, Fig. 3.c, and Fig. 3.d, respectively.

For the second simulation set ($\omega_{in} = 5$ deg/sec, $\theta = \theta_1, \theta_2, \theta_3$) the same data for post and pre-impact were computed and plotted in Fig. 4.a, Fig. 4.b, Fig. 4.c, and Fig. 4.d,

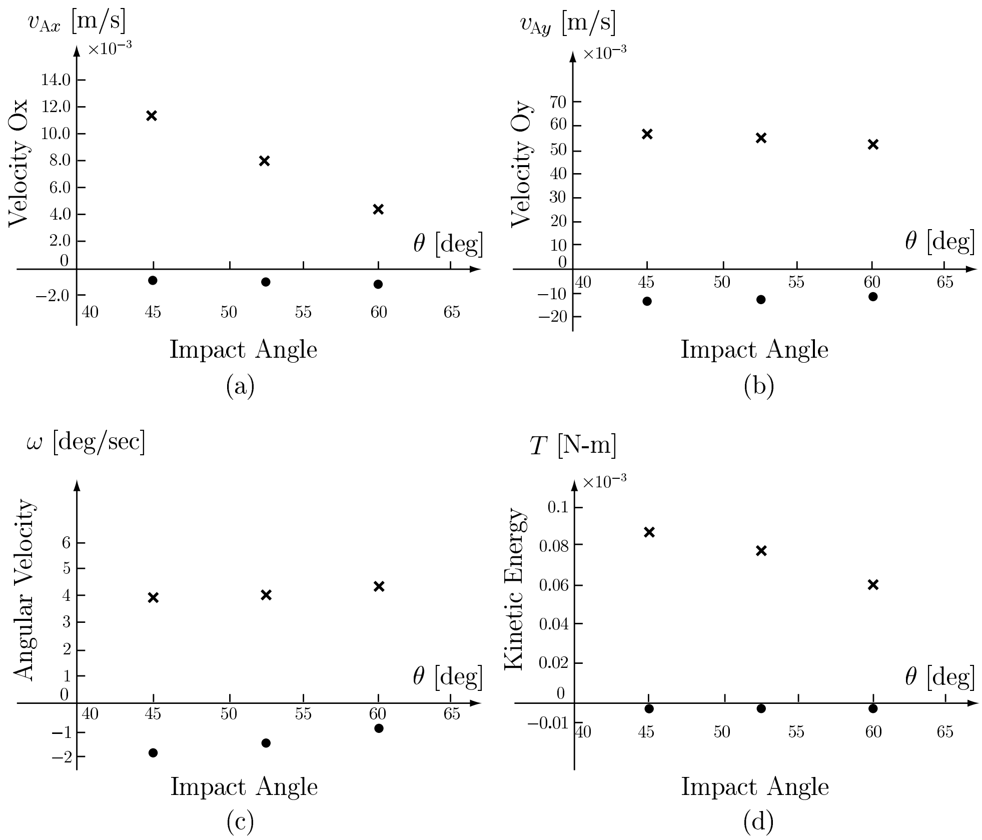

Figure 4: Impact of two link chain with an initial angular velocity $\omega_{in} = 5\ m/s$

respectively. The Ox velocity v_{Ax}, Oy velocity v_{Ay}, angular velocity ω, and kinetic energy T, for post-impact (po.im.) and pre-impact (pr.im.), for the first and second simulation sets are given in Table 1.

SUMMARY

This paper considers the multi-contact, rigid body collisions of kinematic chains in the presence of friction. The solution techniques are based on the differential formulation of the equations of impact with the normal impulsive forces determined by combining the classical Hertzian contact theory and elastic-plastic indentation theory.

Numerical results of the collisions of a two link chain, with two contact points in the presence of gravity have been presented. Dynamic response characteristics as kinetic energy T, angular velocity ω, v_{Ax} and v_{Ay} velocities, for the the post and pre-impact have been computed.

Table 1: v_{Ax} and, v_{Ay} velocity, angular velocity ω, and kinetic energy T

Initial Data	v_{Ax} [m/s]	v_{Ay} [m/s]	ω [deg/s]	T [N − m]
1 deg/s; 45°; pr.im.	1.52729	45.62396	0.95603	0.07226
1 deg/s; 45°; po.im.	−0.57063	−9.28193	−0.05998	0.00324
1 deg/s; 52.5°; pr.im.	1.12426	42.97673	0.96172	0.06379
1 deg/s; 52.5°; po.im.	−0.59133	−8.92203	−0.05193	0.00362
1 deg/s; 60°; pr.im.	0.70064	39.93305	0.97365	0.05344
1 deg/s; 60°; po.im.	−0.60199	−8.46615	−0.04012	0.00416
5 deg/s; 45°; pr.im.	11.40812	56.60626	3.92947	0.08753
5 deg/s; 45°; po.im.	−0.86462	−13.66523	−1.86982	0.00299
5 deg/s; 52.5°; pr.im.	8.013201	54.87833	4.02182	0.07796
5 deg/s; 52.5°; po.im.	−0.99816	−12.91071	−1.50102	0.00302
5 deg/s; 60°; pr.im.	4.42944	52.29797	4.35933	0.06051
5 deg/s; 60°; po.im.	−1.14885	−11.69617	−0.95820	0.00307

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