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ADVANCED EXPERIMENTAL PROCEDURE FOR IN-DUCT AERO-ACOUSTICS

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Abstract

The purpose of this paper is to present a method for characterization of in-duct aero-acoustic sources that can be described as active acoustic two-ports. The method is applied to investigate the sound produced from an orifice plate. The motivation is to obtain better data for the development of improved prediction methods for noise from flow singularities, e.g., in HVAC systems on aircrafts. Most of the earlier works fall into two categories; papers modeling the scattering of acoustic waves and papers modeling the sound generation. Concerning the scattering it is possible to obtain estimates of the low frequency behavior from linear perturbations of the steady state equations for the flow. Concerning the sound generation most of the presented work is experimental and follows a paper by Nelson&Morfey, which present a scaling law procedure for the in-duct sound power based on a dipole model of the source. One limitation with the earlier works is that the sound power only was measured on the downstream side. Also data was only obtained in 1/3-octave bands, by measuring the sound radiated from an open duct termination. Assuming plane waves and linear acoustics the flow duct singularity can be completely modeled as an active 2-port. The experimental determination of its properties is done in a two steps procedure. In the first step the passive data, i.e., the scattering matrix **S**, is determined using external (independent) sources. In the second step the **S** matrix is used and the source vector is determined by testing the system with known acoustic terminations.

1. INTRODUCTION

Sound propagation in duct systems is of importance for engineers working with the aspect of reducing the sound power radiated from the openings or minimizing the sound pressure level within some parts of the system. The sound field in these duct systems depends on their "*passive*" and "*active*" part. The "*passive*" part is controlled by the duct geometry and speed of sound and it determines how the sound propagates through the system. The "*active*" part depends on the acoustic sources within the system and describes how sound energy is generated i.e. the sources. Sources can sometimes be placed outside the duct system considered and act as a boundary

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condition. An example of this is the action of an IC-engine on an exhaust system. These sources are called “external”. In other cases the sources are included within the description of the system. These sources are called “internal” and can for example be a fan, or an orifice. This paper is concerned with the description of “internal” noise produced by flow in ducts, i.e., flow induced noise. This noise source does not involve any moving surfaces such as in fans or loudspeakers. The noise is generated by unsteady flow separation and the paper focuses on broad band noise produced by turbulence. The flow separation concerned here is usually due to sharp edges. Some typical examples are bends, constrictions, expansions, and orifices. Since the duct system in many cases has the purpose of decreasing the sound propagation and to prevent sound radiation from its openings, the flow induced noise imposes a lower limit of possible sound levels. Flow induced noise has been investigated by several authors and some of these publications can be found in Ref. [1-4]. It is quite complex to estimate the noise produced by flow because of the noise mechanism (turbulence). However, one method that has been successful is the scaling law technique [2]. This technique uses a combination of theoretically developed expressions, based on Lighthill´s theory for aeroacoustic sound, in combination with measurement data. The theory has been developed by assuming that flow separation at a distinct point generating a fluctuating force is the main mechanism. This force acts as a fluctuating acoustic dipole source distribution [2-4]. This source is dependent on the surrounding, which in this case is the duct wall. By using the theoretical developed relations and measure the produced noise for different dimensions (same geometry) and flow speeds, a collapse of the data is possible. This collapse of data is the non-dimensional reference spectrum. This spectrum can now, in combination with the theoretical formulation and combined with the flow condition and dimension, be used inversely to predict generated noise for similar cases**.** Oldham&Ukpoho [3] further developed work performed by Nelson&Morfey [2], who studied noise produced in rectangular ducts having constrictions like spoilers and orifices. Oldham&Ukpoho [3] used this work and applied it on circular ducts. They argued that a more general reference spectrum, valid for different geometries but having “similar” flow separation processes, could be found. This should be achieved by using a suitable definition of the Strouhal number. One limitation with the earlier works is that the sound power only was measured on the downstream side and only obtained in 1/3-octave bands, by measuring the sound radiated from an open duct termination. So the aim of this paper is to present a more general experimental method for characterization of in-duct aero-acoustic sources that can be described as active acoustic two-ports. The method is applied to investigate the sound produced from a single orifice plate. It is hoped that the results also will be useful for the development of improved prediction methods for noise from flow singularities.

2. THEORETICAL BACKGROUND

In order to describe the active part of the orifice the source pressure amplitudes p_{a-}^{s} and p_{b-}^{s}, see Figure 1, have to be estimated. These waves represent the outgoing pressure amplitudes from the source process under reflection free conditions (an infinite duct). The source pressure amplitudes can be added to the passive part to form an active 2-port which, assuming that the data is in the frequency domain, is given by [5]

$$\begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix} = \mathbf{S} \begin{pmatrix} p_{a+} \\ p_{b+} \end{pmatrix} + \begin{pmatrix} p_{a-}^{s} \\ p_{b-}^{s} \end{pmatrix}, \tag{1}$$

where, all the data is referred to two reference cross-sections *a*&*b* and **S** is 2x2 matrix represents the passive part. The source strength is described as a vector in equation (1) which represents the stochastic source process. The Fourier transform of such a signal only exist in a generalized sense and therefore one normally describes the source vector via the so called source cross-spectrum matrix [5], which can be defined as:

$$\mathbf{G}^{s} = \begin{pmatrix} p_{a-}^{s} \\ p_{b-}^{s} \end{pmatrix} \begin{pmatrix} p_{a-}^{s} \\ p_{b-}^{s} \end{pmatrix}^{c} = \begin{pmatrix} G_{a-a-}^{s} & G_{a-b-}^{s} \\ G_{b-a-}^{s} & G_{b-b-}^{s} \end{pmatrix} \tag{2}$$

where, $G_{x-y-}^{s} = p_{x-}^{s}\left(p_{y-}^{s}\right)^{c}$, and c denotes the complex conjugate. One method to estimate the source data for flow separation in constrictions is to use the model proposed by Nelson&Morfey [2]. This model is based on theoretical expressions derived using Lighthill's theory and expressed as a scaling law, which must be obtained via measurements.

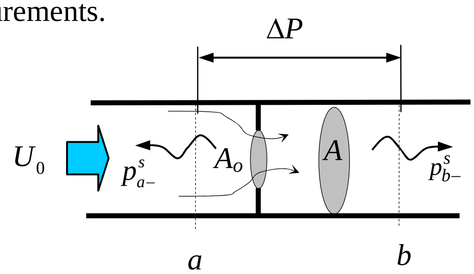

Figure 1 Sound generation by a compact flow obstruction in a duct. The reference sections *a* and *b* are assumed to be just up- and downstream of the obstruction (orifice plate). Small Mach-numbers are assumed and low frequencies (plane waves). This will imply that the source region is acoustically compact.

2.1 A scaling Law for the Source Strength

A modified version of the scaling law derived by Nelson&Morfey will be presented, which is adapted to the 2-port model and the plane wave range. The steady state force $\overline{F}$ created by a flow constriction can be written as:

$$\overline{F} = A \cdot \Delta P = \rho_0 U_0^2 C_L A / 2 . \tag{3}$$

where, $C_L = \left(1/\gamma - 1\right)^2, \gamma = \sigma \Big/ \left(1 + \sqrt{0.5\left(1\text{-}\sigma\right)}\right), \sigma = A_o / A$ is the pressure loss coefficient, vena contracta ratio [1] and area contraction ratio, respectively.
The unsteady dipole force F created by the constriction is assumed to be proportional to the steady force [2]. Since this unsteady force is a broad-band random type of signal it is best described by its spectral density $G_{FF} = \left|F\right|^2$. This spectral density can be written as:

$$G_{FF} = \frac{\overline{F}^2 \cdot K^2(St_c)}{\Delta f_c}, \tag{4}$$

where K is a dimensionless function (spectrum) of the St-number and Δf is a characteristic (Strouhal) frequency scale. This frequency scale is chosen as U_c / D_c, where the flow speed (U) and diameter (D) is representative for the vena contracta [3]. If the source is a compact dipole it will (neglecting the convective effect) generate two plane waves with the same amplitude and opposite phase, i.e., $p^s_{a-} = -p^s_{b-}$. Applying conservation of momentum over the source region implies:

$$(p^s_{a-} - p^s_{b-})A + F = 0. \tag{5}$$

From this result using equations (3) and (4) the plane wave acoustic power radiated in the duct in the up- or downstream direction can be calculated. When we neglect the effect of mean flow (convection) these acoustic powers are the same and equal to:

$$W = \frac{A}{\rho_0\, c_0}\left|p^s_{a-}\right|^2 = \left|\frac{F}{2A}\right|^2 \frac{A}{\rho_0\, c_0} = \frac{\rho_0 c_0^3 M^4 C_L{}^2 D_c A}{16 U_c}\left|K(St_c)\right|^2 , \tag{6}$$

where $M = U_0 / c_0$. It can be shown the result in Equation (6) is unchanged if the convective effect is included in the analysis. The vena contracta ratio γ can be used to calculate U_c / D_c:

$$U_c = \frac{U_0}{\gamma} \;,\; D_c = D\gamma^{1/2} \;. \tag{7}$$

From this we obtain an equation which relates the non-dimensional spectrum K to the measured source data (source strength):

$$K^2\left(St_c\right) = \frac{16\, G^s_{a-a-}}{\rho_0\, c_o^3\, M^3 C_L{}^2 \gamma^{3/2}\, D}, \tag{8}$$

where, $St_c = fD_c / U_c$ and a can be interchanged for b for the downstream side and $G^s_{a-a-} = \left|p^s_{a-}\right|^2$. Equation (8) differs from what is presented in Nelson&Morfey [2], since it given in terms of a spectral density rather than a frequency band (1/3-octave).

3. EXPERIMENTAL PROCEDURES

The active 2-port in equation (1) gives a full representation of the sound generation and scattering properties of a flow duct constriction in the plane wave range. The experimental determination of its properties is best done as a two step procedure [6]. In the first step the passive data, i.e., the scattering matrix **S**, is determined using external (independent) sources. In the second step the **S** matrix is used and the source vector is determined by testing the system with known acoustic terminations.

3.1 Test Rig description

Experiments were carried out at room temperature using the flow acoustic test facility at MWL. The test duct used during the experiments consisted of a standard steel-pipe with a wall thickness of 3 mm. The inner duct diameter was 57 mm and the overall length of the rig was around 7 meters. The test object was a single diaphragm orifice with a concentric hole with a diameter of 30 mm (area contraction ratio 0.28). The thickness of the orifice plate was 2 mm and the hole had sharp edges. Twelve loudspeakers were used as external acoustic sources. The loudspeakers were divided equally between the upstream and downstream side. Each loudspeaker was mounted in a short side-branch connected to the main duct as shown in Figure 2. Six (1-6) condenser microphones (B&K 4938) flush mounted in the duct wall, three upstream and three downstream of the test object, were used to cover the plane wave range in the test duct [7]. The cut-on frequency of the first higher order mode in a circular duct is: $f_{cut-on} = 1.84c\left(1-M^2\right)/\pi D$, where D is the duct diameter, or around 3400 Hz in this case. The flow speed was measured upstream of the test section using a small pitot-tube connected to an electronic manometer, at a distance of 1000 mm from the upstream loudspeaker section. The flow speed was measured in the middle of the duct and before and after each acoustic measurement and the average was used. The passive part (the scattering matrix) was measured using the so called source switching technique, see section 3.2. To ensure a high signal-to-noise-ratio a step sinus excitation was used with 1000 averages in each step.

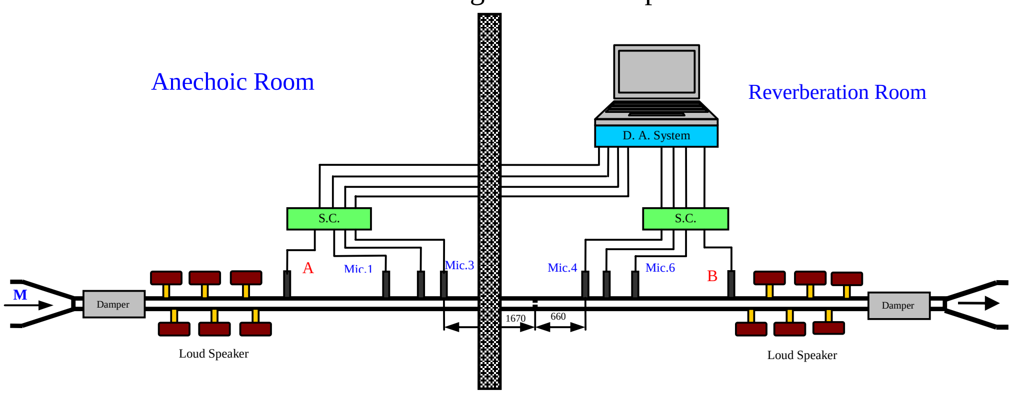

Figure 2 Layout of the test rig used. The rig was operated between two measurement rooms at MWL.

The active part was measured with the external signal (the loudspeakers) on both sides turned off. In order to suppress the local turbulence noise at the microphones, the source data was measured using cross-spectra between the microphones 3&4 and A&B, see section 3.3. For these measurements with a relative poor signal-to-noise-ratio 10000 averages was used to reduce the random errors. Finally all source data (passive and active) were moved to the in- or outlet section of the orifice or orifices. The frequency resolution used in all the measurements was 1.56 Hz except for the validation case where 3.12 Hz was used.

3.2 Passive Data (scattering matrix) Determination

Using pairs of microphones and plane wave decomposition p_+ and p_- the traveling wave amplitudes can be determined [7]. If an independent external source is available we can remove the part of the sound field correlated with the flow generated source by a correlation technique. If the external source is a loudspeaker then this can be realized by measuring the transfer function between the loudspeaker voltage and the traveling wave amplitudes [6]. Using this approach equation (1) reduces to:

$$\begin{pmatrix} h_{ea-} \\ h_{eb-} \end{pmatrix} = \mathbf{S} \begin{pmatrix} h_{ea+} \\ h_{eb+} \end{pmatrix}, \tag{9}$$

where $h_{ex} = p_x / e$ and e is the loudspeaker voltage. To determine the unknown **S** matrix we need to use at least two different external source configurations. Normally this is realized as in Figure 2 by using loudspeakers on each side of the test object. The two cases can then be realized by switching between the loudspeakers on the up- and downstream side.

3.3 Active Data (source cross-spectrum matrix) Determination

In the measurement of the passive data one can also determine the upstream and downstream reflection data, i.e.:

$$\begin{pmatrix} p_{a+} \\ p_{b+} \end{pmatrix} = \underbrace{\begin{pmatrix} R_a & 0 \\ 0 & R_b \end{pmatrix}}_{\mathbf{R}} \begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix} \tag{10}$$

where, R is the reflection coefficient. Inserting this in Equation (1) gives

$$\left(\mathbf{E} - \mathbf{SR}\right) \begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix} = \begin{pmatrix} p_{a-}^{s} \\ p_{b-}^{s} \end{pmatrix} \tag{11}$$

This can be rewritten in terms of the acoustic pressures using:

$$\begin{pmatrix} p_a \\ p_b \end{pmatrix} = \left(\mathbf{E} + \mathbf{R}\right) \begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix} \tag{12}$$

From Equations (11) and (12) one obtains:

$$\begin{pmatrix} p_{a-}^{s} \\ p_{b-}^{s} \end{pmatrix} = \left(\mathbf{E} - \mathbf{SR}\right)\left(\mathbf{E} + \mathbf{R}\right)^{-1} \begin{pmatrix} p_a \\ p_b \end{pmatrix} \tag{13}$$

In principle Equation (13) can be rewritten directly to find an estimate for the source cross-spectrum matrix. However, this estimate will depend on the auto-spectra at the reference sections *a*&*b* and will be affected by local turbulence. It is therefore better to create two estimates based on two different reference cross-sections *a*&*b* and *a´*&*b´* and mix them to avoid this problem [6]. The resulting estimate will then only involve cross-spectra between fluctuating pressures and if the two pairs of reference sections are not too close the local turbulence contributions will be uncorrelated. The resulting equation for the source cross-spectrum estimate then becomes:

$$G^{s} = \mathbf{T}_{+}^{-1}\mathbf{C}'\begin{pmatrix} G_{a'a} & G_{a'b} \\ G_{b'a} & G_{b'b} \end{pmatrix}\mathbf{C}^{c} \tag{14}$$

where, $\mathbf{C} = (\mathbf{E} - \mathbf{SR})(\mathbf{E} + \mathbf{R})^{-1}$, the prim denotes a quantity referred to *a´&b´* and $\mathbf{T}_{+}$ is the transformation matrix for the source strength vector from *a&b* to *a´&b´* [6].

4. RESULTS AND DISCUSSIONS

For the first time a full source characterization for a flow generated source in a duct has been performed, i.e., both the passive and active part in the form of an acoustic 2-port have been determined. The active or source strength part is scaled using the theory proposed by Nelson&Morfey [2], which is modified in order to apply to spectral density estimates. Compared to earlier studies, Nelson&Morfey and Oldham&Upkoho [3], both the up- and downstream source data are obtained. It is found that the downstream source strength, for higher St-numbers, tends to be stronger than the upstream, see Figure 3. This implies that the simple dipole model originally proposed by Nelson&Morfey is not sufficient and further work is needed to explain this result. Interesting results for the passive data are also found and agree, for low frequencies, well with a simple quasi-steady theory as suggested in Refs. [1,8].

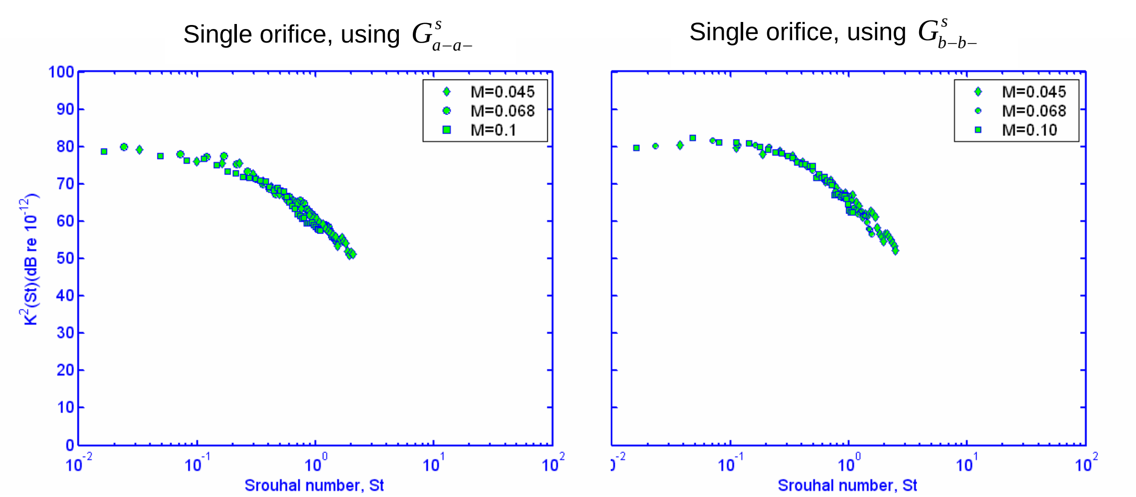

Figure 3 Data collapse for the dimensionless source spectrum up- and downstream side (*a* and *b*) for a single orifice based on Eq. (8).

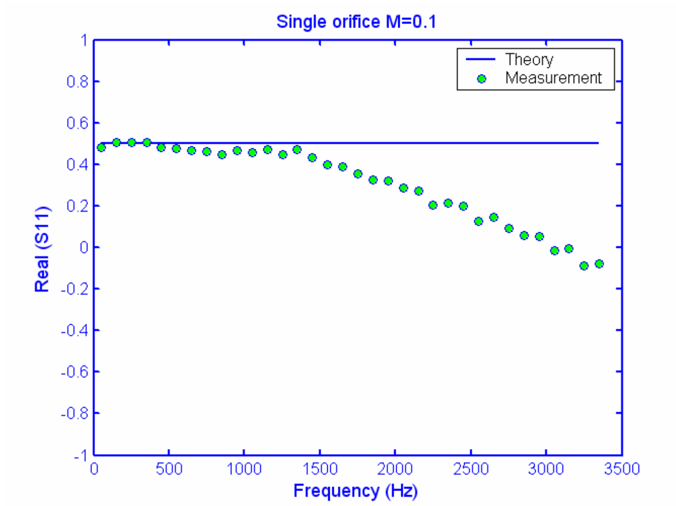

Figure 4 The first element of the scattering matrix S11 (the upstream reflection coefficient) compared with the simple quasi-steady theory from Ref. [8].

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