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PRESSURE WAVES IN A LAYERED ELASTIC TUBE WITH VISCOELASTIC LIQUID

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Abstract
Propagation of acoustic waves in thin-walled two-layered elastic tube with compressible polymeric liquid is investigated. It is assumed that the waveguide consists of two thin circular cylindrical shells with different thicknesses, made from different isotropic elastic materials. The layers of the shell are supposed to be rigidly jointed. Dynamics of the tube wall in the wave is described within Kirchhoff-Love approximation, formulated for a layered cylindrical shell. Dynamics of polymeric liquid in the tube is treated within quasi-one-dimensional approach; the liquid rheology is described by linear hereditary rheological equation. The sound dispersion in the waveguide is studied in a low frequency range. Numerical analysis has shown strong influence of the liquid rheological properties and parameters of the layered shell on dispersion and attenuation of pressure signals.

INTRODUCTION

The problem of acoustic wave propagation in cylindrical layered structures has important engineering, biomedical and technological applications [2], [4], [7]. One of them can be found in the field of acoustic measurements, where integrated thin walled acoustic tube devices were developed and demonstrated to be an attractive sensor candidate [15]. The actual configuration of a tube sensor device is often a two-layer composite shell, which supports motivation for systematic study of sound waves characteristics in layered thin walled tubes. Another widespread application of the layered waveguides is connected with using of different anticorrosion coatings, which can change essentially the acoustic properties of the system. Results on acoustics of cylindrical layered shells are reported in [3], [10]. Sound transmission through layered shells with viscoelastic layers was studied in [6].

The topic of acoustic wave propagation in liquid-filled thin walled cylindrical tubes is widely covered in the literature. However, it has been mostly related to the case of ideal and pure viscous fluids. The results relating to liquids with more complex rheology, particularly, viscoelastic fluids, can be found in [12], [13], [14]. Here approach similar to [12] is used for studying more general configuration of the waveguide, namely, infinite cylindrical shell, made from two thin elastic layers. The shell is filled by compressible viscoelastic liquid (solution of high polymer in a low molecular solvent). The subject of the paper is the low frequency sound propagation in the system.

PROBLEM FORMULATION

Dynamic equations for two-layered thin circular cylindrical shell are written below for the case when the layers are made from different isotropic elastic materials and the package dynamics can be described within Kirchhof-Love approximation [1]. These equations in an axisymmetrical case have the form:

$$\frac{C}{R^2}\frac{\partial^2 u}{\partial\zeta^2}+\frac{C_{12}}{R^2}\frac{\partial w}{\partial\zeta}-\frac{K}{R^3}\frac{\partial^3 w}{\partial\zeta^3}=(h_1+h_2)\cdot\rho_s\cdot\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{C\,w}{R^2}+\frac{C_{12}}{R^2}\frac{\partial u}{\partial\zeta}-\frac{K_{12}}{R^3}\frac{\partial^2 w}{\partial\zeta^2}+\frac{1}{R^3}\cdot\left(\frac{D_1}{R}\frac{\partial^4 w}{\partial\zeta^4}-K\frac{\partial^3 u}{\partial\zeta^3}-K_{12}\frac{\partial^2 w}{\partial\zeta^2}\right)= \tag{2}$$

$$-(h_1+h_2)\cdot\rho_s\frac{\partial^2 w}{\partial t^2}+\Delta p$$

Here u and w are longitudinal and radial displacements of the internal surface of the shell, contacting with liquid, which is chosen for a coordinate surface [1]. The layered tube is related to cylindrical coordinate system with the origin on the tube axis and x, r are the longitudinal and radial coordinates, correspondingly; R is the radius of the coordinate surface; h_1 and h_2 - the thicknesses of the internal and external layers, respectively; ζ is the non-dimensionless axial coordinate ($\zeta=x/R$); Δp - contact pressure, equal to normal stress in the liquid at the pipe wall; t - time; $C, C_{12}, K, K_{12}, D_1$ are rigidities of the layered shell, specified according to the formulas [1]:

$$C=B_{11}^{(1)}h_1+B_{11}^{(2)}h_2;\; C_{12}=B_{12}^{(1)}h_1+B_{12}^{(2)}h_2;\; K=0.5\cdot[B_{11}^{(1)}h_1{}^2+B_{11}^{(2)}(2h_1h_2+h_2{}^2)] \tag{3}$$

$$K_{12}=0.5\cdot[B_{12}^{(1)}h_1{}^2+B_{12}^{(2)}(2h_1h_2+h_2{}^2)];\; D_1=\frac{1}{3}\cdot[B_{11}^{(1)}h_1{}^3+B_{11}^{(2)}(3h_1{}^2h_2+3h_1h_2^2+h_2{}^3)]$$

where the quantities $B_{11}^{(1)}, B_{11}^{(2)}, B_{12}^{(1)}, B_{12}^{(2)}$ are defined as follows:

$$B_{11}^{(i)} = \frac{E_i}{1-\nu_i^2}\ ;\ B_{12}^{(i)} = \nu_i \cdot B_{11}^{(i)};\quad i = 1,\, 2 \tag{4}$$

In (4) E_i, ν_i are Young modules and Poisson coefficients of the layers (index $i=1$ corresponds to the internal layer and $i=2$ - to the external one); ρ_s is the reduced density of the layered shell, introduced by the formula [1]:

$$\rho_s = \frac{\rho_1 h_1 + \rho_2 h_2}{h_1 + h_2} \tag{5}$$

Here ρ_1 and ρ_2 are the layer's densities.

The equations (1), (2) are rewritten in dimensionless form, using the relations

$$\{u_1, u_2, \xi, \varepsilon_1, \varepsilon_2\} = R^{-1}\{w, u, r, h_1, h_2\};\ \ p_c = p_0{}^{-1}\Delta p;\ \ \tau = t/t_0;\ \ t_0 = R\left(\rho_s / p_0\right)^{1/2}$$

$$\left\{C^*, C_{12}^*\right\} = \left(p_0 R\right)^{-1}\{C, C_{12}\};\quad \left\{K^*, K_{12}^*\right\} = (p_0 R^2)^{-1}\{K, K_{12}\};\quad D^* = (p_0 R^3)^{-1} D_1$$

and their solution is searched in the form $\{u_1, u_2, p_c\} = \{\hat{u}_1, \hat{u}_2, \hat{p}_c\}\exp\left[i\left(\Omega\tau - k\zeta\right)\right]$, where Ω and k are dimensionless frequency and wave number, respectively. It leads to the following equations for the complex amplitudes:

$$ik\,(C_{12}^* + K^* k^2)\,\hat{u}_1 + [k^2 C^* - \left(\varepsilon_1 + \varepsilon_2\right)\Omega^2]\,\hat{u}_2 = 0 \tag{6}$$

$$(C^* + 2K_{12}^* k^2 + D^* k^4 - \left(\varepsilon_1 + \varepsilon_2\right)\Omega^2)\,\hat{u}_1 - ik(C_{12}^* + K^* k^2)\,\hat{u}_2 - \hat{p}_c = 0$$

To find the contact pressure here it is necessary to solve the internal dynamic problem for the liquid with the boundary conditions at $r = R$:

$$v_r = \frac{\partial w}{\partial t},\ \ v_x = \frac{\partial u}{\partial t},\ \ \Delta p = \Delta p_f - \tau_{rr},\ \ \Delta p_f = p_f - p_0 \tag{7}$$

Here v_x, v_r are the axial and radial velocity components of the liquid; p_f is the pressure in liquid; p_0 - equilibrium pressure within the tube; τ_{rr} - the normal component of the stress tensor deviator in liquid.

Linearized equations of non-stationary flow of polymeric liquid in the tube are formulated within quasi-one-dimensional approach [12]. The equations of momentum and mass balance for liquid are averaged along cross-section of the tube and written in dimensionless form, using the variables:

$$\overline{P}=P/p_0-1;\quad \overline{V}=Vt_0/R;\quad \overline{\rho}=\frac{\rho}{\rho_{fo}}-1;\quad \overline{\tau}_w=\left(\tau_{xr}\right)_{r=R}/p_0;\quad \overline{v}_R=\left(t_0/R\right)\cdot(v_r)_{r=R}$$

$$\dot{\overline{u}}_2=\left(t_0/R\right)\cdot\frac{\partial u}{\partial t};\quad V=\frac{2}{R^2}\int_0^r V_x r\,dr;\quad P=\frac{2}{R^2}\int_0^r p_f r\,dr;\quad \rho=\frac{2}{R^2}\int_0^r \rho_f r\,dr \tag{8}$$

where ρ_f, ρ_{f0} are the liquid density and its equilibrium value, respectively; τ_{xr} is the tensile stress in the liquid; and $V_x=v_x-\dfrac{\partial u}{\partial t}$ is the relative velocity. Then the equations for the complex amplitudes, introduced by the relations $\left\{\overline{P},\ \overline{V},\ \overline{\rho},\ \overline{v}_R,\ \overline{\tau}_w\right\}=\left\{\hat{P},\ \hat{V},\ \hat{\rho},\ \hat{v}_R,\ \hat{\tau}_w\right\}\exp[i\left(\Omega\tau-k\zeta\right)]$, take the form [12]:

$$i\Omega\hat{V}=i\kappa k\hat{P}+\Omega^2\hat{u}_2+2\kappa\hat{\tau}_w \tag{9}$$

$$i\Omega\hat{\rho}+2i\Omega\,\hat{u}_1-ik\hat{V}=0 \tag{10}$$

$$\hat{P}=\kappa^{-1}c_f^2\hat{\rho} \tag{11}$$

Here c_f is the sound speed in the liquid and $\kappa=\rho_s/\rho_{f0}$.

To close the boundary value problem, formulated above, it is necessary to find the tensile stress amplitude $\hat{\tau}_w$ at the shell boundary in (9). Following [12], the non-stationary friction force at the tube wall can be found from solution, describing pulsating flow of incompressible polymeric liquid in a rigid tube with the same radius. In this case the small input of the cross effect of the liquid's rheology and compressibility [11] and also small influence of the tube's radial deformations on the losses, are neglected. The result has the form [12]:

$$\hat{\tau}_w=-4\overline{\eta}\,D\hat{V};\quad D=-\frac{1}{4}\cdot\frac{\mu T(\mu)}{1-2\mu^{-1}T(\mu)};\quad T(\mu)=\frac{J_1(\mu)}{J_0(\mu)} \tag{12}$$

$$\mu=i\left(i\Omega/\kappa\overline{\eta}\right)^{1/2};\quad \overline{\eta}=\overline{\eta}_{sv}+\left(i\Omega\right)^{-1}G^*;\quad G^*=\int_0^\infty\frac{\left(\Omega\overline{\theta}\right)F\left(\overline{\theta}\right)(1+i\Omega\overline{\theta})}{1+\left(\Omega\overline{\theta}\right)^2}d\overline{\theta}$$

$$\overline{\theta}=\theta/t_0;\quad \overline{\eta}_{sv}=\eta_{sv}/\left(p_0t_0\right)$$

Here G^* is the dimensionless complex dynamic module of the liquid; η_{sv} - the Newtonian solvent viscosity; $F(\theta)$ - the spectrum of relaxation times θ; J_0, J_1 are Bessel functions of the first kind of the zero and first order, respectively. The contact pressure amplitude $\hat{p}_c$ is found from the boundary condition (7):

$$\hat{p}_c = \left(1 - \frac{2i\Omega\kappa\overline{\eta}}{3\overline{c}_f^2}\right)\hat{P} - 2i\Omega\overline{\eta}\hat{u}_1; \qquad \overline{c}_f = c_f t_0 / R \tag{13}$$

DISPERSION EQUATION AND NUMERICAL RESULTS

The dispersion equation for sound waves in the waveguide follows from (6), (9)-(11), (12) and (13). It can be simplified in the case of long waves (k << 1), when the bending stresses in the shell are small with respect to membrane ones [5], with account for the following inequalities: $K^* << C_{12}^*$, $D^* << K_{12}^* << C^*$. The result has the form:

$$A_1 z^2 - B_1 z + C_1 = 0 \tag{14}$$

$$A_1 = -Qi\Omega C^{*2} + i\Omega C_{12}^{*2} + 2\Omega^2 \overline{\eta}\, C^*; \qquad Q = 1 - (\varepsilon_1 + \varepsilon_2) C^{*-1} \Omega^2 \tag{15}$$

$$B_1 = \overline{c}_f^{-2}(N C_{12}^{*2} - N C^{*2} Q) - (\varepsilon_1 + \varepsilon_2) i\Omega Q C^* - i\Omega\kappa^{-1} S C_{12}^* - 2i\Omega\overline{\eta}\,\overline{c}_f^{-2} N C^* +$$

$$2(\varepsilon_1 + \varepsilon_2)\Omega^2\overline{\eta} - 2\kappa^{-1} N S C^*; \quad N = i\Omega + 8\kappa\overline{\eta} D; \quad S = 1 - \tfrac{2}{3} i\Omega\kappa\overline{\eta}\overline{c}_f^{-2}$$

$$C_1 = -(\varepsilon_1 + \varepsilon_2)[\overline{c}_f^{-2}(NQC^* + 2\Omega\overline{\eta} iN) + 2\kappa^{-1} NS]; \quad c = \Omega / k; \quad z = c^{-2}$$

Note that in the case of a single-layered shell the dispersion equation (14) coincides with the dispersion equation, studied in [12].

Equation (14) was studied numerically in the low frequency range for a discrete relaxation spectrum of the liquid

$$(i\Omega)^{-1} G^* = \frac{\overline{\eta}_p - \overline{\eta}_{\mathrm{sv}}}{z(\alpha_1)} \sum_{k=1}^{\infty} \frac{k^{\alpha_1} - i\Omega\overline{\theta}_1}{k^{2\alpha_1} + (\Omega\overline{\theta}_1)^2} \tag{16}$$

Here $z(\alpha_1)$ is the Riemann zeta function of the spectral distribution parameter α_1 ($\theta_k = \theta_1 / \mathrm{k}^{\alpha_1}$) and $\overline{\eta}_p$ - non-dimensional Newtonian viscosity of the polymeric solution. Newtonian viscosity of the solution was calculated from Martin relation [11] $\eta_p / \eta_{sv} = 1 + \beta exp(k_M \beta)$ with k_M = 0.4 and $\eta_{sv} = 0.1 Pa \cdot s$. The relaxation times θ_k were found according to Rouse theory (α_1 = 2) from the equation $\overline{\theta}_1 = 0.608\, \overline{\eta}_{sv} A \exp(k_M \beta)$ with A = 500 (the non-dimensional parameter $A = [\eta] M p_0 / R_G$, where $[\eta]$- characteristic viscosity of the solution, M - molecular mass of the polymer, R_G - universal gas constant, β - reduced polymer concentration).

The results of simulations for rheological description of the liquid, described above, are presented on the Figure 2. The graphs on the Figure 1 correspond to the following fixed values of the rheological parameters: $\eta_p = 0.5 Pa{\cdot}s$, $\eta_{sv} = 0.5{\cdot}10^{-3} Pa{\cdot}s$, $\theta_1 = 10^{-2}\ s$, $\alpha_1 = 2$ (these values were estimated for 2.5% solution of polystyrene with $M = 2{\cdot}10^6$ in toluene [11]). For all curves $\rho_{f0} = 850\ kg/m^3$ and $c_f = 1500\ m/s$. Curves 1-3 on all figures correspond to three pairs of geometrical parameters of the layered shell: $\varepsilon_1 = 0.09,\ \varepsilon_2 = 0.01$; $\varepsilon_1 = 0.05,\ \varepsilon_2 = 0.05$; $\varepsilon_1 = 0.01,\ \varepsilon_2 = 0.09$; respectively (for all plots $\varepsilon_1 + \varepsilon_2 = const = 0.1$). It was assumed that the internal shell is aluminium made ($E_1 = 7 \cdot 10^{10} Pa,\ \nu_1 = 0.34$, $\rho_1 = 2700\ \ kg/m^3$) and the external one is made from polyamide ($E_2 = 3 \cdot 10^9 Pa,\ \nu_2 = 0.42$, $\rho_2 = 1100\ \ kg/m^3$). Note that polyamide, as usually all polymeric materials, demonstrate frequency dependent behaviour at periodic loading. However, in the low frequency range this dependency can be neglected [9], and polyamide can be described by pure elastic model with constant Young module and Poisson coefficient, given above.

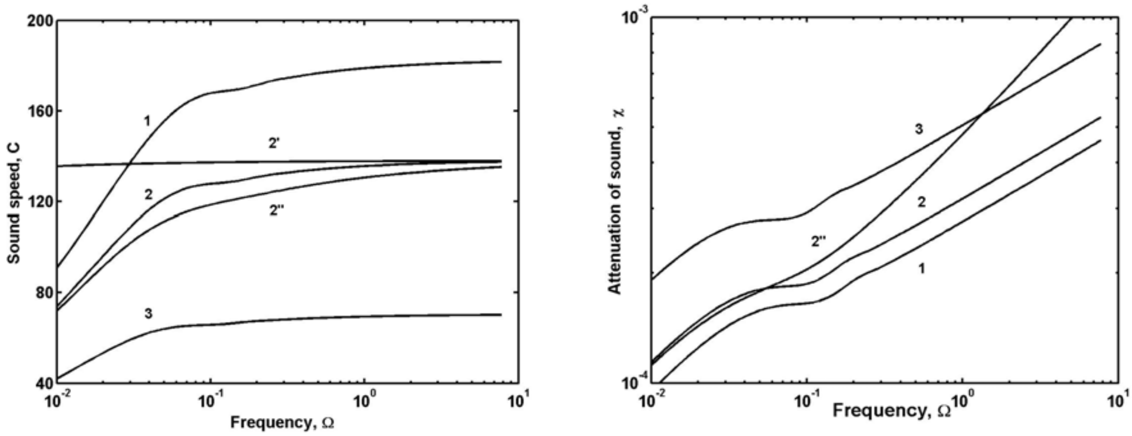
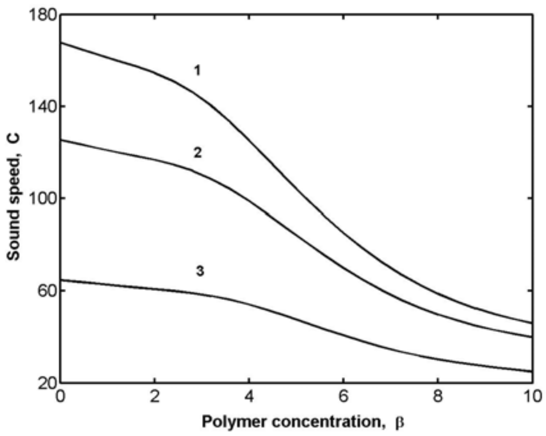


Figure 1 – Dimensionless sound speed and attenuation in the waveguide versus frequency

The plots for non-dimensional sound speed $C = \Omega/\mathrm{Re}\{k\}$ and attenuation of sound $\chi = -\mathrm{Im}\{k\}$ were computed from the equation (14). The lines $2'$, $2''$ in the Fig. 1, 2 correspond to pure solvent ($\overline{\eta} = \overline{\eta}_{sv}$) and pure viscous liquid with viscosity of the solution ($\overline{\eta} = \overline{\eta}_p$), respectively; the other parameters are the same as for the plot 2.

As it follows from the Figure 1, the changes in the shell structure (variation of the ε_1, ε_2 values) influence not only the dispersion but also the attenuation of the wave. For stiffer shell the sound wave velocity is larger and attenuation smaller than for more compliant one. The dispersion sign in the studied frequency range is positive, which is explained by transition from viscosity to inertia dominated flow regime in the tube. Note that positive dispersion of sound in a narrow tube in the low frequency range was directly observed in [8]. Numerical data show also that for the same values of the system parameters the sound speed in a pure viscous liquid with viscosity η_p is essentially less than that one in a similar viscoelastic liquid, which is explained by the frequency dependent dynamic viscosity of polymeric solution.



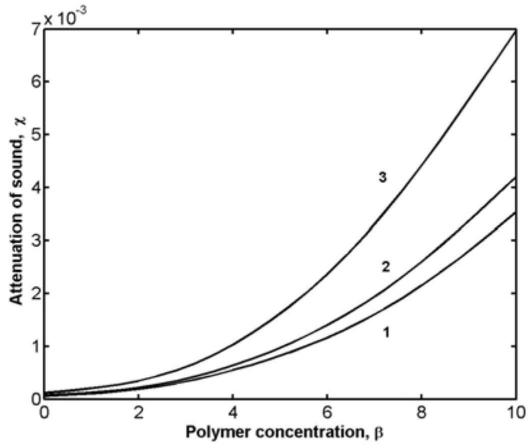


Figure 2 – Dimensionless sound speed and attenuation in the waveguide versus reduced polymer concentration

The plots on the Figure 2 illustrate the influence of the polymer concentration on the sound speed and attenuation in the waveguide. It can be concluded that effect of polymer concentration on the wave velocity is more pronounced in the case of a stiffer shell (with relatively thin polyamide coating). In any case the growth of the reduced polymer concentration leads to increase of the sound attenuation and slows the wave - the result is explained by the losses increase with β.

SUMMARY

The model of low frequency sound propagation in two-layered elastic cylindrical shell with viscoelastic polymeric liquid inside has been developed and investigated. The hydraulic (quasi-one-dimensional) approach was used for solution of internal hydrodynamic problem in order to account for the frequency dependent dissipation at liquid-shell interaction in the wave. The dispersion equation was studied numerically for the shell, consisting from aluminium and polyamide layers with different thicknesses. It was shown that the shell composition influence not only the dispersion, but also the attenuation of the wave. For a stiffer shell the sound wave velocity is larger and attenuation smaller than for more compliant one. The effect of polymer concentration has been studied also; it was shown that growth of reduced polymer concentration leads to increase of the sound attenuation and slows the wave.

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