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ACTIVE CONTROL OF HIGH FREQUENCY VIBRATION IN UNCERTAIN STRUCTURES

A. Gani *, R.S. Langley

Department of Engineering,
Cambridge University,
Trumpington Street, CB21PZ, Cambridge, UK
aga26@eng.cam.ac.uk

Abstract

The active suppression of structural vibration is normally achieved by either feedforward or feedback control. In the absence of a suitable reference signal feedforward control cannot be employed and feedback control is the only viable approach. Conventional feedback control algorithms (e.g. LQR and LQG) are designed on the basis of a mathematical model of the system and ideally the performance of the system should be robust against uncertainties in this model. The aim of this paper is to numerically investigate the robustness of LQR and LQG algorithms by designing the controller for a nominal system, and then assessing (via Monte Carlo simulation) the effects of uncertainties in the system. The ultimate concern is with the control of high frequency vibrations, where the short wavelength of the structural deformation induces a high sensitivity to imperfection. It is found that standard algorithms such as LQR and LQG are generally unfeasible for this case. This leads to a consideration of design strategies for the robust active control of high frequency vibrations. The system chosen for the numerical simulation concerns two coupled plates, which are randomized by the addition of point masses at random locations.

INTRODUCTION

Many situations arise in which it is desirable to reduce the vibration levels of a structure which is subjected to high frequency excitation [1]. In considering vibrational control methods which might be applied, the following physical characteristics of high frequency vibration should be borne in mind:

i. ***A large number of degrees of freedom are required to describe the detailed structural response.*** This is because high frequency vibrations have a short wavelength in comparison to the length scale of the structure. Modelling such vibrations with the finite element method (FEM) requires around five to seven elements per half wavelength, and thus very many degrees of freedom (typically millions) are needed for a complex structure.

ii. ***The system response is very sensitive to uncertainties.*** This is again due to the short wavelength of the vibration. Structural uncertainties can have a significant effect on the exact location of the natural frequencies, and likewise on the detailed spatial distribution of the mode shapes. Two nominally identical structures from the same production line can thus display very different dynamic response properties.

A common approach to the control of vibration is to apply either feedback or feedforward active vibration control (AVC). Most existing work on AVC tends to be focussed on low frequency vibration, often with collocated feedback, as employed for example in references [2-5]. One advantage of feedback control over feedforward control is that there is no requirement for knowledge of the disturbance signal [6], and collocated feedback control has good stability and robustness properties.

The aim of the present paper is to examine the performance of feedback control in the presence of system uncertainties, with the aim of assessing the likely efficacy of this technique when applied to high frequency vibration. Attention is focussed on LQR and LQG controllers, and these are applied to a two plate system which is randomised by a change in location of a number of attached masses. Monte Carlo simulations are used to assess the robustness of a controller designed for a nominal system, and possible alternative control design methods are discussed.

OPTIMAL LQR AND LQG CONTROLLERS

During the last two decades, various types of optimal feedback structural control algorithm have been developed. The main algorithms are: Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG), and the robust control algorithms H_2 and H_∞, together with various adaptive control strategies. The present work is a preliminary study of the effect of structural uncertainties at mid to high frequencies, and only LQR and LQG controllers have been considered thus far. These controllers are summarised in the present section.

For a system with the state-space equation of motion given in Eq. (1) below, the LQR algorithm provides a means of finding the controller that minimizes the cost function given in Eq. (2):

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \ , \quad \mathbf{x}(0) = \mathbf{x}_o \qquad (1)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

$$J = \int_{t_o}^{t_f} [\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}_r\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}_r\mathbf{u}(t)]\, dt \qquad (2)$$

Here $\mathbf{Q}_r$ is the error penalty and $\mathbf{R}_r$ is the control penalty. For a linear time invariant (LTI) multiple-input multiple-output (MIMO) system, the concern is often to minimize an output of the form $\mathbf{y}(t)$=$\mathbf{Cx}(t)$, in which case it is common to take $\mathbf{Q}_r$=$\mathbf{C}^{\mathrm{T}}\mathbf{C}$. If the aim is to minimise all the states, then $\mathbf{y}(t)$=$\mathbf{Ix}(t)$ is selected to treat all states as equally important or $\mathbf{y}(t)$=$\mathbf{Q}_{ii}\mathbf{Ix}(t)$ to impose a different penalty for each state. Assuming [**A, B**] is controllable and [**A, C**] is observable, then the unique optimal control law for LQR is the full state negative feedback given by Eq. (3) below, in which $\mathbf{K}_{lqr} = \mathbf{R}_r^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{X}_{lqr}$. Here $\mathbf{X}_{lqr}$ is a symmetric, positive definite, constant matrix obtained from the Algebraic Riccati Equation (ARE) [8].

$$\mathbf{u}(t) = -\mathbf{K}_{lqr}\mathbf{x}(t) \tag{3}$$

The major drawback of the LQR method is the requirement for all states to be measured, and also external noise is not considered. Clearly, this is not feasible for many practical systems. Hence in the LQG method a Kalman filter is used to estimate the full states from noisy output measurements. The plant state-space model for the standard LQG problem is given in Eq. (4) below. The disturbances **w** *(process noises)* and **v** *(measurement noises)* are uncorrelated, zero-mean, Gaussian, white noise processes with specified co-variance matrices. The block diagram for the LQG controller is given in figure 1. The LQG design process involves two steps: the first is to find $\mathbf{K}_{lqr}$ assuming that all states are known, and the second is to find an optimal state estimator so that $\mathrm{E}\{(\mathbf{x}-\hat{\mathbf{x}})^{\mathrm{T}}(\mathbf{x}-\hat{\mathbf{x}})\}$ is minimized, where $\hat{\mathbf{x}}$ is the estimated state vector. This estimator is independent of the weights $\mathbf{Q}_r$ and $\mathbf{R}_r$. With $\mathbf{e}=\mathbf{x}-\hat{\mathbf{x}}$, the closed loop dynamics for the plant and compensator are given by Eq. (5) below. Since the matrix $\mathbf{A}_{cl}$ is upper-block-diagonal, the closed loop poles of this system are given by the poles of deterministic LQR system together with the poles of the Kalman filter.

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t) + \mathbf{w} \quad , \qquad \mathbf{y}(t) = \mathbf{Cx}(t) + \mathbf{v} \tag{4}$$

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{pmatrix} = \begin{pmatrix} \mathbf{A}-\mathbf{BK}_{lqr} & \mathbf{BK}_{lqr} \\ 0 & \mathbf{A}-\mathbf{LC} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{e} \end{pmatrix} + \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{I} & -\mathbf{L} \end{pmatrix} \begin{pmatrix} \mathbf{w} \\ \mathbf{v} \end{pmatrix} \quad \text{or} \quad \dot{\tilde{\mathbf{x}}} = \mathbf{A}_{cl}\tilde{\mathbf{x}} + \tilde{\mathbf{w}} \tag{5}$$

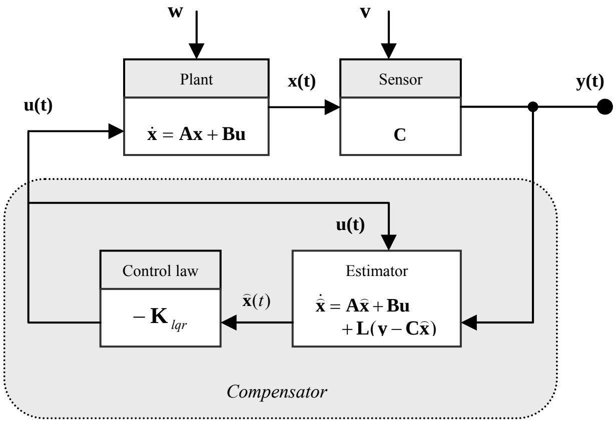

Figure 1- Block diagram for LQG optimal control

SIMULATION RESULTS: LQR IN THE PRESENCE OF STRUCTURAL UNCERTAINTIES

A simulation of the robustness of the LQR algorithm for an example structure is discussed in this section. The system consists of two simply supported plates with distributed point masses and spring couplings, as shown in Figure 2. The random positions of the point masses are perturbed to produce structural uncertainties. For simplicity, point masses are placed only on plate 1. The length, width and thicknesses of the plates are 0.8x1.0x0.005 *m* for plate 1 and *1.0*x*1.2*x*0.005 m* and for plate 2. The density, Young's modulus and Poisson's ratio for both plates are taken as 2700 kgm^{-3}, 7.1×10^{10} and 0.33 respectively.

An LQR controller has been designed for the nominal system given in figure 2 (i.e. a system with the masses in a fixed position), and the robustness of the controller has been investigated by considering its performance for systems with the masses in different positions. The degree of uncertainty is controlled by specifying the number of point masses and their total mass, as a percentage of the bare plate mass.

The stability of the system is assessed from the closed-loop poles (eigenvalues). Table 1 summarizes the simulation results and shows: the number of modal degrees of freedom considered for each plate; the number of point masses added to plate 1; the sum of the point masses, as a percentage of the plate mass; the "% instability", which indicates the number of poles that are unstable. Figure 3 shows the poles for the case of a system with 10 point masses (20% of total plate mass) for both the nominal and the perturbed systems (100 ensembles). Local controller aims to suppress the vibration response of the system at the drive point, whereas a global controller minimizes the overall vibration response of the system. An explanation of the data presented in each subplot is given in table 2. For local control it is clear that as the level of uncertainty increases, the percentage of unstable poles tends to increase. However beyond a certain number of masses the effect is more like a general increase in the plate density, and the number of unstable poles decreases. The global controller has more robust stability since it's designed to minimize the overall response of the system.

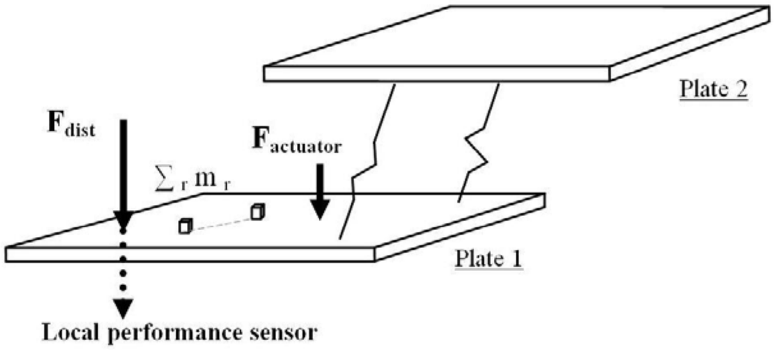

Figure 2 –Coupled-plate system used for perturbation analysis

Table 1: Percentage of instability for coupled-plate with LQR controller

DOF plate-1 and 2	*# point masses*	*% from plate mass*	*Global control (% instability)*	*Local Control (% instability)*
25	5	10	0	55
		20	0	97
		30	0	99
		40	0	63
		70	**11**	**77**
	10	10	0	19
		20	0	62
		30	0	64
		40	0	47
	20	10	0	44
		20	0	7
100	5	10	0	89
		20	0	95
		70	0	96
	10	10	0	72
		20	0	58

Table 2 - The data presented in Figures 3 and 4

Plot	1	2	3
1	*Eigenvalues: nominal system*	*Eigenvalues: nominal system with Global-controller*	*Eigenvalues: nominal system with local-controller*
2	*Eigenvalues: perturbed systems*	*Eigenvalues: perturbed systems with Global-controller*	*Eigenvalues: perturbed systems with local-controller*
3	*Statistical overlap factor:* $S = 2\sigma / \mu$, σ= *standard deviation of perturbed frequency.* μ= *mean spacing between each frequency*	*Histogram plot: Unstable frequencies under global controller*	*Histogram plot: Unstable frequencies under local controller*

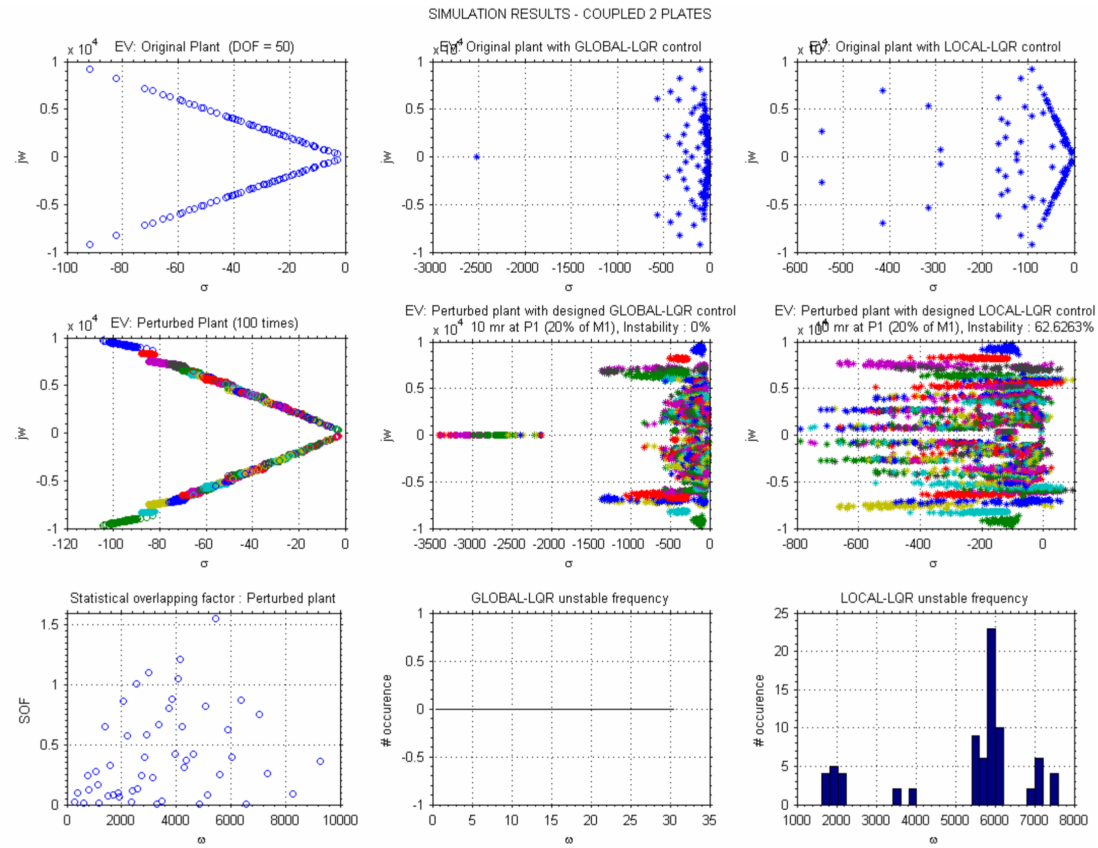

Figure 3 - LQR-10 point masses with total of 20% from plate mass

The local controller was found to be very sensitive to uncertainties and to become unstable with moveable masses having only 1% of the plate mass. In contrast, the global controller was able to accommodate a high level of uncertainty (70 % of total plate mass) as shown in table 1. However, since LQR requires the measurements of all states this result will not be achievable in practice. The simulation result show that, as would be expected, the higher frequencies are the more sensitive to uncertainties.

SIMULATION RESULTS: LQG IN THE PRESENCE OF STRUCTURAL UNCERTAINTIES

Using the same methodology as in the previous section, an LQG controller is designed for a nominal two-plate system and its robustness assessed by perturbing the positions of the masses. Table 3 summarizes the simulation results. In this case the local controller was highly sensitive and could not accommodate even a very small level of uncertainty. Furthermore, the global controller was stable only for mass loading below 1%. Figure 4 (refer to table 2 for details), shows a sample plot of the system eigenvalues for 150 ensembles. It can be noted that the LQG has no global system-independent guarantee of robustness [9].

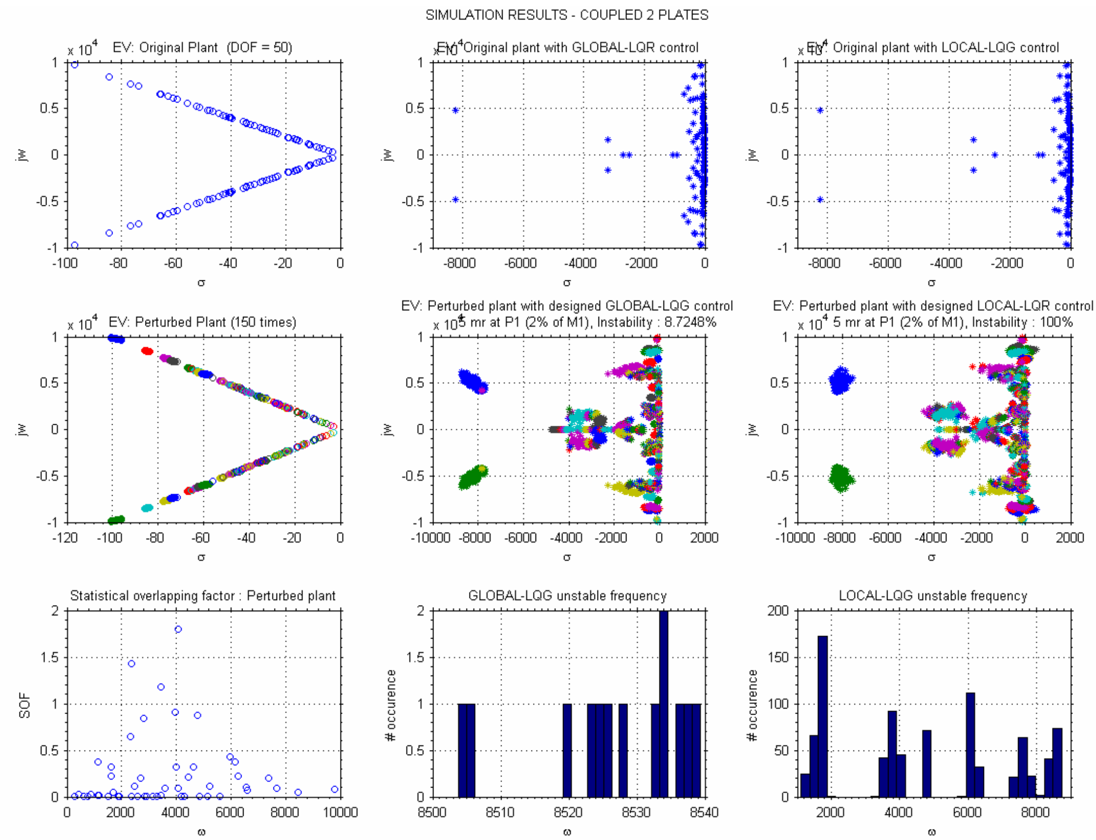

Figure 4 – LQG, 5 point masses with total of 2% from plate mass.

Table 3 - Percentage of instability for single -plate with LQG controller

DOF plate-1 and 2	*# mr*	*% from M*	*Global control (% instability)*	*Local Control (% instability)*
25	5	1	0	100
		2	16	100
		5	30	100
		10	98	100
		20	99	100
	10	2	61	100
		10	91	100
		20	61	100
100	5	1	0	100
		2	14	100
		10	83	100
		20	97	100
		30	100	100
	10	10	85	100
		20	89	100
		30	99	100

As might be expected, the LQG controller performance is much less robust than that of the LQR. Simulation studies with LQR and LQG reveal the system gets unstable at high frequency and this frequency range is very sensitive to structural uncertainties. As can be seen from figures 3 and 4, the open loop poles of the system are stable before and after applying the point mass perturbations. This suggests that, the underlying passive system is stable. However, the closed loop poles are not stable due to the instability introduced by the control system.

CONCLUSIONS

This paper has explored the robustness of LQR and LQG controllers in the presence of structural uncertainties for a coupled plate system. Although some improvement in performance might be expected from robust controllers such as H_∞, the present work has highlighted the extreme difficultly encountered in designing feedback controllers for mid or high frequency structural vibrations in the presence of uncertainty. A revised approach would be to consider local controllers such as skyhook dampers with simple PID control and strong robustness properties. The design problem is then not to design a complex feedback law, but to decide on the optimal location of the devices to be employed. This type of optimization study will require an efficient method for the analysis of mid and high frequency vibration in the presence of the controllers, and the hybrid method [1] offers one such approach. This strategy is being considered in a continuation of the present work.

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