Damping Estimation via Energy-Dissipation Method

Jin-Wei Liang
Mingchi University of Technology
Department of Mechanical Engineering
liangj@ccsun.mit.edu.tw

ABSTRACT

This paper makes use of an energy balance to estimate damping parameters in multiple degree-of-freedom (DOF) vibration systems. The method balances the energy input as registered in the force-displacement relationship of the real system against the energy lost in a theoretical model, consisting of viscous and Coulomb friction components, to develop the identification algorithms. The developed algorithms are applicable in the general and periodic input cases. Numerical investigations applying the algorithms to the simulated data illustrate promising results. In addition, the effects of random noise on the estimation results are numerically investigated.

Keywords: energy dissipation, damping estimation

1. INTRODUCTION

Friction parameter estimation is based on the analysis of measured input and output responses. Our interest is in identifying parameters of basic friction models by making use of vibration properties. For single DOF case, free vibration decrements have been exploited long ago for systems with linear stiffness elements and "small" damping. Free vibration decrements are not applicable if the damping is strong enough to preempt sufficient oscillations. As such, it makes sense to develop schemes to identify damping parameters in forced oscillators.

A scheme for extracting Coulomb and viscous friction parameters from forced oscillations based on the analytical solutions of Den Hartog [1] and Hundal [2] for the nonsticking response to harmonic excitations was formulated in [3]. The limitations of this analytical forced-resonance method are that it is not applicable for damping which is not "small," it relies on analytical solutions of single-DOF linear systems, and it does not treat friction models other than Coulomb plus viscous (see for example, references [4-9]). Thus, an energy balance was proposed as an alternative to identifying friction parameters by using analytical solutions [10].

The energy method balances the energy input as registered in the force-displacement relationship of the real system against that lost theoretically in a damping model with unknown parameters to develop the identification algorithms. Numerical and experimental investigations have shown the reliability and effectiveness of the energy-dissipation method in estimating viscous and Coulomb friction from single DOF vibration systems subjected periodic input [10]. In the current study, the energy-dissipation method will be extended to handle the multiple DOF problems.

Many estimation methods exist in the literatures that can be applied to extract damping information from the input and output measurements of the system. For instance, a direct method was proposed by Mohammad *et al.* in [11] where physical parameters, i.e. mass, stiffness and damping, of linear and non-linear structures can be estimated using measured time data and the least-squares criterion. The method is simple and reliable provided that the system is excited in the range where there are modal frequencies. Chen and Tomlinson [12] proposed estimating damping parameters in nonlinear oscillators by utilizing the acceleration, velocity and displacement output and formulating the output in terms of series of frequency response functions. The model used in [12] can be regarded as a subgroup of the AVD models. On the other hand, a wavelet-based approach for identifying parameters from nonlinear systems has been proposed by Ghanem in [13]. The method relies on a wavelet-based discretization of the non-linear differential equation of motion where orthogonal Daubechies scaling functions are used to track fast variations of the state of the dynamical system. The method might be sensitive to noise since responses are projected onto orthogonal space in order to find the wavelet-Galerkin solution of the differential equation of the system. By using the orthogonal expansion, details of data are zoomed in order to form proper representations in terms of expansion coefficients. As such, high-frequency noise could cause distorted expansion coefficients, which in turn might lead to incorrect estimates.

Recently, Chen in [14] proposed a Haar-wavelet based identification method. The same approach was also adopted in an on-line identification task by the author [15]. The method projects the governing differential equation of motion onto orthogonal Haar spaces so that a set of algebraic equations with the system parameters as unknowns can be obtained. Using the input/output data of the system together with the least-square criterion, the unknown system parameters can be estimated. The method is also prone to noise problem because it relies on accurate approximation of the signal using the orthogonal basis. Different from the expansion methods, the energy-dissipation approach projects the differential equation of motion onto energy spaces. The method is more robust when facing random or high frequency noise because such noise is expected to be filtered through the application of digital integration. The latter is adopted in implementing the identification process of this study.

2. THEORETICAL BASIS

All The energy-dissipation method involves the balance between the energy dissipated by the friction force of a real system against the energy input to the system. This balance results in the "equivalent viscous and Coulomb friction" parameters, similar to the traditional "equivalent viscous damping" concept presented in undergraduate textbook such as [16]. To briefly explain the idea, let's consider a single DOF damped-forced oscillator with the following equation of motion

$$m\ddot{x} + kx + F(x,\dot{x}) = a(t), \tag{1}$$

where *m* is the mass, *k* represents the stiffness, *x* is the displacement, dots indicate derivatives with respect to time, and $a(t)$ represents an excitation. In addition, $F(x,\dot{x})$ denotes the damping force occurring at the contact interfaces. Multiplying Eq. (1) by *dx* and integrating the resultant equation along the motion path *C* brings forth the following equation

$$\int_C \{m\ddot{x}dx + kxdx + F(x,\dot{x})\}dx = \int_C a(t)dx .$$

To facilitate the implementation of integration, the integration variable can be changed to time. Hence

$$\int_t^{t+T_1} \{m\ddot{x} + kx + F(x,\dot{x})\}\dot{x}dt = \int_t^{t+T_1} a(t)\dot{x}dt$$

where T_1 denotes a finite time interval. Now, if the followings are defined

$W_d = \int_t^{t+T_1} F(x,\dot{x})\dot{x}dt$; $W_a = \int_t^{t+T_1} a(t)\dot{x}dt$;

$W_e = \int_t^{t+T_1} \{m\ddot{x} + kx\}\dot{x}dt$,

Eq. (1) becomes

$$W_d = W_a - W_e \tag{2}$$

Equation (2) implies a balance among the dissipated, applied energy and the sum of kinetic and elastic energy. To implement the damping identification, we assume that *x(t)* (and its derivatives) and *a(t)* can be measured. Then, the terms involved in Eq. (2) are quantified while $F(x,\dot{x})$ is expressed using an assumed friction model with unknown parameters. By integrating and balancing Eq. (2), one acquires identification equations for the damping parameters.

When *a(t)* and *x(t)* are periodic, one can integrate Eq. (2) over a cycle of periodic motion. In that case, the contribution of the conservative components of the oscillator " $m\ddot{x} + kx$ " is zero, so that the following energy-dissipation equation can be obtained

$$W_d = \int_t^{t+T} F(x,\dot{x})\dot{x}dt = \int_t^{t+T} a(t)\dot{x}dt = W_a . \tag{3}$$

here *T* represents the fundamental period of the response. To further illustrate the idea, we assume that $F(x,\dot{x})$ is modeled by "viscous plus Coulomb friction" while a harmonic base excitation is used for $a(t)$. Therefore, $a(t) = kY_o \cos\omega t$, and

$$W_d = \int_t^{t+T} \{c\dot{x} + F_k \operatorname{sgn}(\dot{x})\}\dot{x}dt = \int_t^{t+T} \{kY_o \cos\omega t\}\dot{x}dt = W_a \tag{4}$$

Here, c and F_k are the equivalent damping parameters to be identified. Eq. (4) can be used to estimate c and F_k using two excitation levels. Alternatively, more excitation levels can be applied to gain multiple versions of Eq. (4). Suppose that the number of equations is more than the number of unknown parameters, the least-squares criterion can be applied to improve the robustness of the method.

The algorithms outlined above are suitable for estimating damping parameters from a single DOF vibration system. A study focusing on periodic input (and response) has been presented in [10] in which the effectiveness and reliability of the energy-dissipation method were demonstrated. The method is not only valid for the periodic case, it is also applicable in the general-input or input-free case, for which T becomes an arbitrary finite time interval rather than a forcing period. In the input-free case, $a(t)$ vanishes in Eq. (2). The initial states of the system, under this condition, should be large to introduce sufficient conservative energy. Thus, the ensuing free responses will provide information from which the damping parameters are estimated.

The basic concepts and procedures used in the single DOF case can be extended to handle multiple DOF vibration systems. To show how the extension can be accomplished, the following equation of motion of a multi-DOF system is considered

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{k}\mathbf{x} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{D}(\dot{\mathbf{x}})\mathbf{F_k} = \mathbf{a(t)}$$

where $\mathbf{M} \in R^{n\times n}$ represents the diagonal mass matrix, n is the number of degrees of freedom, $\mathbf{k} \in R^{n\times n}$ indicates the symmetric stiffness matrix, $\mathbf{c} \in R^{n\times n}$ is the viscous-damping matrix, and $\mathbf{a(t)} \in R^n$ is the input vector. $\mathbf{F_k} \in R^n$ represents a vector of elements F_{ki} such that $\mathbf{D}(\dot{\mathbf{x}})\ \mathbf{F_k}$ models the Coulomb components locating at certain places of the discrete model of the system. Hence, $\mathbf{D}(\dot{\mathbf{x}}) \in R^{n\times n}$ is a matrix comprising of sign functions of relative velocities occurring at contact interfaces. Since, in general, not every damping components in $\mathbf{c}$ and $\mathbf{F_k}$ are unknowns, Eq. (4) can be reorganized into the following

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{k}\mathbf{x} + \mathbf{E}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{B}(\dot{\mathbf{x}})\mathbf{c}^* + \mathbf{G}(\dot{\mathbf{x}})\mathbf{F}_\mathbf{k}^* = \mathbf{a(t)} \tag{5}$$

in which $\mathbf{E}(\mathbf{x},\dot{\mathbf{x}}) \in R^n$ denotes a vector containing all the "known" damping components multiplied by their corresponding state functions. On the other hand, $\mathbf{c}^* = \left[c_1^*, c_2^*, \cdots, c_Q^*\right]^T$ and $\mathbf{F}_\mathbf{k}^* = \left[F_{k1}^*, F_{k2}^*, \cdots, F_{kS}^*\right]^T$ contain the unknown viscous and Coulomb-friction

parameters which are to be estimated. $\mathbf{B}(\dot{\mathbf{x}}) \in R^{n\times Q}$ and $\mathbf{G}(\dot{\mathbf{x}}) \in R^{n\times S}$ denote state-function matrices which when multiplied separately by $\mathbf{c}^*$ and $\mathbf{F}_\mathbf{k}^*$ model the viscous and Coulomb damping of the system.

To formulate the damping-estimation algorithms, we first assume that *a(t)* and *x(t)* are both periodic with a common fundamental period, *T*. Note that the periodicity condition will be released later on in this study. Multiplying Eq. (5) with $\dot{\mathbf{x}}^T$, where $\dot{\mathbf{x}}_i = [0\cdots\dot{x}_i\cdots 0]^T{}_{n\times 1}$, and integrating the resultant equation over a cycle of periodic motion yields

$$\int_t^{t+T} \dot{\mathbf{x}}^T{}_i \mathbf{M}\ddot{\mathbf{x}}dt + \int_t^{t+T} \dot{\mathbf{x}}^T{}_i \mathbf{k}\mathbf{x}dt + \int_t^{t+T} \dot{\mathbf{x}}^T{}_i \mathbf{E}(\mathbf{x},\dot{\mathbf{x}})dt + \int_t^{t+T} \dot{\mathbf{x}}^T{}_i \mathbf{B}(\dot{\mathbf{x}})\mathbf{c}^* dt + \int_t^{t+t} \dot{\mathbf{x}}^T{}_i \mathbf{G}(\dot{\mathbf{x}})\mathbf{F}_\mathbf{k}^* dt = \int_t^{t+T} \dot{\mathbf{x}}^T{}_i \mathbf{a}(\mathbf{t})dt \quad (6)$$

Due to the fact that $\mathbf{M}$ is a diagonal matrix and $\dot{\mathbf{x}}_i$ possesses only one non-zero element, the following equation can be obtained

$$\int_t^{t+T} \dot{x}_i m_i \ddot{x}_i dt + \int_t^{t+T} (\sum_{j=1}^{n} k_{ij} x_j)\dot{x}_i dt + \int_t^{t+T} e_i(\mathbf{x},\dot{\mathbf{x}})\dot{x}_i dt + \int_t^{t+T} (\sum_{j=1}^{Q} b_{ij}(\dot{\mathbf{x}})c_j^*)\dot{x}_i dt + \int_t^{t+t} (\sum_{j=1}^{S} g_{ij}(\dot{\mathbf{x}})F_{kj}^*)\dot{x}_i dt = \int_t^{t+T} a_i(t)\dot{x}_i dt \quad (7)$$

where m_i represents the "*i*-th" diagonal element of M. $\{k_{ij}, j=1,\ldots,n\}$, $\{b_{ij}, j=1,\ldots,Q\}$ and $\{g_{ij}, j=1,\ldots,S\}$ individually denote the "*i*-th" row vector of matrices **k,** **B**, and **G**, whereas a_i and e_i are the "*i*-th" element of vectors a(t) and $\mathbf{E}(\mathbf{x},\dot{\mathbf{x}})$, respectively.

Since the integration of conservative components over a cycle of periodic motion are zeros, we have $\int_t^{t+T} \dot{x}_i m_i \ddot{x}_i dt = 0$ and $\int_t^{t+T} k_{ii} x_i \dot{x}_i dt = 0$. Eq. (7) can then be recast into

$$\int_t^{t+T} (\sum_{j=1}^{Q} b_{ij}(\dot{\mathbf{x}})c_j^*)\dot{x}_i dt + \int_t^{t+t} (\sum_{j=1}^{S} g_{ij}(\dot{\mathbf{x}})F_{kj}^*)\dot{x}_i dt = \int_t^{t+T} a_i(t)\dot{x}_i dt - \int_t^{t+T} (\sum_{\substack{j=1\\ j\neq i}}^{n} k_{ij} x_j)\dot{x}_i dt - \int_t^{t+T} e_i \dot{x}_i dt$$

or equivalently,

$$c_1^* \int_t^{t+T} b_{i1}\dot{x}_i dt + c_2^* \int_t^{t+T} b_{i2}\dot{x}_i dt + \cdots + c_Q^* \int_t^{t+T} b_{iQ}\dot{x}_i dt + F_{k1}^* \int_t^{t+T} g_{i1}\dot{x}_i dt + F_{k2}^* \int_t^{t+T} g_{i2}\dot{x}_i dt + \cdots + F_{kS}^* \int_t^{t+T} g_{iS}\dot{x}_i dt \quad (8)$$
$$= \int_t^{t+T} a_i(t)\dot{x}_i dt - \int_t^{t+T} (\sum_{\substack{j=1\\ j\neq i}}^{n} k_{ij} x_j)\dot{x}_i dt - \int_t^{t+T} e_i \dot{x}_i dt$$

The procedures described above can be repeated for every single DOF of the system to result in n equations similar to Eq. (8). To that end, the following identification equations can be obtained

$$c_1^*\tau_{i1} + c_2^*\tau_{i2} + \cdots + c_Q^*\tau_{iQ} + F_{k1}^*\gamma_{i1} + F_{k2}^*\gamma_{i2} \cdots + F_{kS}^*\gamma_{iS} = \beta_i \quad i = 1,2\cdots n \quad (9)$$

where

$$\tau_{ij} = \int_t^{t+T} b_{ij}(\dot{\mathbf{x}})\dot{x}_i dt \qquad j = 1,2\cdots Q \quad (10)$$

$$\gamma_{ij} = \int_t^{t+T} g_{ij}(\dot{\mathbf{x}})\dot{x}_i dt \qquad j = 1,2\cdots S \quad (11)$$

$$\beta_i = \int_t^{t+T} a_i(t)\dot{x}_i dt - \int_t^{t+T} (\sum_{\substack{j=1\\ j\neq i}}^{n} k_{ij} x_j)\dot{x}_i dt - \int_t^{t+T} e_i(\dot{\mathbf{x}})\dot{x}_i dt \quad (12)$$
$$i = 1,2\cdots n$$

It should be emphasized that the algorithms presented in Eqs. (9)-(12) are only suitable for the periodic case in which both input and response are periodic data. For the case where $a(t)$ is a general input, the periodicity of the signal no longer holds. In that case, the integrating durations are arbitrary, and the contribution of conservative terms is not zero. Then, the expression of Eq. (12) becomes

$$\beta_i = \int_t^{t+T} a_i(t)\dot{x}_i dt - \int_t^{t+T} m_i \ddot{x}_i \dot{x}_i dt - \int_t^{t+T} (\sum_{j=1}^{n} k_{ij} x_j)\dot{x}_i dt - \int_t^{t+T} e_i(\dot{\mathbf{x}})\dot{x}_i dt \qquad i = 1,2\cdots n \quad (12a)$$

Here, the integrating duration, T, is an arbitrary finite time interval rather than a forcing period. Eqs. (9)-(11) and (12a) can be adopted to estimate damping parameters in a general-input problem. For the input-free case, $a(t)$ vanishes in Eq. (12a). Large initial states are required under such a condition to introduce sufficient free responses from which the damping information can be extracted.

Additionally, in order to measure "the goodness of fit" between the estimated system response and the simulated or experimental one, a normalized mean square error (MSE) of "acceleration" signal is adopted. This error criterion is defined as the following [11]

$$\mathbf{MSE}_i = \left[\frac{\left\{\sum_{j=1}^{N_p} (\ddot{x}_{ij} - \tilde{\ddot{x}}_{ij})^2\right\}}{N_p \sigma_{\ddot{x}_i}^2}\right] \times 100, \quad i = 1,2,....n \quad (13)$$

where $\ddot{x}$ is the simulated or measured acceleration, $\tilde{\ddot{x}}$ is the estimated one. N_p is the number of data points, and $\sigma_{\ddot{x}}^2$ is the variance of the simulated or measured acceleration.

In the next section, numerical examples focusing on

periodic and general input are adopted to validate the proposed method. The input-free scheme will be used in the experimental investigations which are currently undergoing. In numerical studies, the effects of random noise on the estimation results are also explored.

. APPLICATION OF THE METHOD TO SIMULATION DATA

Two numerical examples with known parameters subjected to periodic or general (random) input are selected here to validate the proposed schemes. In implementing, the differential equations of motion are first formulated in matrix form similar to that of Eq. (5). A stiff, low-order ordinary differential equation (ODE) solver of Bogacki and Shampire (ode23tb) was then adopted to numerically integrate the ODEs at a constant sampling rate after converting the ODEs to first-order form [17]. Simpson's rule was adopted to obtain τ_{ij}, γ_{ij}, and β_i from signals of adequate lengths.

3.1 Numerical study of a 2-DOF system

To validate the method, a two-DOF vibration system, whose schematic diagram is presented in Figure 1, is first explored. According to Figure 1 and the contexts of Eq. (5), the following differential equation of motion can be obtained

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} k+k_1 & -k_1 \\ -k_1 & k_2+k_1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}+ \begin{bmatrix} \dot{x}_1-\dot{x}_2 & 0 \\ \dot{x}_2-\dot{x}_1 & \dot{x}_2 \end{bmatrix}\begin{Bmatrix} c_1^* \\ c_2^* \end{Bmatrix}+\begin{bmatrix} \mathrm{sgn}(q) & 0 \\ -\mathrm{sgn}(q) & \mathrm{sgn}(\dot{x}_2) \end{bmatrix}\begin{Bmatrix} F_{k1}^* \\ F_{k2}^* \end{Bmatrix}=\begin{Bmatrix} kw(t) \\ 0 \end{Bmatrix} \tag{14}$$

where $q=\dot{x}_1-\dot{x}_2$ representing the relative velocity between m_1 and m_2, $w(t)=W_0\cos\omega t$ is the harmonic base excitation, and c_1^*, c_2^*, F_{k1}^*, and F_{k2}^* denote the un-identified damping parameters. Obviously, the "viscous plus Coulomb friction" model is adopted here to describe the damping behaviors of the contact.

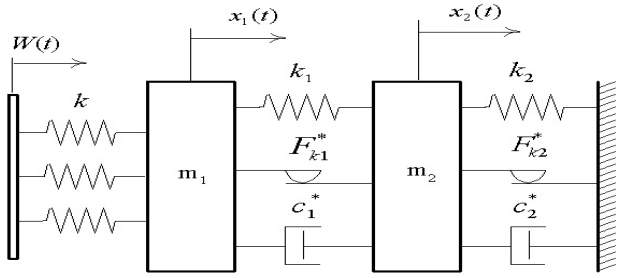

Figure 1 The two DOF system used in the parameter estimation process.

Based on Eq. (14) and the contexts of Eqs. (9)-(12), the followings are obtained

$$\tau_{11}c_1^*+\gamma_{11}F_{k1}^*=\beta_1 \tag{15}$$

$$\tau_{21}c_1^*+\tau_{22}c_2^*+\gamma_{21}F_{k1}^*+\gamma_{22}F_{k2}^*=\beta_2 \tag{16}$$

where $\tau_{12}=\gamma_{12}=0$, and

$$\tau_{11}=\int_t^{t+T}(\dot{x}_1-\dot{x}_2)\dot{x}_1dt\,;\quad \gamma_{11}=\int_t^{t+T}\mathrm{sgn}(\dot{x}_1-\dot{x}_2)\dot{x}_1dt\,;$$

$$\beta_1=\int_t^{t+T}(kW_0\cos\omega t+k_1x_2)\dot{x}_1dt \tag{17}$$

$$\tau_{21}=\int_t^{t+T}(\dot{x}_2-\dot{x}_1)\dot{x}_2dt\,;\quad \tau_{22}=\int_t^{t+T}(\dot{x}_2)^2dt\,;$$

$$\gamma_{21}=-\int_t^{t+T}\mathrm{sgn}(\dot{x}_1-\dot{x}_2)\dot{x}_2dt \tag{18}$$

$$\gamma_{22}=\int_t^{t+T}\mathrm{sgn}(\dot{x}_2)\dot{x}_2dt\,;\quad \beta_2=\int_t^{t+T}k_1x_1\dot{x}_2dt \tag{19}$$

It is worth to note that both of the differential equations appearing in Eq. (14) are of second order. Therefore, each of them can be used to solve for two unknown parameters provided that persistent excitation conditions are satisfied [18]. The maximal number of unknowns that can be estimated in this example is four. However, one can also choose to use both ODEs of Eq. (14) with various excitation levels so that more than four algebraic equations are obtained. In that case, more robust estimations can be gained through the application of the least-squares criterion. We recommend the latter approach. To that end, Eqs. (15) and (16) are recast into the following

$$A\underline{P}=\underline{U} \tag{20}$$

where

$$A=\begin{bmatrix} \tau_{11}^1 & 0 & \gamma_{11}^1 & 0 \\ \tau_{21}^1 & \tau_{22}^1 & \gamma_{21}^1 & \gamma_{22}^1 \\ \vdots & \vdots & \vdots & \vdots \\ \tau_{11}^k & 0 & \gamma_{11}^k & 0 \\ \tau_{21}^k & \tau_{22}^k & \gamma_{21}^k & \gamma_{22}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tau_{11}^l & 0 & \gamma_{11}^l & 0 \\ \tau_{21}^l & \tau_{22}^l & \gamma_{21}^l & \gamma_{22}^l \end{bmatrix}_{2l\times 2}\quad;\qquad \underline{P}=\begin{bmatrix} c_1^* \\ c_2^* \\ F_{k1}^* \\ F_{k2}^* \end{bmatrix};$$

$$\underline{U}=\begin{bmatrix} \beta_1^1 \\ \beta_2^1 \\ \vdots \\ \beta_1^k \\ \beta_2^k \\ \vdots \\ \beta_2^l \end{bmatrix}_{2l\times 1} \tag{21}$$

Here, the superscript "k" corresponds to different excitation level. Hence, there are totally "l" excitation levels applied in Eq. (21). The optimal solution for vector P, in the least-squares sense, becomes

$$\underline{P} = (A^T A)^{-1} A^T \underline{U}\,, \tag{22}$$

Eq. (22) provides an estimate of the unknown parameters, along with a residual $\underline{r} = A\underline{P} - \underline{U}$ that is generally nonzero. The residual can serve as an indicator of the quality of the damping model. In this case, it tells whether "viscous plus Coulomb" model can adequately describe friction behaviors occurring at different contact interfaces. In addition, since the coefficients are acquired by digital integration, high-frequency noise is expected to be filtered, but low frequency noise might have more influence.

Next, numerical experiments are conducted to illustrate the reliability of the proposed method. The simulations were carried out in accordance with Eq. (14) together with the following parameter values: $\omega = 8\pi$, $m_1 = m_2 = 1.0$, $k = 10.0$, $k_1 = k_2 = 20$, $c_1^* = c_2^* = 6.0$, $F_{k1}^* = F_{k2}^* = 10.0$. Three excitations corresponding to $W_{01} = 40.0$, $W_{02} = 80.0$, and $W_{02} = 180.0$ were applied to gain the integrated coefficients. Based on these data, the least-squares estimates obtained from Eq. (22) were: $\tilde{c}_{1LS}^* = 5.9993$; $\tilde{c}_{2LS}^* = 6.0089$; $\tilde{F}_{k1LS}^* = 9.9422$; $\tilde{F}_{k2LS}^* = 9.9462$. The mean of absolute values of the residuals (errors in each of Eq.(20)) normalized by the right-hand side of Eq. (20) was 1.11*e*-3, for numerically integrated noise-free numerical data with a perfect model. Moreover, the MSE values calculated from the simulated and estimated acceleration responses were $MSE_1 = 1.67\times10^{-6}$ % and $MSE_2 = 1.32\times10^{-5}$ %, respectively, where the subscript denotes the DOF the motion was referred to. Evidently, the estimation accuracies are promising which shows the effectiveness of the method.

In order to examine whether the estimation accuracy can withhold when noise are present, we added to the simulated force input, velocity and displacement responses a white random noise of 5% of their root mean squares (rms) respectively. In processing the contaminated data, we would still treat the signals as periodic ones although they have definitely lost that property in a more rigorous sense. Features of the contaminated signal can be observed in Figure 2 in which the upper plot shows the force input (corresponding to $W_o = 180$) while the lower plot shows $x_1(t)$.

Upon choosing one complete cycle of the contaminated signals and performing the required integrations in accordance with Eqs. (17)-(19), one obtained the parameter estimations. The results were: $\tilde{c}_{1LS}^* = 5.84$; $\tilde{c}_{2LS}^* = 5.96$; $\tilde{F}_{k1LS}^* = 13.73$; $\tilde{F}_{k2LS}^* = 10.29$ with the mean of the absolute values of the normalized residuals being 1.75*e*-2. At the first glance, the estimation errors seem to be substantial. However, when one proceeded to examine the discrepancy between the simulated and estimated acceleration signals as shown in Figure 3, one actually

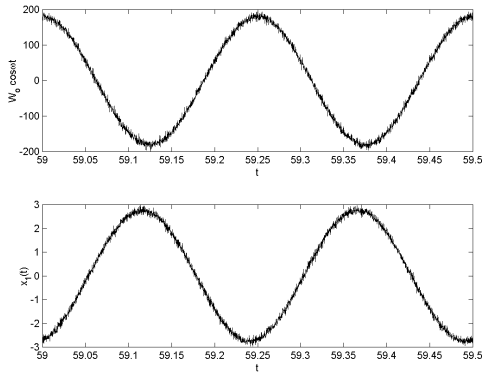

Figure 2 The contaminated signals (solid line) obtained by adding a white random noise to the simulated data (dashed line) which have been masked by the noise.

found that they were almost indistinguishable, especially for the motion of mass 1. Note that the comparison tests demonstrated in Figure 3 show the acceleration data to which we added no random noise. Therefore, both the estimated and simulated data seem to be cleaner than that presented in Figure 2. The associated MSE values for these acceleration data were MSE_1=0.0011% and MSE_2=0.132% respectively. The low MSE values and the close features appearing in the cures of Figure 3 indicate that variations of dry-friction force have small impact on the response of the system. Moreover, in Fig. 2, one also sees that the random noise added on the signals are severe which may not often been seen in the real situations. To this end, the reliability of the energy-dissipation method subjected to low-level random noise can be assured. In the next section, a 3-DOF numerical system is investigated.

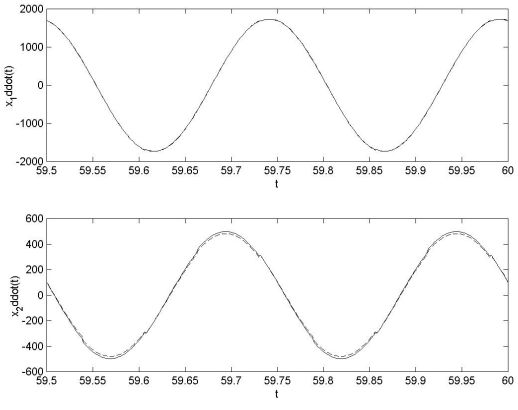

Figure 3 Comparison between the estimated (dashed line) and the simulated (solid line) acceleration responses with 5% noise level, the 2-DOF system.

3.2 Numerical study of a 3-DOF system: periodic excitation case

The 3-DOF numerical example, with its schematic presented in Fig. 4, has been studied in [11]. According to Figure 4, the following equations of motion can be obtained

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{Bmatrix}+\begin{bmatrix} (c_{11}^*+c_{12}^*) & -c_{12}^* & 0 \\ -c_{12}^* & (c_{12}^*+c_{23}^*) & -c_{23}^* \\ 0 & -c_{23}^* & (c_{23}^*+c_{33}^*) \end{bmatrix}\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{Bmatrix}$$
$$+\begin{bmatrix} (k_{11}+k_{12}) & -k_{12} & 0 \\ -k_{12} & (k_{12}+k_{23}) & -k_{23} \\ 0 & -k_{23} & (k_{23}+k_{33}) \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}+\begin{Bmatrix} 0 \\ 0 \\ \mathrm{sgn}(\dot{y}_3) \end{Bmatrix}F_k^*=\begin{Bmatrix} f(t) \\ 0 \\ 0 \end{Bmatrix} \quad (23)$$

Note that four viscous parameters, c_{11}^*, c_{12}^*, c_{23}^*, and c_{33}^*, and one Coulomb parameter, F_k^*, are to be estimated. Eq. (23) is converted into the following form

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{Bmatrix}+\begin{bmatrix} (k_{11}+k_{12}) & -k_{12} & 0 \\ -k_{12} & (k_{12}+k_{23}) & -k_{23} \\ 0 & -k_{23} & (k_{23}+k_{33}) \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}$$
$$+\begin{bmatrix} \dot{y}_1 & \dot{y}_1-\dot{y}_2 & 0 & 0 \\ 0 & \dot{y}_2-\dot{y}_1 & \dot{y}_2-\dot{y}_3 & 0 \\ 0 & 0 & \dot{y}_3-\dot{y}_2 & \dot{y}_3 \end{bmatrix}\begin{Bmatrix} c_{11}^* \\ c_{12}^* \\ c_{23}^* \\ c_{33}^* \end{Bmatrix}+\begin{Bmatrix} 0 \\ 0 \\ \mathrm{sgn}(\dot{y}_3) \end{Bmatrix}F_k^*=\begin{Bmatrix} F_0\cos\omega t \\ 0 \\ 0 \end{Bmatrix} \quad (24)$$

According to Eq. (24) and the contexts of Eqs. (9)-(12), the following identification equations are obtained

$$\tau_{11}c_{11}^*+\tau_{12}c_{12}^*=\beta_1 \quad (25)$$

$$\tau_{22}c_{12}^*+\tau_{23}c_{23}^*=\beta_2 \quad (26)$$

$$\tau_{33}c_{23}^*+\tau_{34}c_{33}^*+\gamma_{31}F_k^*=\beta_3 \quad (27)$$

where

$$\tau_{11}=\int_t^{t+T}\dot{y}_1^2dt \quad ; \quad \tau_{12}=\int_t^{t+T}\dot{y}_1(\dot{y}_1-\dot{y}_2)dt \quad ;$$

$$\beta_1=\int_t^{t+T}\dot{y}_1(F_o\cos\omega t+k_{12}y_2)dt \quad (28)$$

$$\tau_{22}=\int_t^{t+T}\dot{y}_2(\dot{y}_2-\dot{y}_1)dt \quad ; \quad \tau_{23}=\int_t^{t+T}\dot{y}_2(\dot{y}_2-\dot{y}_3)dt \quad ;$$

$$\beta_2=\int_t^{t+T}\dot{y}_2(k_{12}y_1+k_{23}y_3)dt \quad (29)$$

$$\tau_{22}=\int_t^{t+T}\dot{y}_3(\dot{y}_3-\dot{y}_2)dt; \quad \tau_{34}=\int_t^{t+T}\dot{y}_3^2dt \quad ;$$

$$\tau_{35}=\int_t^{t+T}\dot{y}_3\,\mathrm{sgn}(\dot{y}_3)dt \; ; \quad \beta_3=\int_t^{t+T}\dot{y}_3(k_{23}y_2)dt \quad (30)$$

Numerical validations were performed with respect to the 3-DOF system shown in Fig. 4 together with the following parameters: $m_1=1.5$, $m_2=1.3$, $m_3=2.0$ kg, $k_{11}=k_{12}=k_{23}=k_{33}=1\times10^4$ N/m, $c_{11}^*=c_{12}^*=c_{23}^*=c_{33}^*=20$ Ns/m, and $F_k^*=1.0$ N. The same data were also applied in the study performed in [11] in which a direct parameter estimation method was proposed.

The procedures taken in the previous 2-DOF cases were applied here. It should be emphasized that two excitation levels, separately corresponding to 5 Hz and 10 Hz, were adopted in this example. The approach differed from the 2-DOF case where the system was excited at one single frequency and three different levels. Forcing the system at two different frequencies, both near resonance, could improve the estimation a bit. Although, in fact, accuracies obtained from exciting the system solely at one frequency were also acceptable. For instance, the estimates obtained using the mixed-frequency excitation were: $\tilde{c}_{11}^*=20.04$, $\tilde{c}_{12}^*=20.05$, $\tilde{c}_{23}^*=20.19$, $\tilde{c}_{33}^*=20.09$, $\tilde{F}_k=1.01$.

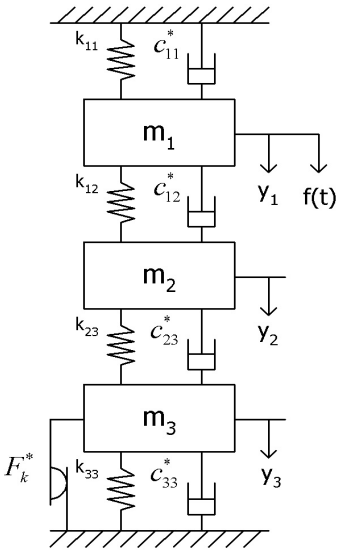

Figure 4 Three-DOF system with friction link between mass 3 and ground.

The mean of absolute values of the normalized residuals was 7.94*e*-4, for numerically integrated noise-free numerical data with a perfect model. The MSE values associated with the simulated and estimated acceleration signals were: $\mathbf{MSE_1}=0.0012\%$, $\mathbf{MSE_2}=0.0013\%$, $\mathbf{MSE_3}=0.0013\%$. The effectiveness of the proposed method is again verified by the low values of MSE and the small mean of the normalized absolute residuals..

In contrast, if two excitation levels, $F_0=100$ and 200, corresponding to the same frequency, $\omega=20\pi$, were applied, the resultant estimates turned out to be: $\tilde{c}_{11}^*=19.26$, $\tilde{c}_{12}^*=18.16$, $\tilde{c}_{23}^*=23.41$, $\tilde{c}_{33}^*=20.80$, $\tilde{F}_k=1.015$, while the mean of the absolute residuals was 9.49*e*-5. The corresponding MSE values obtained based on these estimates were: $\mathbf{MSE_1}=0.0038\%$, $\mathbf{MSE_2}=0.0014\%$, $\mathbf{MSE_3}=0.0029\%$. Obviously, the estimation accuracies obtained from the mixed-frequency case are better (especially the viscous damping estimate) than that of the single-frequency case. Although the latter ones are also acceptable which could be verified by the low MSE values associated with the estimates. Our speculation is that forcing the system with more than one frequency, near resonance, might excite richer responses that contains more information regarding the system's characteristics, including damping. Thus, better estimates can be gained from these rich responses. A thorough investigation might be required

to conclude the statement.

Similar to the 2-DOF case, we then added to the simulated force input, velocity and displacement responses a white random noise of 5% of their individual root-mean-squares values. The excitation conditions were the same as those of the clean-data case. The estimates obtained from these contaminated data were: $\tilde{c}_{11}^{*} = 19.70$, $\tilde{c}_{12}^{*} = 17.25$, $\tilde{c}_{23}^{*} = 18.45$, $\tilde{c}_{33}^{*} = 20.08$, $\tilde{F}_k = 0.85$. The mean of the normalized absolute residuals was 3.57*e*-2, while the corresponding MSE values based on these erroneous-opted estimates were: $\mathbf{MSE}_1 = 0.029\%$, $\mathbf{MSE}_2 = 0.032\%$, $\mathbf{MSE}_3 = 0.035\%$. These data when compared to those associated with the clean-data case again indicate that the method is robust to random noise. The statement can be verified by the low MSE values and the extremely close curves shown in the comparison plot of Fig. 5.

To summarize, we have shown that the energy-dissipation scheme works very well in identifying damping from multi-DOF vibration systems subjected to periodic input, whether the data are clean or contaminated. In order to show that the method also works in the general-input situations, the periodicity conditions of the input (and response) will be released in the next section in which a band-passed random input is considered.

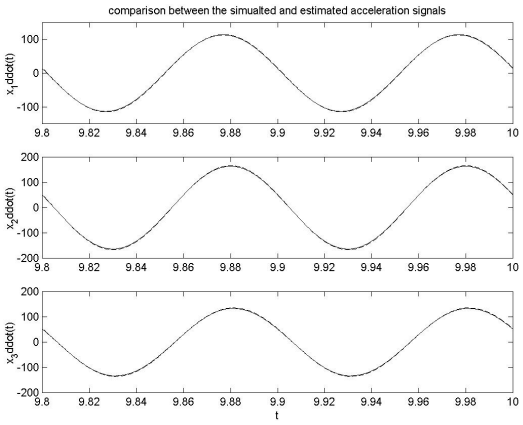

Figure 5 Comparison between the estimated (dashed line) and the simulated (solid line) acceleration responses with 5% noise level, the 3-DOF system.

3.3 Numerical study of a 3-DOF system: non-periodic excitation case

A couple of changes have to be made before implementing the identification process of non-periodic data. These include: (1) the contribution of the conservative components can no longer be ignored and (2) the integration time interval will be an arbitrary finite one instead of a complete forcing period. In numerical experiments, the system was excited at co-ordinate 1 by using two band-pass limited (in the frequency range 5-50 Hz) white noise signals with zero mean and of r.m.s. level 10 and 40, respectively. With the same parameter values as in the periodic input case, Fig. 6 demonstrates the random features of both the input and responses of the system. Based on the simulated data and Eqs. (28)-(30), the integrated coefficients were obtained by using the procedures similar to those taken in the previous periodic cases. Then, the damping estimates of the noise-free, random-excited data were obtained, which were: $\tilde{c}_{11}^{*} = 20.0002$, $\tilde{c}_{12}^{*} = 20.0010$, $\tilde{c}_{23}^{*} = 19.9990$, $\tilde{c}_{33}^{*} = 20.0036$, $\tilde{F}_k = 0.999995$. In addition, the mean of the normalized absolute residuals was 9.37*e*-6, whereas the MSE values obtained in this case were $\mathbf{MSE}_1 = 0.029\%$, $\mathbf{MSE}_2 = 0.047\%$, $\mathbf{MSE}_3 = 0.091\%$.

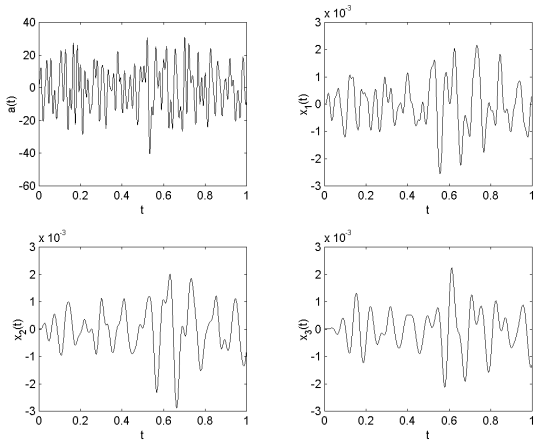

Figure 6 The input and responses of the 3-DOF vibration system subjected to general input.

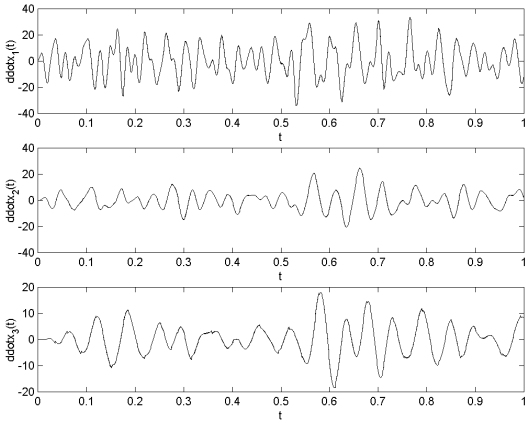

Figure 7

Obviously, the estimates obtained in the general-input case are more accurate than those of the periodic case. The statement can be verified by directly comparing the estimates obtained here to those associated with Table 2. Other evidence can be observed in Figure 7 in which indistinguishable features between the estimated and simulated acceleration are presented. The better estimates from general input might be reasonable since the band-passed white noise input has broader (and higher) bandwidth than that of the periodic one. It is conceivable that such excitation can excite richer responses from the system. Thus, more information is provided from which the damping

parameters are extracted. Nonetheless, the accuracy associated with the periodic input is also very encouraging.

Similar to the previous cases, we added to the simulated force input, velocity and displacement responses a white random noise of 5% of their individual root-mean-squares values. The resultant damping estimates obtained from these contaminated data were $\tilde{c}_{11}^{*} = 19.9986$, $\tilde{c}_{12}^{*} = 20.0010$, $\tilde{c}_{23}^{*} = 19.9990$, $\tilde{c}_{33}^{*} = 20.0004$, $\tilde{F}_{k} = 0.99995$, while the mean of the normalized absolute residuals was 2.32*e*-6. The MSE values obtained in this case were $\mathbf{MSE}_1 = 0.037\%$, $\mathbf{MSE}_2 = 0.074\%$, $\mathbf{MSE}_3 = 0.107\%$. The results indicate that the band-passed random input is almost free from the effects of the added white noise. Therefore, the accuracies of the estimates and the associated MSE values are more-or-less unaltered after the signals were superimposed with random noise.

4. CONCLUSION

In this paper the energy-dissipation method is extended to identify damping parameters from multi-DOF vibration systems. The method balance the input energy against the energy dissipated by a theoretical friction model, consisting of viscous and Coulomb components. Through numerical investigations, the method has been shown to be effective and reliable in estimating damping parameters from clean and contaminated signals. In comparison with those of the periodic input, the estimation accuracy is better if a band-passed white noise is adopted as the excitation. In addition, the method is robust when high frequency random noise is present in the measurements. This is especially true when the band-passed random input is of concern. The method is also applicable in the input-free situation, which will be adopted in the experimental investigations. The experimental study is currently undergoing.

5. ACKNOWLEDGEMENT

This research is supported by National Science Council, Taiwan, Republic of China, under grant number NSC-94-2212-E-131-002.

6. REFERENCES

[1]. J. P. Den Hartog, Forced vibration with combined Coulomb and viscous damping, *Transactions of the American Society of Mechanical Engineering* 53 (1931) 107-115.

[2]. M. S. Hundal, Response of a base excited system with Coulomb and viscous friction, *Journal of Sound and Vibration* 64 (1979) 371-378.

[3]. J.-W. Liang and B. F. Feeny, Identifying Coulomb and viscous friction in forced dual-damped oscillators, *Journal of Vibration and Acoustics* 126 (1) (2004) 118-125.

[4]. R. A. Ibrahim, Friction-induced vibration, chatter, squeal, and chaos: part I- mechanics of friction, Friction-Induced Vibration, Chatter, Squeal, and Chaos, *ASME Proceedings* DE-49 (1992) 107-121.

[5]. B. Armstrong-Helouvry, P. Dunpont, C. Canudas De Wit, A survey of models, analysis tools and compensation methods for the control of machines with friction, *Automatica* 30(7) (1994) 1083-1138.

[6]. J. T. Oden, J. A. C. Martins, Models and computational methods for dynamic Friction phenomena, *Computer Mechanics in Applied Mechanics and Engineering* 52(1-3) (1985) 527-634.

[7]. J.-W. Liang, B. F. Feeny, Dynamical friction behavior in a forced oscillator with a compliant contact , *Journal of Applied Mechanics* 65(1) (1998) 250-257.

[8]. N. Hinrichs, M. Osetreich, K. Popp, On the modeling of friction oscillator, *Journal of Sound and Vibration* 216(3) (1998) 435-459.

[9]. D. A. Haessig, B. Friedland, On the modeling and simulation of friction, *Journal of Dynamic Systems, Measurement and Control* 113 (1991) 354-362.

[10]. J.-W. Liang B. F. Feeny, Balancing energy to estimate damping parameters in forced oscillators, *Journal of Sound and Vibration* (in press).

[11]. K. S. Mohammad, K. Worden, G. R. Tomlinson, Direct parameter estimation for linear and non-linear structures, *Journal of Sound and Vibration* 152(3) (1992) 471-499.

[12]. Q. Chen, G. R. Tomlinson, Parametric identification of systems with dry friction and non-linear stiffness using a time series model, *Journal of Vibration and Acoustics* 118 (1996) 252-263.

[13]. R. Ghanem, F. Romeo, A wavelet-based approach for model and parameter identification of non-linear systems, *International Journal of Non-Linear Mechanics* 36 (2001) 835-859.

[14]. S.-L. Chen, H.-C. Lai, K.-C. Ho, Identification of linear time varying systems by Haar wavelets, submitted to *Int. J. System Sci*.

[15]. J.-W. Liang, On-line identification via Haar wavelet expansions, submitted to *International Journal of Adaptive Control and Signal Processing*.

[16]. W. T. Thomson, M. D. Dahleh, *Theory of Vibration with Applications*, Prentice-Hall, New Jersey.

[17]. P. Bogacki, L. F. Shampine, A 3(2) pair of Runge-Kutta formulas, *Applied Mathematics Letters* 2 (1989) 1-9.

[18]. H. K. Khalil, *Nonlinear Systems*, Prentice-Hall, New Jersey.

[19]. J.G.F. Francis, “The QR Transformation I,” *Comput. J.*, vol. 4, pp. 265-271, 1961.