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COMPACT STRUCTURAL-ACOUSTIC COUPLED MODELS VIA MODEL ORDER REDUCTION (MOR)

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Abstract

A reduced order model is developed for low frequency, fully coupled, undamped structural acoustic analysis of interior cavities, backed by flexible structural systems. The reduced order model is obtained by applying a projection of the coupled system matrices, from a higher dimensional subspace to a lower dimensional subspace, whilst preserving some essential properties of the coupled system. The basis vectors for projection are computed efficiently using the block Arnoldi algorithm, which generates an orthogonal basis for the Krylov subspace. Two computational test cases are analyzed, and the computational gains and the accuracy compared with the direct method in ANSYS. It is shown that reduced order model results in a very significant reduction in simulation time, while maintaining the desired accuracy of the state variables under investigation. The method could prove as a valuable tool in the analysis of complex coupled structural acoustic systems, and their subsequent optimization or sensitivity analysis, where, in addition to fast analysis, a fine frequency resolution is often required.

INTRODUCTION

In a modern passenger vehicle or a commercial airplane, the noise, vibration and harshness (NVH) performance is one of the key parameters which the customer uses to assess product quality. In order to gain competitive advantage, manufacturers are striving to reduce NVH levels. As a result, design engineers often seek to evaluate the low frequency NVH behaviour of automotive/aircraft interiors using coupled finite element-finite element (FE/FE) or finite element-boundary element (FE/BE) discretized models. Due to the coupling between the fluid and structural domains in the coupled FE/FE formulation, the resulting mass and stiffness matrices are no longer symmetrical. With wavelengths decreasing for increasing frequency, the model size drastically increases with frequency. This presents as a major problem where optimization is required, especially when there are a large number of design

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variables to be optimized. Therefore, generation of compact models, for fast coupled structural acoustic analysis is of great interest to the NVH community.

The two most popular approaches currently used to reduce the computational time of such coupled problems are the mode superposition and the component mode synthesis (CMS) method. The reader is referred to [1], for a review of other approaches to reduce computational time. More recently, however, model order reduction (MOR) via implicit moment matching, has received considerable attention among mathematicians and the circuit simulation community [2, 3]. It has been shown in various engineering applications [4, 5] that the time required to solve reduced order models via MOR is significantly small when compared to solving original higher dimensional model, whilst maintaining the desired accuracy of the solution. The aim of MOR is to construct a reduced order model, from the original higher dimensional model, which is a good representation of the system input/output behavior at certain points in the frequency domain. The reduction is achieved by applying a projection from a higher order to a lower order space using a set of Krylov subspaces, generated by the Arnoldi algorithm. Additionally, the reduced model preserves certain essential properties such as maintaining the second order form and stability.

The paper focuses on the application of such Krylov based MOR techniques to undamped, fully coupled structural acoustic problems. An open source software *mor4ansys* [10] is used to generate the reduced order model from an ANSYS higher dimensional model. The harmonic simulation of the reduced order model is performed in the MATLAB/Mathematica V5.0 environment. It is shown that the reduced order model speeds up the simulation by orders of magnitude, without any significant loss of solution accuracy.

MODEL ORDER REDUCTION FOR SECOND ORDER SYSTEMS

After discretization in space of a general mechanical system model, one obtains a system of second order ordinary differential equations in matrix form as follows:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Fu(t) \qquad (1)$$
$$y(t) = L^T x(t)$$

Where *(t)* is the time variable, *x(t)* is the vector of state variables, *u(t)* is the input force vector, and *y(t)* the output measurement vector. The matrices *M*, *C* and *K* are mass, damping and stiffness matrices, F and L are the input distribution matrix and output measurement matrix at certain points respectively. A harmonic simulation, assuming $\{F\} = F_0 e^{i\omega t}$ and ignoring damping in (1) yields:

$$[-\omega^2[M] + [K]]\,\{x\} = \{F\} \qquad (2)$$
$$y(\omega) = L^T x(\omega)$$

Where, ω denotes the circular frequency, and, $\{x\}$, $\{F\}$ denote complex vectors of state variables and inputs to the system respectively. The idea of model reduction is to find a lower dimensional subspace $V \in \Re^{Nxn}$, and,

$x = Vz + \varepsilon$ where, $z \in \Re^n$, $n << N$ (3)

such that the time dependent behaviour of the original higher dimensional state vector x can be well approximated by the projection matrix V in relation to a considerably reduced vector z of order n with the exception of a small error $\varepsilon \in \Re^N$. Once the projection matrix V is found, the original equation (2) is projected onto it. The projection produces a reduced set of system equations, in second order form, as follows:

$$(-\omega^2[M_r]+[K_r])\{z\}=\{F_r\} \quad (4)$$
$$y_r(\omega) = L_r^T z(\omega)$$

Where the subscript *r* denotes the reduced matrix and:

$$M_r = V^T MV,\ K_r = V^T KV,\ C_r = V^T CV,\ F_r = V^T F,\ L_r = V^T L.$$

It is worth noting that $y_r(\omega) \approx y\ (\omega)$. Due to its low dimensionality, the solution to (4) is much faster than the original higher dimensional model. The input and output vectors maintain the same size as (2). There exist several methods to choose *V*. In this work, we choose the projection matrix *V* to be a Krylov subspace in order to provide the moment matching property [2, 3].

Model order reduction for coupled structural acoustic systems:

For a coupled structural acoustic case, we start off from Cragg's pressure formulation [8]:

$$\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{pmatrix} Cs & 0 \\ 0 & Ca \end{pmatrix}\begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix} \quad (5)$$
$$y(t) = L^T \begin{Bmatrix} u \\ p \end{Bmatrix}$$

Where, M_s is the structural mass matrix, M_{fs} is the coupled mass matrix, M_a is the acoustic mass matrix, C_s is the structural damping matrix, C_a is the acoustic damping matrix, K_s is the structural stiffness matrix, K_{fs} is the coupled stiffness matrix, K_a is the acoustic stiffness matrix, F_s is the structural force vector, *y(t)* the output measurement vector and *u*, *p* are the displacements and pressures at nodal co-ordinates respectively. Ignoring damping for the structure and fluid, the coupled equations become:

$$\left[-\omega^2\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix} + \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix} \right]\begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix} \quad (6)$$

It can be seen that (6) is similar to (2). In this case, the approximation becomes:

$$\begin{Bmatrix} u \\ p \end{Bmatrix} = \{x\} = Vz + \varepsilon \qquad (7)$$

Combining (5) and (7), the transfer function of the system H*(s)* = (*Y(s)* / *U(s)*) via the Laplace transform can be written as:

$$H(s) = L^T (s^2 M_{sa} + sC_{sa} + K_{sa})^{-1} F_{sa} \qquad (8)$$

Ignoring damping, and expanding (8) using the Taylor series about $s = 0$ results in:

$$H(s) = \sum_{i=0}^{\infty} (-1)^i L^T (K_{sa}{}^{-1} M_{sa})^i K_{sa}{}^{-1} F_{sa} s^{2i} = \sum_{i=0}^{\infty} m_i s^{2i} \qquad (9)$$

Where $m_i = (-1)^i L^T (K_{sa}{}^{-1} M_{sa})^i K_{sa}{}^{-1} F_{sa}$ are called the moments of $H(s)$ and,
$M_{sa} = \begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}, K_{sa} = \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}, F_{sa} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix}$.

By matching some of these moments of the higher dimensional system about *s=0*, the reduced order model can be constructed, as it directly relates the input to the output of the system. Theoretically, any expansion point within the frequency range of interest can be used, and a real choice depends on required approximation properties. However, explicitly computing such moments tends to be numerically unstable [2, 3]. So we try to implicitly match these moments of via the Arnoldi process. Su and Craig [6], showed that if the projection matrix *V* is chosen from a Krylov subspace of dimension *q*,

$$\mathrm{K}_q(K^{-1}M, K^{-1}F) = span\{K^{-1}F, (K^{-1}M)K^{-1}F, ...(K^{-1}M)^{q-1}K^{-1}F\} \qquad (10)$$

then, the reduced order model matches *q+1* moments of the higher dimensional model. Loosely speaking, if the q^{th} vector spanning the Krylov sequence is present in matrix V, we match the q^{th} moment of the system. The block vectors $K^{-1}F$ and $K^{-1}M$ can be interpreted as the static deflection due to the force distribution *F*, the static deflection produced by the inertia forces associated with the deflection $K^{-1}F$ respectively.

THE ARNOLDI ALGORITHM

To avoid numerical problems while building up the Krylov subspace, an orthogonal basis is constructed for the given subspace. This is done using the block Arnoldi algorithm. Given a Krylov subspace $K_q\ (A_1,\ g_1\ )$, the Arnoldi algorithm finds a set of vectors with norm one i.e. that is orthogonal to each other, given by:

$$V^T V = I \quad \text{and} \quad V^T A_1 V = H_q \qquad (11)$$

Where $H_q \in \Re^{qxq}$ is a block upper Hessenberg matrix and $I_q \in \Re^{qxq}$ is the identity matrix. Figure: 1 describes the implemented algorithm, which is used to generate the

Arnoldi vectors for the coupled structural acoustic system. For multiple inputs, the block version of the algorithm can be found in [3]. In this case:

$$Colspan(V) = K_q(K_{sa}^{-1}M_{sa}, K_{sa}^{-1}F_{sa})$$
$$V^T K_{sa}^{-1} M_{sa} V = H_q \text{ and } V^T V = I \tag{12}$$

The discussion of the block version of the algorithm, which is used to generate the Arnoldi vectors for the coupled structural acoustic system (Multiple input, Multiple output) is quite involved, and the reader is referred to [3] for a detailed discussion. It can be seen from the algorithm, that in each step, one vector orthogonal to all previously generated vectors are constructed and normalized. Due to the iterative property of the algorithm, it is possible to produce reduced order model of lower dimension than initially specified, by just discarding the columns in matrix *V* and subsequently the rows and columns of the reduced matrices.

Algorithm: 1:

Input: System Matrices K_{sa},M_{sa}, F_{sa}, L and *n* (number of vectors), expansion point $s = (\omega_E + \omega_B)/2$
Output: *n* Arnoldi vectors

0. Set $v_i = g$
1. For $i = 1 \rightarrow n, do$:
   - 1.1 Deflation check: $t_{i,i-1} = \| v_i \|$
   - 1.2 Normalization: $v_i = v_i^* / t_{i,i-1}$
   - 1.3 Generation of next vector: $v_{i+1}^* = Av_1$
   - 1.4 Orthogonalization with old vectors: for j=1 to i:
     - 1.4.1 $t_{j,1} = v_j^T v_{i+1}^*$
     - 1.4.2 $v_{j+1}^* = v_{i+1}^* - t_{j,i} v_j$
2. Discard resulting H_q, and project $M_{sa}, K_{sa}, F_{sa}, L$ onto V to obtain reduced system matrices $M_{Rsa}, K_{Rsa}, F_{Rsa}, L_r$ where the subscript *Rsa* represents the reduced structural acoustic matrices.

Figure: 1: Arnoldi Process [2] [3].

NUMERICAL TEST CASES

Test Case - 1:

The first example we consider in this paper is an academic test case, rather than an industrial application. The test model is a 1m x 1m clamped undamped aluminium plate backed by a rigid walled rectangular cavity of dimension 1m x 1m x 1m. All the other sides of the cavity are assumed to be rigid. A total of 8400 elements were used for the model. A force excitation of 1N is applied on one of the nodes on the plate as shown in *Figure: 2(a)*. The coupled equations are solved using two approaches: (a) The direct method and (b) MOR via Arnoldi. 35 vectors were generated using the Arnoldi algorithm as described in the previous section. The reduced order model was

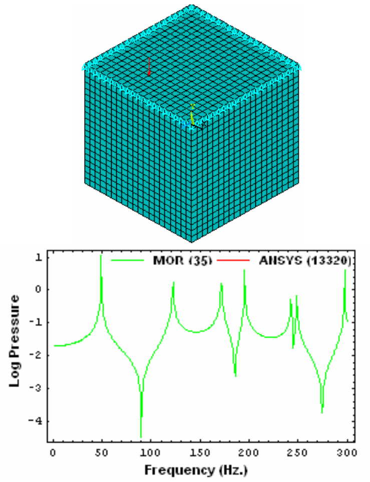

Figure 2 – (a) Top: Coupled FE model (b) Bottom: Noise Transfer Function

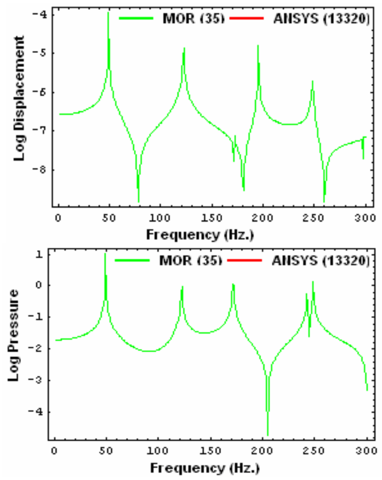

Figure 3 – (a) Top: Receptance (b)Bottom: Noise Transfer Function

set up and solved in Matlab/Mathematica V5.0 environment. The displacement amplitudes of the plate and the noise transfer function (Pressure/Force) computed on certain points inside the fluid domain are specified as outputs for the analysis. The transfer functions are shown from *Figure: 2(b), 3(a)* and *3(b).*

Test Case - 2:

A model structure, made of simple beams and plates was manufactured to test new algorithms and techniques. The structure was modelled using a top-down modelling approach. The structural model was divided into seven areas. Four of these areas: one corresponding to the vehicle roof, firewall, floor pan and back plate, were meshed using four noded quadrilateral shell elements with six degrees of freedom at each node. A total of 3706 shell elements were adequate to capture the dynamic behaviour of the structure. The acoustic model was modelled using eight noded acoustic brick elements with one pressure degree of freedom at each node. Two faces of the acoustic model were assumed to be fully reflective i.e. rigid walls. The coupled model was excited at two locations as shown in *Figure: 4(a).* 170 vectors were generated using the multiple input multiple output Arnoldi algorithm. The noise transfer functions are shown from *Figure: 4(b), 5(a)* and *5(b).* All computations described in this paper were performed on a Pentium 3GHz 2GB RAM machine.

COMPUTATIONAL TIMES

To evaluate the computational gains achieved by using reduced order models via the Arnoldi process, the computational times required to solve the higher dimensional ANSYS model and the reduced order model are compared. Table: 1 shows the computational time required for test cases 1 and 2.

Model	Elements	DOF's	ANSYS Direct	MOR via Arnoldi	Reduction
TC^1	8400	11427	2906 s	27.8 s	99.04 %
TC^2	14220	29413	16530 s	169.4 s	98.97 %

Table 1 – Computational Times; TC^1: Test Case-1; TC^2: Test Case-2.

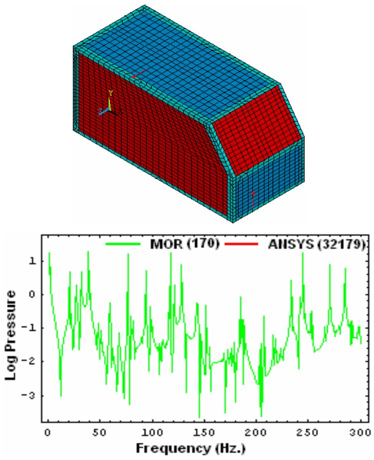

Figure 4 – (a) Top: Coupled FE model
(b) Bottom: Noise Transfer Function

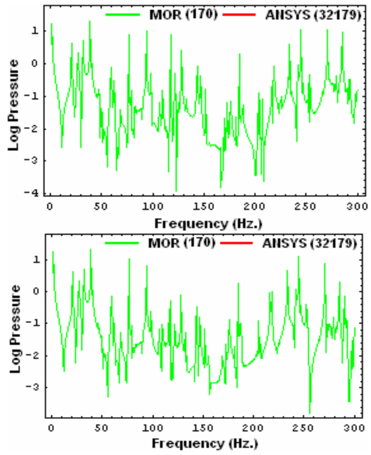

Figure 5 – (a) Top: Noise Transfer Function
(b) Bottom: Noise Transfer Function

For the reduced order model, the computational time is a combination of four steps (a) Running a Stationary solution and generating matrices (b) Reading matrices and generating of Arnoldi vectors (c) Projection to second order form and (d) Simulation of the reduced order model. The spilt computational times for test case 1 and 2 are shown in *Table: 2*. It can be seen that significant speed up is achieved by the use of model order reduction. Another interesting feature of the Arnoldi process is that, the computational times do not depend on the frequency resolution for the reduced order model, while, an almost linear increase can be observed with increasing substeps for the higher dimensional ANSYS model. This is particularly useful, for complex structural acoustic systems, where a higher frequency resolution is often desired.

Model	**ANSYS Stationary**	**Read Matrices , Arnoldi Vector Generation**	**Projection of Matrices**	**Reduced model Simulation**	**Total: MOR via Arnoldi**
TC^1	6 s	21.3 s (35 Vectors)	0.4 s	0.2 s	27.8 s
TC^2	4 s	144.7 s (170 Vectors)	14.7 s	6 s	169.4 s

Table 2 – MOR Split Computational Times; TC^1: Test Case-1; TC^2: Test Case-2.

SUMMARY

A new method to develop efficient reduced order model for fully coupled structural acoustic problems has been outlined. The basis vectors for model reduction are computed by applying the Arnoldi algorithm, which computes the projection vectors spanning the Krylov subspace, to match the maximum number of moments of the system. The two test cases used in this paper, show that good approximation properties can be obtained by projecting the higher dimensional system to a lower dimension and matching the low frequency moments of the system. In the test cases shown, the moments are matched at approximately half of the analysis range $s = (\omega_E + \omega_B)/2$. The choice of s is often an open question. If a Taylor series expansion is considered around a higher frequency, a reduced order model could be obtained with better approximation properties around that frequency range. In addition to this, a reduced order model could be calculated, which matches moments around several expansion points, with each expansion point requiring a separate factorization. However, for the test cases analyzed in this paper, a single expansion point yields very good approximation properties. *Figure*: *2(b)*, *3(a), 3(b), 4(b)*, *5(a)* and *5(b)* indicates that there is almost no difference between results from ANSYS calculated and the reduced order model. While there exist several methods to choose basis vectors, we have chosen these vectors to span the Krylov subspace. Compared to the computing eigen modes and eigen vectors of the system matrices, computing vectors spanning the Krylov subspace is much faster and efficient, since a normal modal analysis of a complex structural or an acoustic system tends to be computationally expensive. In fact, there is no guarantee that the computed modes included for the mode superposition via a modal analysis would be enough for the time/harmonic analysis, and often an approximate guess of modes within the *2n* range are computed for projection, *n* being end frequency [7]. *Figure: 2(b)*, *3(a)* and *3(b)* show that the reduced order model accurately captures the dynamic behaviour of the coupled higher dimensional system, indicated by peaks at ~170Hz and ~240Hz, which correspond to the acoustic modes of the cavity. Also, a complete approximation of the output is guaranteed by the Arnoldi process. Although this has not been verified explicitly in either of the test cases, existing literatures show that a complete match is specific to the Arnoldi process (e.g. [9]). For both test cases, 25 outputs were chosen for the analysis, which included both normal displacements on the structural portion of the model and pressure levels in the fluid domain. The number of vectors needed to accurately represent the system was 35 and 170 for test cases 1 and 2 respectively. The difference in the number of vectors needed can be attributed to the nature of coupled models itself and its resulting transfer function. It is also worth noting that the process of computing the minimum number of required vectors can be completely automated. The reduced order modelling framework outlined in this paper could serve as an excellent alternative to many other reduction techniques, particularly for low frequency vibro-acoustic optimization, where reduction of computational time is often sought.

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