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HOLOGRAPHIC RECONSTRUCTION OF THE VIBRO-ACOUSTIC FIELD OF AN ENGINE USING THE INVERSE BEM AND EQUIVALENT SOURCES

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Abstract

In this paper, the vibro-acoustic source field of an engine was reconstructed from the measured pressure data of array microphones using the combined technique of equivalent source method (ESM) and inverse boundary element method (BEM). The engine structure was relatively large when the smallest wavelength of interest was considered. Accordingly, the number of boundary nodes for modelling the complicated engine geometry became very large. Because the precision of the holographic reconstruction depends on the number of field data in general, many measured pressure data were used in applying the technique. In the solution, smaller number of measurement was actually made in the field than the number of boundary element nodes. The other required field data were obtained by using the ESM employing a distribution of acoustic multi-pole and multi-order virtual sources inside the engine. After obtaining the sufficient field data, the vibro-acoustic source field of the engine surface was restored using the inverse BEM technique. As a result for a six-cylinder engine, a satisfactory reconstruction of surface acoustic parameters was obtained with an error less than about 20% when the frequency range of interest was up to the third harmonics of the firing frequency.

INTRODUCTION

Identification of source distribution is the effective starting condition for the machine noise control. As one of the indirect methods for the source identification, the BEM-based NAH technique has been recently highlighted [1]. Reconstruction of a large and complex-shaped source requires a large number of field measurement points. As a solution to this problem, smaller number of measurement was made in the field than the required measurement data; the rest was obtained by using the equivalent source method (ESM) employing a distribution of acoustic multi-pole sources inside the source structure.

ESM was suggested [2] as an alternative to BEM that can be also used for the reconstruction of sound field without numerical modelling and source information. ESM is based on the principle of wave superposition in the calculation of sound radiation and scattering from an object, which can be modelled as a combination of elementary acoustic sources such as monopoles, dipoles, etc. It can be said that the Helmholtz equation least-squares (HELS) method [3] is conceptually a special case of ESM using a single-point multi-pole source.

In this paper, in order to obtain the over-determined hologram data with a small number of actual measured data, the generalized ESM with multi-point, multi-pole fictitious sources was formulated and used for reconstructing the radiated sound field of an irregular source structure, like an automotive engine. For suppressing the adverse effect of evanescent waves in the near-field during the inversion process, spatial filtering of coefficients and wave-vector components by regularization scheme was adopted [4]. The effective independence (EfI) method [1] was also employed to determine the optimal position sets of sensors in the field as well as the equivalent sources inside the source structure. After obtaining the sufficient field data for assuring the precision of the result, the vibro-acoustic source field could be efficiently restored using the BEM-based NAH technique; both the regenerated field data and the actually measured data were used as input. We applied the developed combined method to a 2.5L automotive engine with 6-cylinder.

SUMMARY OF FORMULATIONS

BEM-based acoustical holography

Taking the inverse of the discrete form of Kirchhoff-Helmholtz integral equation [1,5], the surface velocity can be reconstructed from the field pressure using the least-square solution and the singular value decomposition (SVD) as

$$\mathbf{v}_S = (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H \mathbf{p}_f = \mathbf{W} \Lambda^{-1} \mathbf{U}^H \mathbf{p}_f , \tag{1}$$

where $\mathbf{G}_1$ is the vibro-acoustic transfer matrix, $\mathbf{p}_f$ the field pressure vector, and $\mathbf{v}_S$ the velocity vector on the surface, the operator H makes the matrix in a Hermitian form, the diagonal elements λ_i of the matrix Λ represent the singular values of the matrix $\mathbf{G}_1$, and $\mathbf{U}$, $\mathbf{W}$ are the unitary singular matrices.

Equivalent source method

When the multi-point, multi-pole equivalent sources are adopted for modelling a vibro-acoustic source, the radiated field pressure at the position $\mathbf{r}_m$ can be expressed by a superposition of spherical functions ψ_j with respect to each equivalent source position $\mathbf{r}_e$ as

$$p_f(\mathbf{r}_m) = \sum_{e=1}^{E}\sum_{j=1}^{J} C_j^e \psi_j(\mathbf{r}_m - \mathbf{r}_e), \tag{2}$$

where E is the number of equivalent sources inside the actual vibro-acoustic source and J is the total number of radiation functions. For M measurement points, one can rewrite Eq. (2) in a matrix form as

$$\mathbf{p}_f = \mathbf{\Psi}_1\mathbf{C}_1 + \cdots + \mathbf{\Psi}_J\mathbf{C}_J \equiv \mathbf{\Phi}\mathbf{D}, \tag{3}$$

where $\mathbf{\Psi}_j$ and $\mathbf{C}_j$ represent the spherical function matrix related to the jth expansion term and corresponding coefficient vector, respectively. The total number of coefficients of multiple equivalent sources is Q (=$E\times J$). The first term $\mathbf{\Psi}_1$ on the right-hand side of Eq. (3) is the system matrix consisting of monopoles only. High-order multi-poles are added up as the order of expansion increases. By using the SVD technique, the coefficients can be obtained by

$$\hat{\mathbf{D}} = \mathbf{W}_\Phi \Lambda_\Phi^{-1} \mathbf{U}_\Phi^H \tilde{\mathbf{p}}_f, \tag{4}$$

where $\tilde{\mathbf{p}}_f$ denotes the measured pressure vector at M field points. Once the coefficient of each spherical function is determined by Eq. (4), one can regenerate arbitrary field pressures at M_2 points outside the minimal spherical surface as

$$\hat{\mathbf{p}}_{f_2} = \mathbf{\Phi}_{M_2}\hat{\mathbf{D}}, \tag{5}$$

where $\mathbf{\Phi}_{M_2}$ represents the spherical function matrix for M_2 field points.

Improvement of reconstruction accuracy

The matrix $\mathbf{\Phi}$ is ill-conditioned in general. Consequently, the condition number of the matrix $\mathbf{\Phi}$ is far larger than unity. During the inversion, high-order multi-poles and wave-vector components, which are closely correlated with evanescent waves, affect the reconstruction result significantly. Because the measured field pressures are always contaminated by noise, small fast-varying noise signals, both measurement noise and high-order wave components can become a big contributor after the inversion. For this reason, the reconstructed results will be severely distorted from the actual target field.

Spherical radiation functions are directly associated with the acoustic radiation modes of multi-poles because each multi-pole has its unique directivity pattern. Therefore, a spatial coefficient filtering technique is considered, which truncates a part of high-order multi-poles. The truncated solution after eliminating the column vectors related to acoustic multi-poles of order q+1 to Q can be expressed as

$$\hat{\mathbf{D}}_q = (\mathbf{\Phi}\mathbf{F}_1^q)^+ \tilde{\mathbf{p}}_f \equiv (\mathbf{\Phi}_q)^+ \tilde{\mathbf{p}}_f, \tag{6}$$

where

$$\mathbf{F}_1^q = diag(1,\cdots 1,0,\cdots,0),\ 0 \le rank(\mathbf{F}_1^q) = q \le Q\,. \tag{7}$$

Here, the operator '+' signifies the pseudo-inversion of a matrix and $\mathbf{\Phi}_q$ denotes the truncated spherical function matrix to the order q. The optimal parameter q_{op} can be obtained by various parameter-choice tools [4]. In this paper, the generalized cross validation (GCV) technique is employed to determine the optimal parameter q_{op}. Spatial filtering of wave-vectors having small singular values of spherical function matrix with a predetermined q_{op} can be thought of. By using a regularization technique, the solution can be given by

$$\hat{\mathbf{D}}_{q_{\rm op}}^{\alpha} = \mathbf{F}_1^{q_{\rm op}} \mathbf{W}_\Phi \mathbf{F}_2^{\alpha} \Lambda_\Phi^{-1} \mathbf{U}_\Phi^H \tilde{\mathbf{p}}_f \equiv (\mathbf{\Phi}_{q_{\rm op}}^{\alpha})^+ \tilde{\mathbf{p}}_f\,, \tag{8}$$

where

$$\mathbf{F}_2^{\alpha} = diag(F_1^{\alpha}, F_2^{\alpha}, \cdots, F_{q_{\rm op}}^{\alpha}),\ F_q^{\alpha} = 1-(1-\beta\lambda_q^2)^{\alpha+1}\,. \tag{9}$$

Here, the diagonal elements of $\mathbf{F}_2^{\alpha}$ represent the wave-vector filter coefficients at the αth iteration step and β denotes the convergence factor.

Hybrid holographic reconstruction

Once the coefficients of equivalent sources $\hat{\mathbf{D}}_q^{\alpha}$ are optimally determined from measured field pressures, then arbitrary field pressures at M_2 points can be regenerated with the matrix $\mathbf{\Phi}_{M_2}$. Combining original measured pressures with regenerated field pressures, Eq. (5) can be merged with the conventional holography equation using BEM. Consequently, the reconstructed surface velocity can be expressed as

$$\hat{\mathbf{v}}_S = \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix}^+ \begin{Bmatrix} \tilde{\mathbf{p}}_f \\ \hat{\mathbf{p}}_{f_2} \end{Bmatrix} = \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix}^+ \begin{Bmatrix} \mathbf{I}_M \\ \mathbf{\Phi}_{M_2} \mathbf{D}_q^{\alpha} \end{Bmatrix} \tilde{\mathbf{p}}_f\,, \tag{10}$$

where $\mathbf{G}_2$ represents the transfer matrix between regeneration points and nodal points of the vibrating source. In this way, the size of global matrix is virtually enlarged.

APPLICATION TO AN AUTOMOTIVE ENGINE

Measurement setup and BE model

A 2.5L automotive engine was mounted on three rigid columns and driven by an engine-dynamo system in a semi-anechoic chamber. The intruding noise from the transmission was excluded by lead-wrapping. Because a 6-cylinder engine was considered, engine noise was dominant at the firing frequency and its harmonics (C3, C6, C9, etc.). For the application of the proposed method, the target engine was modelled 654 linear triangular boundary elements having 329 nodes. The maximum characteristic length of the BEM model was 181.5 mm, which limits the applicable high frequency to about 470 Hz considering the $\lambda/4$-criterion for linear elements. To use as small measurement data as possible, ESM technique was considered to regenerate the sound field to obtain the satisfactory number of data for the reconstruction. Pressures were measured at M = 150 points, which were selected by the EfI method [1]. Locations of equivalent sources were also determined by the EfI technique. Initially, there were 500 candidate equivalent sources randomly distributed inside of the source. The EfI value of each equivalent source could be calculated with the matrix $\mathbf{\Phi}$ in Eq. (3). Finally, 100 equivalent sources were chosen.

Field regeneration using ESM

For a fixed running condition at 3000 rpm, field pressures at 1440 points could be calculated from the final set of equivalent sources (E=100). *Figure 1* shows the percentage regeneration error (in L2 norm) with respect to the measured field pressures with increasing the number of equivalent sources at 150 Hz (C3). One can see that the reconstruction result becomes better as the number of equivalent sources increases. Note that the regenerated field from one equivalent source (E=1), equivalent to the HELS method [3], does not converge to the measured value. This is because the measurements were not taken over a spherical surface. Consequently, the resolution of the reconstruction would not be improved even though the high-order multi-poles were adopted. However, the result for the case with E=100 agrees far better with the measured value than the case using small number of equivalent sources. This means that the radiated sound field can be recovered by the distribution of relatively simple sources such as monopoles and dipoles. Actually, the Kirchhoff-Helmholtz integral equation for irregular source geometry has a very similar meaning with this. From *Fig. 2*, one can say that reconstruction results are satisfactory and *Fig. 1* shows that the error converges to a certain value when the adopted number of equivalent sources is more than about E = 40 in this case. When the frequency range of interest was spanned up to the third harmonic of firing frequency (C9) and 100 equivalent sources were used, we observed a satisfactory reconstructed result with an average error about 12%.

Reconstructed surface velocity and power contribution

Combining the BEM-based NAH (sometimes called, the NAH based on the inverse BEM) with the field data regenerated by the ESM, the source parameters can be reconstructed by using the backward BEM calculation. *Figure 3* illustrates the reconstructed surface velocity distribution calculated from 1440 measured pressures and 1440 regenerated pressures at 150 Hz. In this case, the relative error between two results was about 16 %. Once the surface parameters were identified, the field parameters, i.e., sound intensity and radiated power from the source surface, could be easily predicted by using the forward BEM calculation. *Figure 4* depicts the percentage contribution of each engine component to the total radiation power. This figure reveals that the sound power from front-end accessory drive (FEAD) part is the largest of all engine components at 150 Hz in 1/3-octave band.

CONCLUSIONS

The sound field radiated from an automotive engine was regenerated in a holographic way by using the multi-point equivalent sources. Regenerated field data combined with measured data were, in turn, used in the reconstruction of the source parameters using the BEM-based NAH technique. This method can supplement the BEM-based NAH that requires a lot of computational and measurement effort. We have applied the suggested hybrid holography technique to a 6-cylinder automotive engine and a satisfactory reconstructed result was obtained with an average error less than about 20% for the frequency range of 120~450 Hz (firing frequencies) at 3000 rpm.

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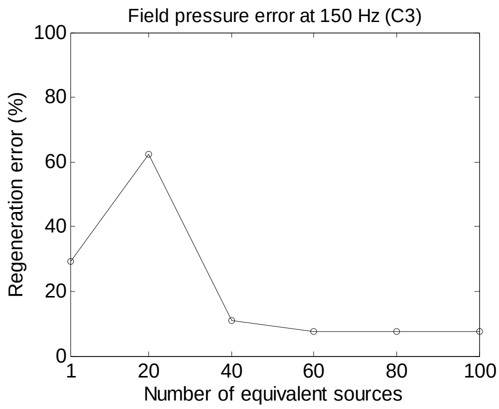

Figure 1 – Regeneration error of field pressures at 150 Hz with increasing the number of equivalent sources.

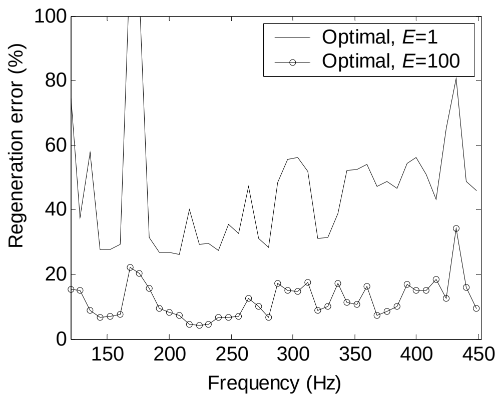

Figure 2 – A comparison of the regeneration error of field pressures varying the frequency.

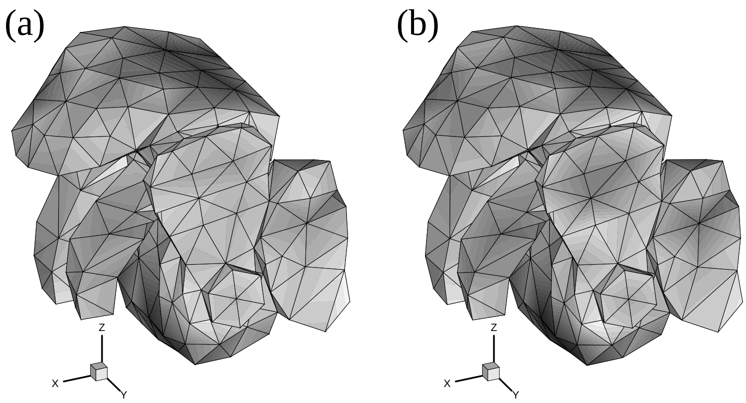

Figure 3 – Reconstructed surface velocity distribution at 150 Hz by combining the BEM-based NAH with (a) 1440 actual measured pressures and (b) 1440 regenerated pressures.

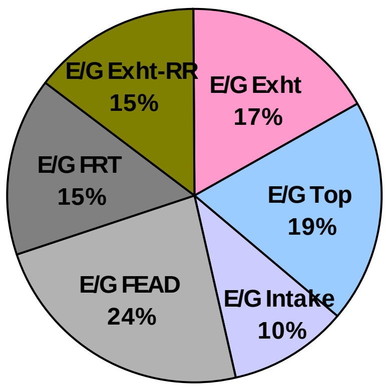

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Figure 4 – Contribution of each engine component to the total radiation power at 150 Hz in 1/3-octave band.