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POLYNOMIAL APPROACH TO DESIGN OF FEEDBACK VIRTUAL-MICROPHONE ACTIVE NOISE CONTROL SYSTEM

Marek Pawelczyk*

Institute of Automatic Control, Silesian University of Technology
ul. Akademicka 16, 44-101 Gliwice, Poland
marek.pawelczyk@polsl.pl

Abstract

Classical active noise control systems can provide high noise attenuation at the close vicinity of the so-called error microphone. If the zone of quiet is required at a more distant location dedicated Virtual-Microphone Control systems can be applied. The aim of the paper is to present further development of this concept. However, contrary to most of the available solutions, the considered structure is not based on the Internal Model Control structure, where estimate of the output disturbance drives the control filter. Instead, estimated residual signal at the virtual microphone constitutes the controller input. This allows directly modeling the so-called difference path and taking benefits of its limited changes compared to changes of the paths from the secondary source to the real and virtual microphones. The optimal H_2 controller is designed using the polynomial approach. Therefore, parametric models of the paths as well as the disturbance are necessary. The disturbance-shaping filter is split using a Diophantine equation. To cope with non-minimum phase character of the path models a transfer function is factorized into inner and outer parts. Finally causal part of the controller is extracted. An equation for the spatial attenuation gradient due to change of the virtual path is also derived. Results of experimental verification of the discussed algorithm applied to control noise in an active headrest system are presented.

INTRODUCTION

In classical active noise control (ANC) systems the residual (output) signal at the error microphone providing information about the acoustic wave interference is minimized. The secondary sound operates at the same time on the acoustic noise at other positions. In the worst case this can result in sound reinforcement at the position

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where noise attenuation is of utmost interest, e.g., at the user ear. In many applications placing another microphone directly at the ear is not accepted. To overcome this problem the idea of Virtual-Microphone Control (VMC) systems was proposed and extensively studied for recent years (see, e.g., [1]-[6], [10], [11]). The purpose is to minimize the effect of the noise at the desired location while performing measurements of sound interference results at another location.

This paper presents another VMC algorithm in the series of VMC algorithms developed by the author [4], [5], [6]. It is proposed for a plant, for which the distance between the real (error) and virtual (placed in the area of interest) microphones is much less than the smallest wavelength in the primary acoustic noise. Such constraint permits to make the assumption that the primary noise at positions of these microphones is the same and can be considered as the disturbance $d(i)$, where i is the discrete-time instant [3], [6], [7].

A wide-sense stationary stochastic disturbance can be modeled as

$$d(i) = F(z^{-1})e(i) \tag{1}$$

where $e(i)$ is a zero-mean wide-sense stationary white noise signal and $F(z^{-1})$ is a minimum phase disturbance-shaping filter obtained, e.g., by spectrally factoring the Power Spectrum Density (PSD) of the disturbance. To be strict, $F(q^{-1})$ should be used in the above relation to distinguish between the operator and the transfer function of complex variable z^{-1}. Nevertheless, in this paper, similarly to a number of publications, the same notation with omitted argument is used both for the signal operation and transfer function bearing the difference in mind.

The plant to be controlled is considered in terms of the path from the secondary source to the real microphone (real path) and the path to the virtual microphone (virtual path), including the electronic components necessary for discrete-time control, i.e., analog anti-aliasing and reconstruction filters, A/D and D/A converters. Both the paths are generally non-minimum phase and have considerable delays. It is assumed that the plant paths, their models and control filter are linear and time-invariant and they are represented by rational transfer functions. It is additionally assumed without loss of generality that F is an FIR filter.

VIRTUAL-MICROPHONE CONTROL SYSTEM

The system structure, similar to that of [3] is considered as presented in Figure 1. For this system an estimate of the residual signal at the virtual microphone, $\hat{y}_v(i)$, constitutes the control filter input. However, the approach to control is completely different in this paper. The objective is to minimize the variance of $\hat{y}_v(i)$ (the command signal is equal to zero), i.e.,

$$L = E\{\hat{y}_v^2(i)\}, \tag{2}$$

where $E\{.\}$ stands for the expectation operator.

Instead of modeling the real and virtual paths separately it is justified to directly model the difference path

$$\Delta S = S_r - S_v \tag{3}$$

as the so-called difference filter $\widehat{\Delta S}$ by using the difference of measurements done with the real and virtual microphones. The advantage of such notion is that for many active control applications, e.g., for the active headrest, the change of S_r and S_v is generally in the same 'direction'. Consequently, the change of the difference path is often much smaller than changes of the paths themselves.

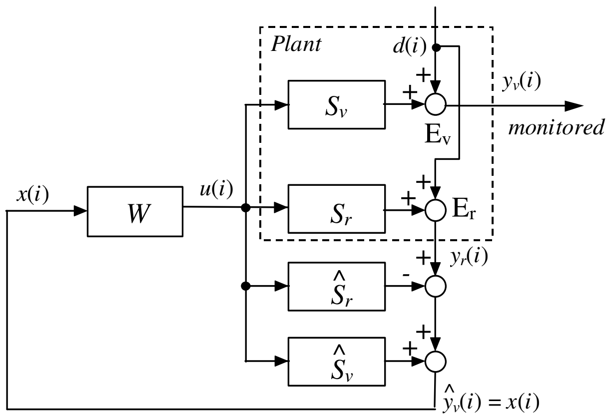

Figure 1 – The VMC1 system with the FXLMS algorithm

With this notation the following are valid:

$$\hat{y}_v(i) = \frac{1}{1+W\widehat{\Delta S}}\, y_r(i), \tag{4}$$

$$u(i) = \frac{W}{1+W\widehat{\Delta S}}\, y_r(i) = -Hy_r(i), \tag{5}$$

$$y_r(i) = d(i) + S_r u(i), \tag{6}$$

In (5) H is the overall controller in the negative feedback notation. Substituting for $u(i)$ from (5) into (6) gives

$$y_r(i) = \frac{1+W\widehat{\Delta S}}{1+W(\widehat{\Delta S}-S_r)}\, d(i) = V_r d(i), \tag{7}$$

where V_r represents the Real-Output Sensitivity Function. Taking (4) and (7) together and noticing that, after (3),

$$\widehat{\Delta S} - S_r = \hat{S}_r - S_r - \hat{S}_v = \widehat{\Delta S} - \Delta S - S_v \,, \tag{8}$$

the signal being controlled is

$$\hat{y}_v(i) = \frac{1}{1 + W(\widehat{\Delta S} - \Delta S - S_v)} d(i) \,. \tag{9}$$

Making the following substitution

$$S_2 = \widehat{\Delta S} - \Delta S - S_v \tag{10}$$

and taking (1) into account allows expressing this problem in a compact form

$$\hat{y}_v(i) = \frac{F}{1 + WS_2} e(i) \,. \tag{11}$$

Thus, the signal $\hat{y}_v(i)$ can be interpreted as the output of a negative feedback control system with plant S_2, controller W and output disturbance $Fe(i)$. Equation (11) can be rewritten to

$$\hat{y}_v(i) = -WS_2 \hat{y}_v(i) + Fe(i) \,. \tag{12}$$

Assuming that k is the time delay of S_2 the disturbance-shaping filter is now proposed to be split using the following Diophantine equation

$$F = F_1 + z^{-k} F_2 \,, \tag{13}$$

$$\begin{cases} \deg F_1 = k - 1 \\ \deg F_2 = \deg F - k \end{cases} . \tag{14}$$

Substituting for F in (12) gives

$$\hat{y}_v(i) = \left[-WS_2 \hat{y}_v(i) + z^{-k} F_2 e(i) \right] + \left[F_1 e(i) \right] , \tag{15}$$

where the two components separated by the squared brackets are uncorrelated because the discrete time delay of S_2 is equal to k and W is assumed to be causal.

Thus, (2) can be written as

$$L = E\left\{\left[-WS_2\hat{y}_v(i) + z^{-k}F_2e(i)\right]^2\right\} + E\left\{\left[F_1e(i)\right]^2\right\}. \quad (16)$$

The second term on the RHS does not depend on the control signal. Therefore, the optimal filter minimizing the cost function is determined by

$$W_{opt} = \arg\min_W E\left\{\left[-WS_2\hat{y}_v(i) + z^{-k}F_2e(i)\right]^2\right\}. \quad (17)$$

Taking (11) into account gives

$$W_{opt} = \arg\min_W E\left\{\left[\left(-WS_2\frac{F}{1+WS_2} + z^{-k}F_2\right)e(i)\right]^2\right\}. \quad (18)$$

The minimum value of the term within the curly brackets is for the control filter satisfying

$$-W_{opt}S_2F + z^{-k}F_2 + z^{-k}F_2W_{opt}S_2 = 0. \quad (19)$$

Taking (13) into account and rearranging gives finally

$$W_{opt} = \frac{z^{-k}F_2}{S_2F_1} = \frac{F - F_1}{S_2F_1}. \quad (20)$$

Unfortunately, such filter is not causal-stable for non-minimum phase transfer function S_2 (this is the case for the ANC system) and some filters F_1. To solve this problem the methodology commonly used for the feedforward system can be applied (see, e.g., [6], [8], [9]). Therefore, the transfer function S_2F_1 is factorized into the inner (unitary all-pass) and outer (minimum phase) parts [6], [8], [9].

$$S = S_2F_1 = S^{(i)}S^{(o)}. \quad (21)$$

Then, applying the factorization to (19) the (sub-) optimal stable and causal control filter can be derived

$$W_{opt+} = \frac{1}{S^{(o)}}\left\{\frac{F - F_1}{S^{(i)}}\right\}_+ \quad (22)$$

and the optimal estimated residual signal at the virtual microphone, (11), is

$$\hat{y}_v(i)_{opt} = \frac{F}{1+\frac{S_2}{S^{(o)}}\left\{\frac{F-F_1}{S^{(i)}}\right\}_+} e(i). \tag{23}$$

The symbol $\{\cdot\}_+$ denotes causal part of $\{\cdot\}$.

For stability of the system the following characteristic equation (see (10))

$$1+W\left(\widehat{\Delta S}-\Delta S-S_v\right)=0 \tag{24}$$

cannot have unstable zeros or $W(\widehat{\Delta S}-\Delta S-S_v)$ cannot encircle the Nyquist point, assuming the control filter is stable. Otherwise, the spectral or inner-outer factorizations should be regularized [8].

The user is interested in noise control at the ear (virtual microphone) located often at a position different than that where the models have been acquired. Therefore, it is necessary to examine the Virtual-Output Sensitivity Function, V_v. Combining (5), (7) and (10) results in

$$u(i)=\frac{W}{1+WS_2}d(i). \tag{25}$$

It follows from Figure 1 that

$$y_v(i)=d(i)+S_v u(i). \tag{26}$$

Taking (25) and (26) together gives finally

$$y_v(i)=\frac{1+W\left(S_2+S_v\right)}{1+WS_2}d(i)=\frac{1+W\left(\widehat{\Delta S}-\Delta S\right)}{1+W\left(\widehat{\Delta S}-\Delta S-S_v\right)}d(i)=V_v d(i). \tag{27}$$

The Virtual- and Real-Output Sensitivity Functions for respective control systems can be related as (see (27) and (7))

$$V_v=V_r\left(1-\frac{\Delta S\, W}{1+\widehat{\Delta S}\, W}\right). \tag{28}$$

The noise attenuation level as a function of frequency takes the following form for the virtual microphone:

$$J_v(\omega) = -10\log_{10}\left(\frac{S_{y_v y_v}(e^{\omega T_S})}{S_{dd}(e^{\omega T_S})}\right) = -20\log_{10}\left(\left|V_v(e^{-j\omega T_S})\right|\right) \text{ [dB]}, \tag{29}$$

where $S_{dd}(e^{\omega T_S})$ and $S_{y_v y_v}(e^{\omega T_S})$ stand for the PSDs of the disturbance and the virtual-output signals, respectively. Analogous equation is valid for the real microphone. Taking (28) and (29) into account the spatial attenuation gradient due to change of the plant response can be expressed as

$$\Delta J(\omega) = J_v(\omega) - J_r(\omega) = -20\log_{10}\left(\left|1 - \frac{\Delta S\, W}{1 + \widehat{\Delta S}\, W}\right|\right). \tag{30}$$

SIMULATIONS

To verify performance of the proposed control system it has been employed for noise control in the right channel of the active headrest system. Geometry of the experimental setup providing data for the simulations is presented in Figure 2. 2 kHz sampling frequency and 4th order analog Butterworth filters with 650 Hz cut-off frequencies have been applied. The plant paths are non-minimum phase and have considerable discrete time delays – 3 samples for the real path and 4 samples for the virtual path, for the nominal position of the head as shown in Figure 2. The noise has been generated by passing a zero-mean white noise signal by a band-pass filter. PSD of the noise together with PSDs of the residual signal at the virtual and real microphones are presented in Figure 3, respectively. The attenuation levels are 8.6 and 3.7 dB, respectively. The results demonstrate that the proposed control system successfully shifts the zone of quiet to the desired location.

SUMMARY

In this paper a feedback Virtual Microphone Control system has been designed for acoustic noise control. Due to non-minimum phase character of the acousto-electric plant the factorization approach has been used. The equation for the attenuation gradient has also been derived. Results of the numerical verification performed using the data acquired on the active headrest confirmed good performance of the control system at the virtual microphone and much lower performance at the real microphone. This conclusion justifies the interest in VMC systems for this type of applications.

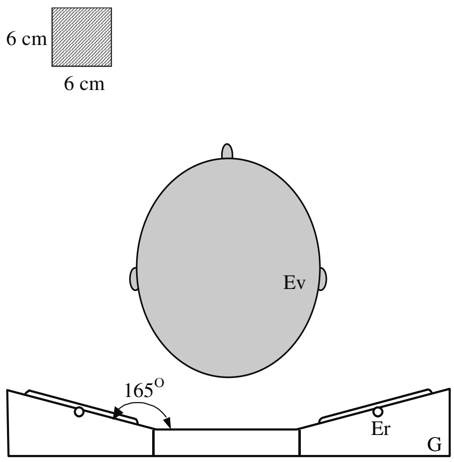

Figure 2 – Experimental set-up of the headrest

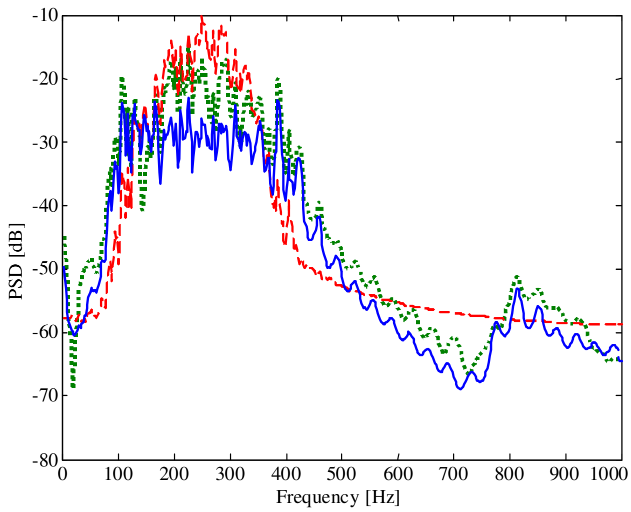

Figure 3 – Power Spectral Densities of the signals: noise (dashed, red), virtual output (solid, blue), real output (dotted, green)

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