ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

A NEW EVOLUTIONARY OPTIMIZATION METHOD WITH AN EXAMPLE OF A CONNECTING-ROD MODEL

Masaaki Okuma *[1], Minoru Yoshida[1], and Takeshi Nakahara[1]

[1]Department of Mechanical and Aerospace Engineering,
Tokyo Institute of Technology,
2-12-1-I3-15, O-okayama, Meguro-ku, Tokyo, 152-8552, JAPAN
mokuma@mech.titech.ac.jp

Abstract

In this paper, the authors present a new structural optimization method based on FE modeling, modal analysis and an evolutionary optimization algorithm with respect to dynamic strength and rigidity of structure. The method consists of two optimization processes in outline. The first process is to find the best topology by evolutionarily growth of structure. A initial structure having simple and slender shape will be appropriately available for the optimization. The method can make such an initial structure grow evolutionarily. The second process is to optimize the best size of the topologically optimized structure in the first process, according to externally applied force and load. The method computes some number of natural frequencies and natural modes, and the deflection of the structure due to externally applied force. The outline of the algorithm is presented in the form of flowchart at first. A demonstration of a connecting-rod model is presented under evaluation of not only strength and deflection but also fatigue durability.

INTRODUCTION

Michell truss in 1904 may be one of very early achievements categorized as evolutionary structural optimization. It was solved analytically without computer. Optimum layout problems such as location of voids in structures started in 1970s. Since 1980's, the growth of digital computer power has made it possible to develop various evolutionary structural optimization methods based on numerical analysis for more practical engineering purposes. Examples are the cellular automata method, the homogenization method and the bi-directional evolutionary structural optimization method[1-6]. Most of the papers regarding such methods showed two-dimensional and three-dimensional examples being started with initial structures fully filled with structural elements in design space[1-4]. Roughly describing, resultant structures are obtained by the removal of a number of unnecessary structural elements from initial structures although the methods have the function to attach new elements again to the place where old elements were removed. The bi-directional method was proposed as a kind of evolutionary structural optimization with the enhancement of attaching new element process.

This paper presents a new evolutionary structural optimization method for obtaining light structures under considering double-disciplinary requirements about statics and dynamics. The method normally starts using an initial structure that is very simple and slender because we think it is very practical. The simple structures can grow up to sophisticated structures using comparatively less computational resources than conventional ones. In static structural optimization, sophisticated structures mean the light structures having appropriate strength and rigidity against applied external static forces. Mises yield criterion is used to judge yielding condition in this paper, although any yield criterion can be available. Structures are reinforced at yielding place by attaching new finite elements on the local surface areas. On the other hand, appropriate number of finite elements are removed from the places with some degrees of allowance to yielding. In dynamic structural optimization, sophisticated structures mean the light structures having its natural frequencies of interest as closely as possible to design target frequencies or having those at higher frequencies than required minimum threshold frequencies. Two kinds of sensitivity functions are used for these optimization objectives in this paper. One is a pseudo sensitivity function about Rayleigh quotient and the other one is strain energy with respect to the removal and attachment of elements. Note that good handling of the “mode-switching” problem is a key point to make a success of dynamic optimization. “Mode-switching” is a phenomenon that changes the order of natural modes. “Mode-switching” really occurs when some finite elements are removed from or attached to the objective structures in evolutionary optimization process. The natural frequency of each mode shape is strongly affected and changed by the “mode-switching”. Consequently, the evolutionary optimization often becomes unstable in the case of using only conventional evaluation parameters. Then, in the present method, the problem is prevented by the combination of the abovementioned two kinds of sensitivity functions. In this paper, an integration method based on the abovementioned techniques for the static and the dynamic optimization is presented because in practice dynamic structural optimization cannot be realized without consideration of strength. In addition, many structural parts in machines dynamically move and/or oscillate so that dynamics must be considered in addition to strength. Therefore, the integration is very important to develop such methods for practical use. This paper presents a concise explanation of the method and a example numerical of a connecting rod model for small passenger car class engines.

Outline of Theory

Figure 1 is the flowchart of the algorithm. The optimization is subject to three kinds of constraints. The one is stress constraint, i.e., yield constraint. Mises yield criterion is applied to the application in this paper. The second one is a deflection constraint. The deflection at some points of interest in the structure are restricted to have equal or smaller than maximum allowable designed deflection against external forces. The third one is natural frequency constraint. Some natural frequencies of interest are restricted to become equal or higher than thresholds that are set as acceptable minimum designed frequencies for each. For the most typical case, the first natural frequency is to be restricted.

\Mises yield criterion is reviewed as

$$\sigma_{eq,i} = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{x,i} - \sigma_{y,i}\right)^2 + \left(\sigma_{y,i} - \sigma_{z,i}\right)^2 + \left(\sigma_{z,i} - \sigma_{x,i}\right)^2 + 6\left(\tau_{xy,i}^2 + \tau_{yz,i}^2 + \tau_{zx,i}^2\right)} \quad (1)$$

where σ and τ represent tensile stress and shearing stress respectively, the subscript i means the element numbering, and the subscripts x, y and z denote three directions of Cartesian coordinate system. Eq.(1) is used for the judgment of both strength and static deflection.

For dynamics the following pseudo sensitivity of eigenvalues are used:

$$\begin{aligned} \Delta\lambda_{k,l} &= \frac{L_k - S_{k,l}}{E_k - T_{k,l}} - \lambda_k \\ &= \lambda_k\big|_{new\ structure} - \lambda_k\big|_{current\ structure} \end{aligned} \quad (2a)$$

$$\begin{aligned} \Delta\lambda_{k,l} &= \frac{L_k + S_{k,l}}{E_k + T_{k,l}} - \lambda_k \\ &= \lambda_k\big|_{new\ structure} - \lambda_k\big|_{current\ structure} \end{aligned} \quad (2b)$$

where L_k and E_k represent the strain energy and the kinetic energy stored in the structure in the k-th natural mode respectively, $S_{k,l}$ and $T_{k,l}$ denote the strain energy and the kinetic energy stored in element $No.l$ in the k-th natural mode respectively, and λ_k represents the eigenvalue of the k-th natural mode. Eq.(2a) is used for removing an element and Eq.(2b) is used for attaching an element. The algorithm is concisely presented in the form of the flowchart in Fig.1.

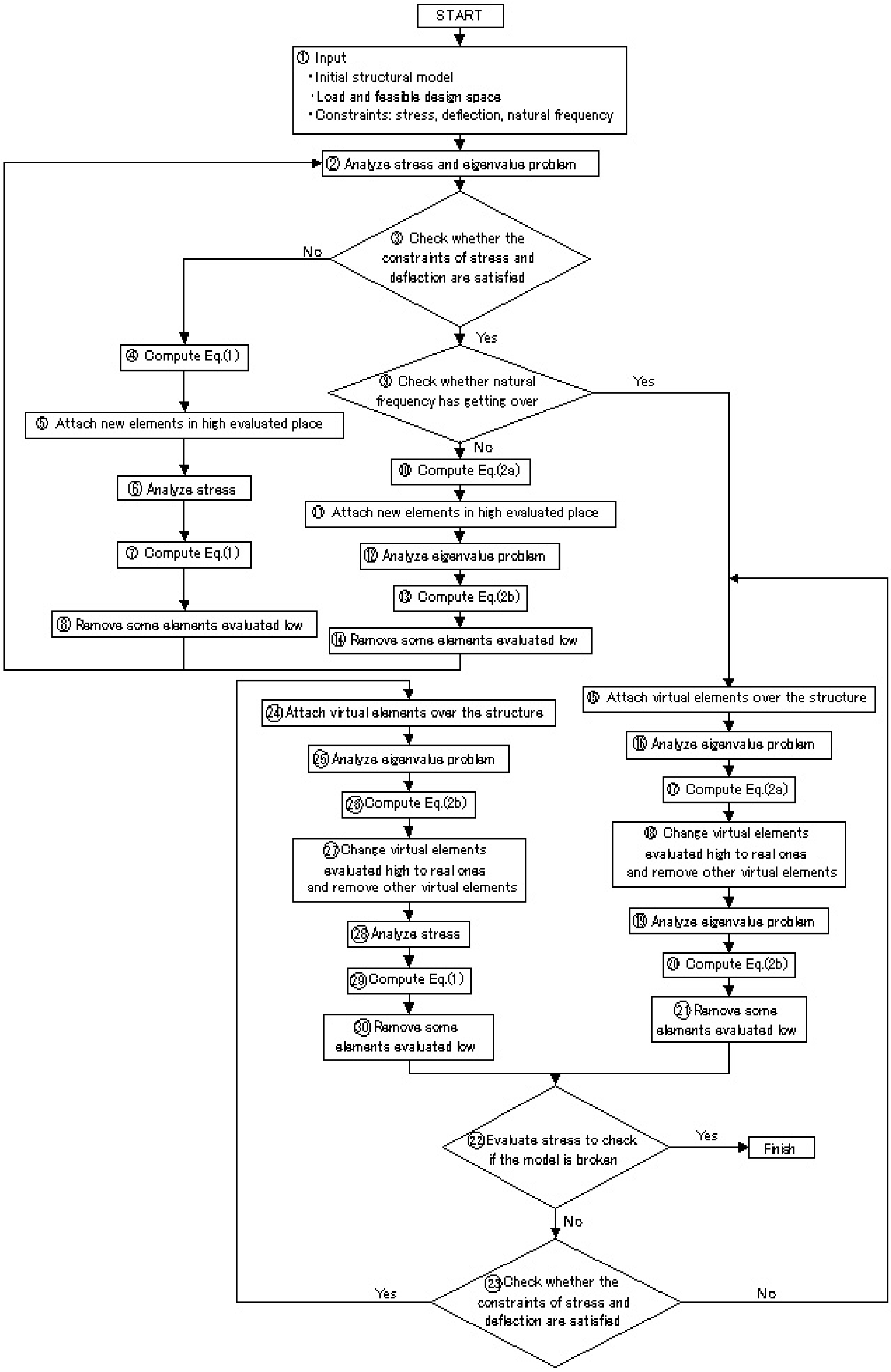

Fig.1 Flowchart of the Algorithm

OPTIMIZATION OF A CONNECTING-ROD MODEL

Figure 2 shows an actual connecting rod and its FE model to be optimized. This is the initial model for the optimization.

Motion analysis of Piston-Crank Mechanism

This analysis is carried out in order to compute the inertia load of the connecting rod. Piston-crank motion under a constant engine speed is modeled and computed using the piston-crank model shown in Fig.3. The derivation of the equations of motion is skipped here because of an orthodox analytical method. Runge-Kutta method is used for the numerical analysis. Two points of modeling are noted here.

Firing force applied to the piston head is assumed as

$$F = P_{\max} \exp\{-\alpha(\theta - \theta_0)\} \times S \tag{3}$$

where P_{max} is the Force amplitude, θ_0 is the firing delay angle from $\theta = 0$ with respect to the crank position. The motion of the piston-crank mechanism generated due to the firing force.

The connecting rod is simply modeled as shown in Fig. 4 for this analysis. Two equivalent mass M_1 and M_2 are firstly expressed by

$$M_1 = M_c \frac{a}{L} \quad \text{and} \quad M_2 = M_c \frac{b}{L} \tag{4}$$

according to the mass equivalence. This simple modeling cannot always represent the inertia moment equivalence so that a parameter Q is defined and used in this paper to satisfy the inertia moment equivalence. The parameter is expressed by

$$Q = M_c k^2 - (M_c \frac{ab^2}{L} + M_c \frac{a^2 b}{L}) = M_c (k^2 - ab) \tag{5}$$

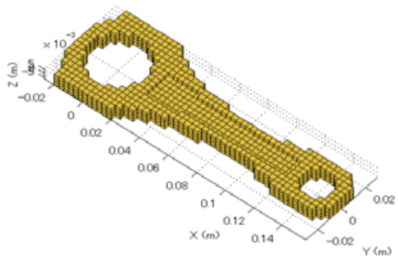

Fig. 2 Specimen of Connecting Rod and its FE modeling

where $M_c k^2$ is the real inertia moment about the center of mass of the connecting rod, and $ab^2Mc/L + a^2bMc/L$ represents the inertia moment of the simple model shown in Fig.4. The parameter is a compensation parameter about inertia moment. Figure 5 shows an example of the piston-crank mechanism motion and the inertia load analyzed using the modeling method. This motion analysis is implemented into the evolutionary optimization. The moment of 50Nm is set as the external moment resistance applied to the crankshaft. The motion analysis is carried out for the period of 2 cycles of the crankshaft rotation under the initial condition in which the rotational speed is set 10000rpm and the initial crank angle $\theta = -\pi(rad)$.

Fatigue Analysis

Fatigue analysis is based on the mean principal stress σ_a and the stress amplitude σ_m which are derived from the maximum principal stress $\sigma_{\max}$ and the smallest principal stress σ_{min}. In this study, the maximum principal stresses are obtained in the motion analysis for two cycles. The instant principal stresses are computed by

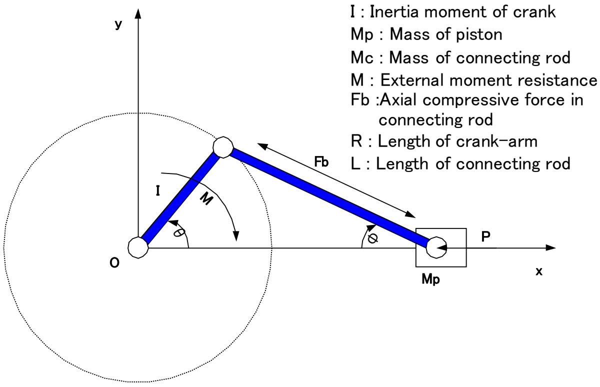

Fig.3 Piston-crank Model to analyze inertia load

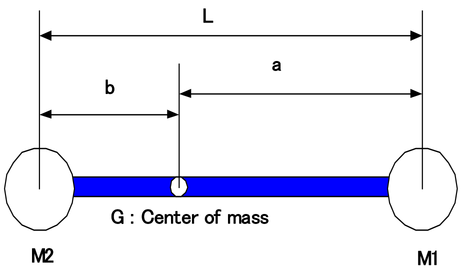

Fig.4 Simple modeling of Connecting rod for motion analysis

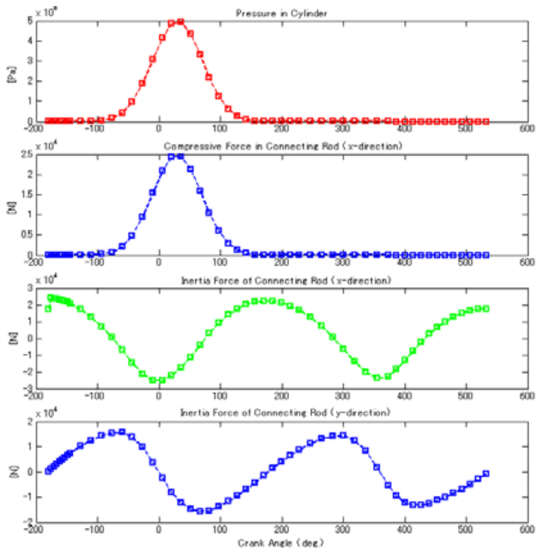

Fig.5 An Example of Motion and Inertia Load Analysis

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma \end{vmatrix} = 0 \qquad (6)$$

The mean stress and the stress amplitude are defined by

$$\sigma_a = (\sigma_{\max} + \sigma_{\min}) / 2 \quad \text{and} \quad \sigma_m = (\sigma_{\max} - \sigma_{\min}) / 2 \qquad (7)$$

The fatigue limit σ_{wR} is set to be $0.5\sigma_b$ where σ_b is the tensile strength. The safety factor is set to be 2 for the tensile strength and the true rupture stress. Figure 6 is a fatigue limit chart about the initial model. The horizontal axis is the scale of the mean stress. The vertical axis is the scale of stress amplitude. The straight solid line denotes the fatigue limit, which can be expressed by

$$\sigma_{wR}\,\sigma_m + \sigma_T\,\sigma_a - \sigma_{wR}\,\sigma_T = 0 \qquad (8)$$

where σ_T and σ_{wR} are the real rupture stress and the fatigue limit stress, respectively. The bended solid line expresses the yield stress. Many small circles represent the stress amplitude vs the mean stress with respect to the maximum and the minimum principal stress about all finite elements of the connecting rod model in two cycles of piston-crank motion. Figure 6 shows that fatigue breakdown occurs in the initial connecting rod in this case study condition.

Figure 7 is the schematics of the resultant FE model after 84 iterations of the optimization. Figure 8 shows the schematics of the cross-sections, which shows the optimized structure is a hollow structure. Figure 9 is a fatigue limit chart about the optimized model. Table 1 lists the comparison of the structural specifications between the initial model and the optimized one. The optimization finds an appropriately good connecting rod model that is lighter and keeps enough dynamic strength.

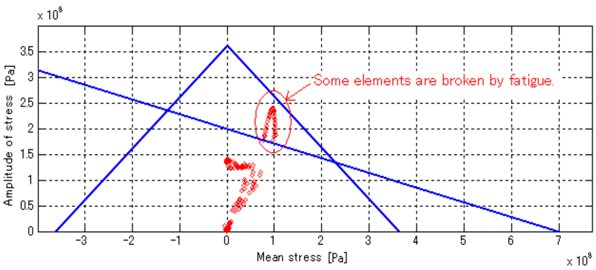

Fig.6 Fatigue limit chart about the initial connecting rod model

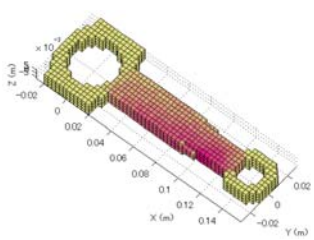

Fig. 7 Optimized FE model

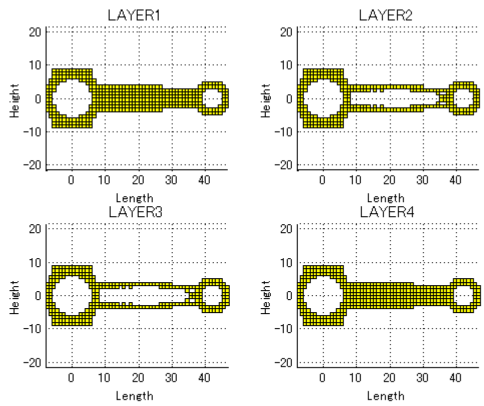

Fig.8 Skematics of Cross-sections of Optimized Model

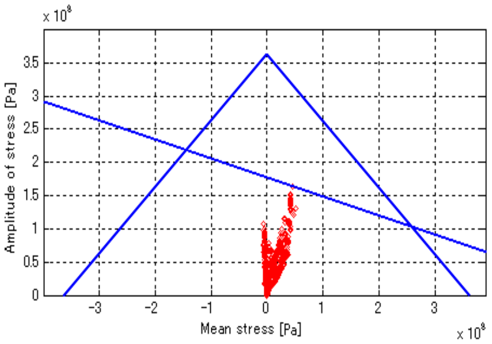

Fig.9 Fatigue limit chart about the resultant model

Table1 Structural Specifications

Items	Initial Model	Constraints (H)	Opt.Model
A	298.1	440.3	404.4
B	336.5	203.8	152.9
C	1.121	1.691	2.371
D	0.11013	0.04220	0.04215
E	568	2437	2830
F	1988	5101	5153
G	3426	5455	5819

A:Mass(kg)
B:Largest equivalent stress (Yield stress:363) [Mpa]
C:Safety rate about Fatigue Breakdown
D:Change of the distance between two centers of journal holes [mm]
E:1st Natural Freq. [Hz]
F:2nd Natural Freq. [Hz]
G:3rd Natural Freq. [Hz]

SUMMARY

This paper presented a new evolutionary structural optimization algorithm and a basic example about optimizing a connecting rod model under the consideration of dynamic strength, deflection and fatigue constraints. The application demonstrated the validity of the proposed algorithm.

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