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DIRECT METHODS *VERSUS* MODAL METHODS IN THE IDENTIFICATION OF RIGID BODY PROPERTIES

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Abstract
The identification of the rigid body properties of a structure is an important issue in various structural dynamic applications, namely in structural modification (coupling/uncoupling), optimisation and vibration control. In most situations, the only way to go is through experimental measurements, i.e., through an inverse process where from the frequency response functions (FRFs) one estimates the desired dynamic properties.

The objective of the present work is to evaluate the performance of two different kind of identification methods that use receptance FRFs as inputs, although in different manners; in one of them the properties are evaluated directly from the measurement data – direct methods – whereas the other type involves a modal identification procedure as an intermediate step – modal methods. A car wheel is the selected actual structure to illustrate the performance of both techniques.

INTRODUCTION

There are various possibilities for the evaluation of the rigid body properties of a structure (mass, centre of mass location and inertia tensor). The theoretical techniques based on sophisticated computational commercial software packages are quite valuable, though often not very practical, especially if the structure is highly complex or if some geometrical or mass properties are not known. In such cases, one has to adopt a different strategy, using experimental methods, classified as static or dynamic. The former class of methods is based on the static equations of motion whereas the latter class is based on the dynamic equations of motion. As a result, with static methods it is only possible, at most, to determine four of the ten rigid body properties (mass and the three centre of mass coordinates). When it is necessary to evaluate all the ten properties one must use the dynamic-based methods [1, 8], either in the time or frequency domain. In the last few years, the authors have dedicated their study to frequency domain methods, which can be classified into *Inertia Restraint Methods* (IRM) [1-3], *Direct Physical Parameter Identification Methods* (DPPIM) [4,5] and *Modal Methods* (MM) [6, 7].

The present paper is dedicated to the evaluation of the performance, advantages and disadvantages of the DPPIM and MM methods. These two kinds of methods use the same input data: the receptance FRFs on the rigid body frequency range.

The *DPPIM* methods allow for the determination of the mass, stiffness and damping parameters through the direct use of measured FRFs available from tests undertaken on "quasi-free" or "softly" suspended structures. Mangus *et al* [4] and Huang and Lallement [5] are amongst those who have developed such techniques.

The *MM* methods are based on the orthogonality relationship between the mass matrix and the rigid body modes. Both *DPPIM* and *MM* methods require the same support conditions and are particularly adequate when the rigid body and the flexible mode shapes are not well separated. For the *MM* methods, a modal identification procedure is required to extract all the modal properties of the rigid body modes.

THEORY

In the present work the determination of the ten inertia parameters follows closely the works of Conti and Bretl [6] and Toivola and Nuutila [7] for the *MM* methods and the works of Huang and Lallement [5] for the *DPPIM* methods.

Modal Methods

The *MM* methods developed by Conti and Bretl [6] and Toivola and Nuutila [7] use the measured responses due to the excitation applied in various points and directions. These methods require as many exciter locations/directions as needed to significantly excite all the six rigid body modes. In theory, one excitation is enough, in contrast with the minimum three excitations required by the *IRM* methods. The measured FRFs for each excitation condition are used to extract the modal properties (natural frequencies, damping ratios and mode shapes), using appropriate identification methods. If the origin of the physical coordinate system is established as a reference, the orthogonality property for mass-normalised mode shapes can be expressed as:

$$\underset{(6\times6)}{[\phi_0]^T}\,\underset{(6\times6)}{[M_0]}\,\underset{(6\times6)}{[\phi_0]}=\underset{(6\times6)}{[I]} \tag{1}$$

where $[M_0]$ and $[\phi_0]$ represent, respectively, the mass matrix and the mass-normalised mode shape matrix with respect to the origin. The first three rows of $[\phi_0]$ represent translational motion and the last three the rigid body rotations. From (1),

$$[M_0]=[\phi_0]^{-T}[\phi_0]^{-1} \tag{2}$$

$[\phi_0]$ is invertible, since the six rigid mode shapes are linearly independent vectors. However, from the identification process, one obtains the mode shapes with respect to the physical coordinate system where the measurements are taken, i.e., $[\phi]$ rather than $[\phi_0]$. The relationship between both matrices is given by:

$$\underset{(3N\times 6)}{[\phi]} = \underset{(3N\times 6)}{[R_0]}\underset{(6\times 6)}{[\phi_0]} \tag{3}$$

where $[R_0]$ represents the transformation matrix of the rigid body modes related to N tri-axial accelerometers [8]. In general, equation (3) is solved in a least squares sense:

$$[\phi_0] = \left([R_0]^T [R_0]\right)^{-1} [R_0]^T [\phi] \tag{4}$$

Numerically we may be tempted to conclude that $[\phi_0]$ can be obtained if $[R_0]$ contains at least data from 2 triaxial measurement points. However, Lee et al [2] proved that a minimum of 3 triaxial measurement points are needed to evaluate $\left([R_0]^T [R_0]\right)^{-1}$. In the present paper, the authors carried out various measurements on different locations for the triaxial accelerometers; the best results were obtained when the 3 accelerometers formed a regular triangle, confirming the conclusions of Lee *et al* [2]. Inverting Eq. (4) one obtains $[\phi_0]^{-1}$, and using this result in Eq. (2) one calculates $[M_0]$. Therefore, the resulting matrix $[M_0]$ represents the best least squares mass matrix related to the origin, based on the 6 extracted mode shapes at N measured response locations, where the components of the inertia tensor are referred to the assumed original coordinates. The mass of the body can be calculated as the average of the first 3 diagonal terms of $[M_0]$. Equating the non-zero elements of the upper right quadrant of $[M_0]$ to the same elements in the mass matrix referred to the centre of mass, 6 equations are obtained [7, 8]. Once the coordinates of the centre of mass are known, the moments and products of inertia relatively to the centre of mass can be easily obtained.

Conti and Bretl [6] state that all the 6 rigid body modes must be identified before estimating the mass matrix. However, there might be difficulties in exciting or identifying some of the modes. Toivola and Nuutila [7] partially solve the problem as only 21 independent orthogonality conditions are required, i.e., only 4 modes are needed. In fact, from the orthogonality condition between 2 modes (Eq.(1)), one has:

$$\{\phi_0\}_i^T [M_0] \{\phi_0\}_j = \delta_{ij} \quad \text{with} \quad \delta_{ij} = \begin{cases} 1\,, i = j \\ 0\,, i \neq j \end{cases} \tag{5}$$

Taking into account the nature of the rigid body mass matrix, it turns out that if n modes are estimated there are $n(n+1)/2$ independent equations. As there are 10 unknowns, Toivola and Nuutila method [7] only requires at least 4 modes. If 5 or 6 modes can be obtained, the resulting equations are solved in a least squares sense. In this method only one mass value is obtained. The components of the inertia tensor are referred to the origin of the coordinate system and should be transformed to the centre of mass using the coordinate transformation as presented in [7].

Direct Physical Parameters Identification Methods

Huang and Lallement [5] propose a method for separating the mass, stiffness and damping matrices, allowing the independent identification of these three types of parameters. Such a separation reduces the number of unknown parameters and thus improves the numerical conditioning. The formulation of this method leads to:

$$\left[Y_1\right]^T\left[K_0\right]\left[Y_2\right]-\left[S_1\right]\left[Y_1\right]^T\left[M_0\right]\left[Y_2\right]\left[S_2\right]=\left[\alpha_{ij}\right] \tag{6}$$

$$\left[\bar{Y}_1\right]^T\left[K_0\right]\left[Y_2\right]-\left[\bar{S}_1\right]\left[\bar{Y}_1\right]^T\left[M_0\right]\left[Y_2\right]\left[S_2\right]=\left[\beta_{ij}\right] \tag{7}$$

$\left[S_1\right]=diag\left(\mathrm{i}\,\omega_i^{(1)}\right)\in C^{m,m}$ and $\left[S_2\right]=diag\left(\mathrm{i}\,\omega_j^{(2)}\right)\in C^{n,n}$ are diagonal matrices of the chosen frequencies taken before (*m*) and after (*n*) each natural frequency, respectively. $\left[Y_1\right]\in C^{6,m}$ and $\left[Y_2\right]\in C^{6,n}$ are formed by the responses of the first and second series of frequencies. $\left[\alpha_{ij}\right]\in C^{m,n}$ and $\left[\beta_{ij}\right]\in C^{m,n}$ are built as in [5]. Elliminating $\left[K_0\right]$ between (6) and (7) leads to:

$$\left(\left[Y_1\right]^T\left(\left[\bar{Y}_1\right]^T\right)^+\left[\bar{S}_1\right]\left[\bar{Y}_1\right]^T-\left[S_1\right]\left[Y_1\right]^T\right)\left[M_0\right]\left[Y_2\right]\left[S_2\right]=\left[\alpha_{ij}\right]-\left[Y_1\right]^T\left(\left[\bar{Y}_1\right]^T\right)^+\left[\beta_{ij}\right] \tag{8}$$

where $^+$ means pseudo-inverse. Separating (8) into its real and imaginary parts, a linear system of equations in the unknown 10 properties is obtained [5].

CASE STUDY

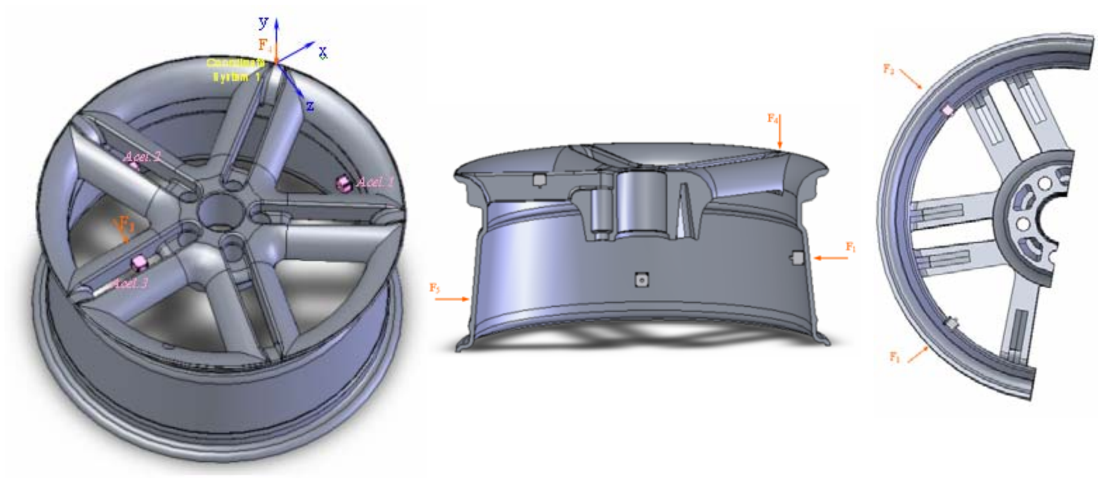
Theoretical Modelling

Figure 1 shows the structure that represents the experimental case study - a car wheel. The theoretical inertia parameters of the actual tested structure were obtained using the *SolidWorks* modulation software; those values are assumed as correct for the sake of comparison. As it can be observed, the structure presents a high degree of symmetry that causes small values for the products of inertia relative to the selected system axes.

Experimental set-up and data assessment

Fig. 1 shows the three measurement points where the triaxial accelerometers have been located as well as the five excitation forces. These forces have been applied through an impact hammer with a rubber tip. The FRFs have been measured and processed in the range *0-12.5Hz*.

The wheel was suspended by flexible springs with negligible mass. The origin of the coordinate system (*Coordinate System 1*) was selected according to Almeida *et al* [8], Fig. 2 represents the 9 FRFs obtained for all the excitation forces.



$Mass = 14.81199\ Kg$

$Center\ of\ mass\ coordinates:\ (\ mm\ )$

$x_g = -175.830 \quad y_g = -67.439 \quad z_g = 169.854$

$Center\ of\ mass\ inertia\ tensor:\ (\ Kg \cdot m^2\ )$

$J_{g_{xx}} = 0.31915007 \quad J_{g_{xy}} = 0.00006793 \quad J_{g_{xz}} = 0.00037792$

$J_{g_{yx}} = 0.00006793 \quad J_{g_{yy}} = 0.50621325 \quad J_{g_{yz}} = -0.00009403$

$J_{g_{zx}} = 0.00037792 \quad J_{g_{zy}} = -0.00009403 \quad J_{g_{zz}} = 0.31895250$

Figure 1 – Actual tested structure.

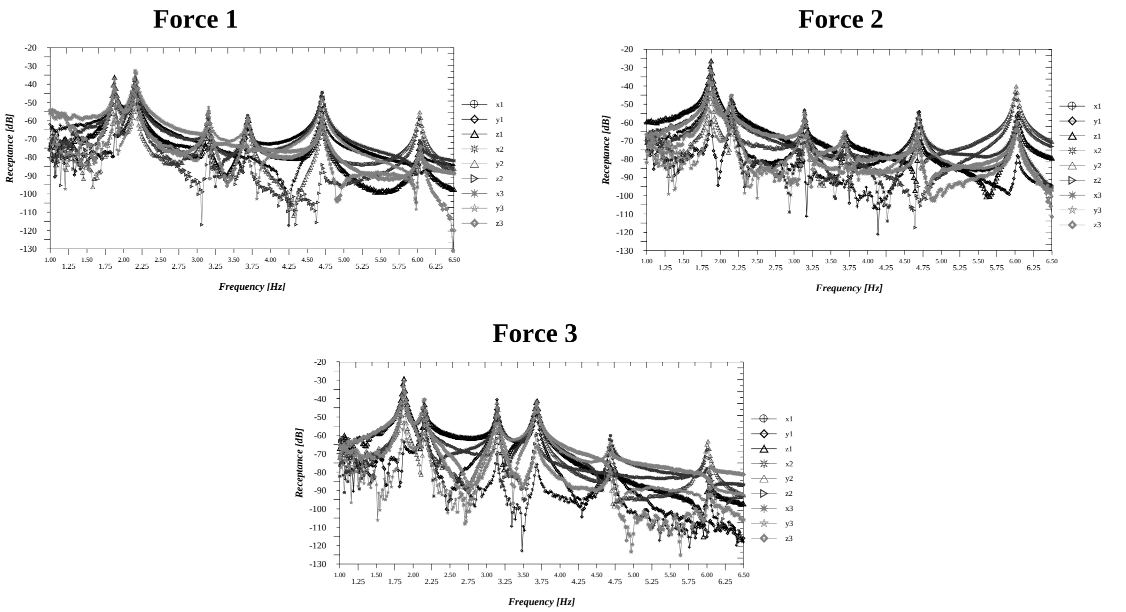
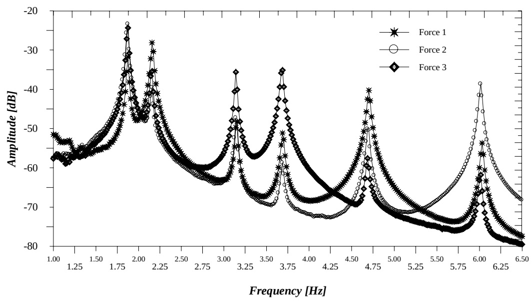


Figure 2 - Receptance FRFs for applied forces 1, 2 and 3.

Application of Modal Methods

From Fig. 2, one can see that for each force some modes are more excited than others. To understand the level of excitation of the modes the authors developed the *Norm Indicator (NI)* [8-10], Fig. 3. The identification process was performed by using a *Single Input Multiple Output* technique of *MODENT* [11].

According to [8-10], the accuracy of the results increases with the excitation of the rigid body modes. The mode shape matrix relative to the 9 measurement directions should be constructed considering both the information of the *NI* (Fig. 3) and the modal matrices obtained for each force. The results are presented by *Set NI*.



$$Set\ \ NI\ \ - \begin{cases} 1^{st}\text{ mode - } force\,2 \\ 2^{nd}\text{ mode - } force\,1 \\ 3^{rd}\text{ mode - } force\,3 \\ 4^{th}\text{ mode - } force\,3 \\ 5^{th}\text{ mode - } force\,1 \\ 6^{th}\text{ mode - } force\,2 \end{cases}$$

Figure 3 - Results of the application of Norm Indicator to each force.

The relative errors for the mass, vector of centre of mass coordinates $\{C_g\}$ and vector of moments and products of inertia $\{J_g\}$ as well as the *summation* of all the parameter errors obtained with *Conti-Bretl* and *Toivola-Nuutila* methods, for the chosen origin, are presented in Fig. 4.

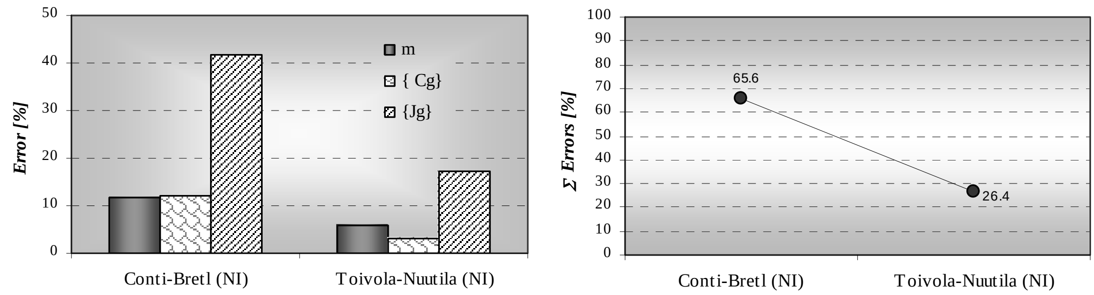


Figure 4 - Results obtained with the application of Conti-Breti and Toivola- Nuutila methods.

Remarks:
The research carried out by the authors in [8-10] and the conclusions of this study confirm that *Toivola-Nuutila*'s method produces better results than *Conti-Bretl*'s.

Application of the Direct Physical Parameters Identification Method

This method has been evaluated by the application of four different sets of forces, three of them formed by three forces and one by four forces, as presented Fig. 5.

	Sets			
Point (force direction)	**A**	**B**	**C**	**D**
1 (x)				
2 (z)				
3 (z)				
4 (y)				
5 (x)				

Figure 5 - Table of sets of applied forces to the wheel.

For each set of forces, nine FRFs were collected w.r.t. the measuring coordinate system. A subsequent transformation leads to the six FRFs $\left(x_0, y_0, z_0, \theta_{0x}, \theta_{0y}, \theta_{0z}\right)$ at *Coordinate System 1*. The *NI* is then applied to identify the excitation level of the rigid body modes and of the natural frequencies (Fig.6 shows the results for set *A*).

The values of the displacements and rotations of the two frequencies immediately before and after each rigid body natural frequency are then used. The relative errors for the mass, centre of mass coordinates $\{C_g\}$ and moments and products of inertia $\{J_g\}$ as well as the *summation* for each set of forces are shown in Fig. 7.

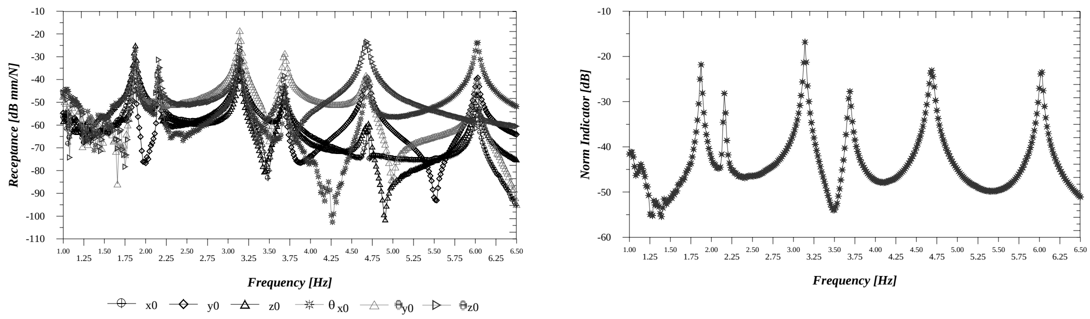


Figure 6 - FRFs for set A and NI values obtained for the frequency range studied.

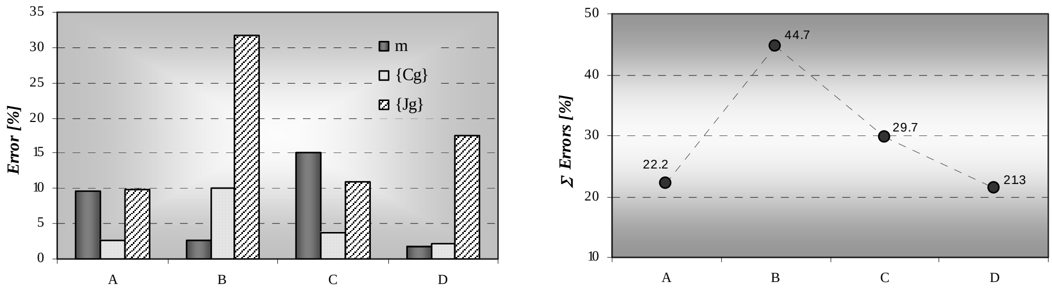


Figure 7 – Results with the application of DPPIM for the sets of applied forces.

Remarks:

- The level of excitation of each mode plays an important role in the estimation of the inertia parameters.

- The *NI* indicator is a valuable tool for the diagnosis of failed estimations of the inertia parameters, as it reveals the poor excited modes at the origin.
- It is concluded that adding a force to the initial sets may improve the results.

CONCLUSIONS

1. Both types of methods give good results provided that all the six rigid body modes are well excited and if the measured FRFs are not too noisy.
2. Within the *MM* methods, *Toivola-Nuutila*'s is better than the original *Conti-Bretl* method. Better results can also be obtained taking rigid body modes from several forces. The *Norm Indicator* is crucial for the selection of the modes.
3. For the *DPPIM* it can be said that the addition of an extra force to the initial data set may improve the results and that the *NI* is a valuable tool in the diagnosis of failed estimations of the inertia parameters, as it reveals the poor excited modes.
4. The *DPPIM* is easier to handle than the *MM* methods because it does not require the application of modal identification techniques to the measured FRFs.
5. A definite conclusion on the best method is not possible, as it is always case-dependent; it is advisable to use more than one method, to compare results.

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