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VIBRATION ATTENUATION IN ROTATING BEAMS WITH PERIODICALLY DISTRIBUTED PIEZOELECTRIC CONTROLLERS

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Abstract

Periodic beams have proven to possess vibration attenuation properties that could be tailored to cover specific frequency bands. Experiments were performed on the properties of rotating periodic beams and were highly encouraging to proceed for further investigations. On the other hand, piezoelectric patches bonded to the surface of a vibrating structure have shown great potential for vibration damping when shunted to a passive RL circuit that dissipates the electric energy or connected to a closed loop control system.

In the present study, combining the characteristics of periodic rotating beams to those of passively shunted piezoelectric patches to damp the vibration of a rotating Euler-Bernoulli will be introduced using finite element analysis.

The results proved the ability of the piezoelectric pairs to damp the vibrations of certain frequency band. Moreover, the periodic nature of the structure introduces vibration attenuation in other frequency ranges.

INTRODUCTION AND LITERATURE SURVEY

The term "Periodic Structure" is used to describe structures that consist of a set of identical parts, cells, connected together. Periodic Structures have drawn the attention of researchers since the mid-sixties [1-6] because of their high ability to attenuate vibrations. Meanwhile, special attention is given to rotating beams [7-13] as rotating beams had wide range of engineering applications. On the other hand, the controlled vibrations [14-25] received great attention due to attractive dynamic characteristics of the controlled structures. Attempts were made to use the advantages of both periodic structures and piezoelectric patches in field of controlled vibration attenuation [16, 18 and 25]. In this paper, we will present an attempt to demonstrate the ability of periodic piezoelectric patches with shunted circuits to attenuate vibration.

ROTATING BEAM WITH SHUNTED PIEZOELECTRIC PAIRS DISTRIBUTED PERIODICALLY

Deriving weak form from Hamilton Principle

Hamilton's Principle states that:

$$\delta\Pi = \delta\int_{t_1}^{t_2}(U - T - W)dt = 0 \tag{1}$$

Where *Π, U, T* and *W* are total energy, the potential energy, kinetic energy and external work respectively. *δ(.)* denotes the first variation.

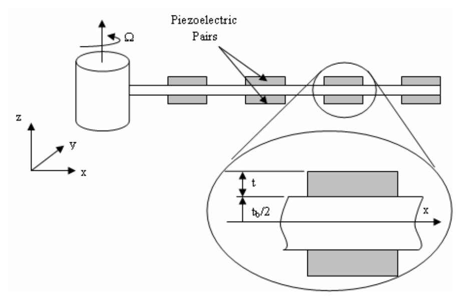

Figure 1: Rotating beam with shunted piezoelectric pairs distributed periodically

The variation of the potential energy for the beam shown in Figure 1 and using piezoelectric constitutive relations [25] in term for potential energy, one gets:

$$\delta U = \int_0^L Q^D I_y\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 \delta w}{\partial x^2}\right)dx + \int_{Vol} zh\delta D\frac{\partial^2 w}{\partial x^2}dV + \int_{Vol} zhD\frac{\partial^2 \delta w}{\partial x^2}dV + \beta\int_{Vol} D(\delta D)dV \tag{2}$$

Where Q^D, *Iy, w, D* and *β* are the beam modulus of elasticity, moment of inertia, mechanical displacement, the electric displacement, and the reciprocal of dielectric constant, respectively. The kinetic energy may be given by:

$$\therefore \delta T = \int_0^L \rho A\left(\frac{\partial w}{\partial t}\right)\left(\frac{\partial \delta w}{\partial t}\right)dx \tag{3}$$

Where ρ, and A are the mass density of the material and beam cross-sectional area respectively. The variation of external work exerted on the system is caused by three sources; the shunt circuit, the external force and the beam rotation. [22-25]:

$$\delta W = \int_A \delta D\left(-L\ddot{D}A - R\dot{D}A\right)dA + \int_A \delta w F dA - \int_0^L f_C(x)\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial \delta w}{\partial x}\right)dx \tag{4}$$

Where L, and R are the shunt circuit inductance and resistance respectively. F is external force respectively. f_c is the centrifugal force due to rotation.

Applying Finite Element model to Hamilton's Principal

The above expressions can be written in the form:

$$\{\delta W_b\}^T\left([m_b]\{\ddot{W}_b\} + [k_b]\{W_b\} + [k_{bD}]\{W_D\} + [S]\{W_b\} - f\right) = 0 \tag{5}$$

$$\{\delta W_D\}^T\left([m_D]\{\ddot{W}_D\} + [C_D]\{\dot{W}_D\} + [k_{Db}]\{W_b\} + [k_D]\{W_D\}\right) = 0 \tag{6}$$

Where:

$$\begin{gathered}
[m_b] = \int_0^L \rho A \lfloor N_w(x)\rfloor^T \lfloor N_w(x)\rfloor dx\{\ddot{W}_b\}dx,\ [k_b] = \int_0^L Q^D I_y \lfloor N_{w,xx}(x)\rfloor^T \lfloor N_{w,xx}(x)\rfloor dx, \\
[k_{bD}] = \int_{Vol} zh \lfloor N_{w,xx}(x)\rfloor^T \lfloor N_D(x)\rfloor dV,\ [S] = \Omega^2 \int_0^L f_c(x) \lfloor N_{w,x}(x)\rfloor^T \lfloor N_{w,x}(x)\rfloor dx, \\
f = \int_A \lfloor N_w(x)\rfloor^T F dA
\end{gathered} \tag{7}$$

$$\begin{gathered}
[m_D] = AL\int_A \lfloor N_D(x)\rfloor^T \lfloor N_D(x)\rfloor dA,\ [C_D] = AR\int_A \lfloor N_D(x)\rfloor^T \lfloor N_D(x)\rfloor dA \\
[k_{Db}] = \int_{Vol} zh \lfloor N_D(x)\rfloor^T \lfloor N_{w,xx}(x)\rfloor dV,\ [k_D] = \beta \int_{Vol} \lfloor N_D(x)\rfloor^T \lfloor N_D(x)\rfloor dV
\end{gathered} \tag{8}$$

Next, arrange expression (5) and (6) in matrix form:

$$\begin{bmatrix} [m_b] & 0 \\ 0 & [m_D] \end{bmatrix}\begin{Bmatrix} \ddot{W}_b \\ \ddot{W}_D \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [C_D] \end{bmatrix}\begin{Bmatrix} \dot{W}_b \\ \dot{W}_D \end{Bmatrix} + \left(\begin{bmatrix} [k_b] & [k_{bD}] \\ [k_{Db}] & [k_D] \end{bmatrix} + \begin{bmatrix} [S] & 0 \\ 0 & 0 \end{bmatrix}\right)\begin{Bmatrix} W_b \\ W_D \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{9}$$

The above matrix equation is the rotating sandwich element equation. The equations of left element, (base material only) obtained by eliminating W_D.

Transfer Matrix Analysis for Cell

Since the structure is periodic, the pass-stop bands can be obtained by studying the equations of one cell only. The cell matrices are constructed by assembling the individual element matrices providing that mechanical degrees of freedom of both elements are followed by electrical degrees of freedom of sandwich element. The transfer matrix analysis is used to derive a direct relation between the forces and displacements at left end of cell, input, and the right end of the cell, output. This analysis is applied on cell dynamic stiffness matrix defined by:

$$K_{Dynamic} = (K + S) - \omega^2 M \tag{10}$$

Where *K*, *S* and *M* are the stiffness, motion-induced stiffness and mass matrices respectively. Since the dynamic stiffness matrix contains electrical degrees of freedom, they should be condensed first before applying the transfer matrix analysis. Putting the cell dynamic stiffness matrix on form:

$$\left(\begin{bmatrix} [k_b] & [k_{bD}] \\ [k_{Db}] & [k_D] \end{bmatrix}\right)_{Dynamic} \begin{Bmatrix} W_b \\ W_D \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{11}$$

Where k_b, k_D, and k_{bD}, and k_{Db} are the bending stiffness, electrical stiffness electro-mechanical coupling matrices respectively. The above equation can be rewritten as:

$$\left(k_b - k_{bD}(k_D)^{-1} k_{Db}\right) W_b = f \tag{12}$$

The above relation will be considered the starting point for the transfer matrix analysis as follows; Consider the cell sketched in Figure 2. It is required to find the input/output relation that takes the form:

$$\begin{Bmatrix} W_5 \\ f_5 \end{Bmatrix} = e^{\mu} \begin{Bmatrix} W_1 \\ -f_1 \end{Bmatrix} \tag{13}$$

Where $W_i^T = \{w_i \quad w_i'\}$, $f_i^T = \{F_i \quad M_i\}$, *i =1, 5*. For ease of demonstration, 3-node elements are assumed.

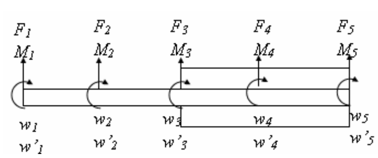

Figure 2: Sketch of forces applied on one cell

Equation (12) can be written as:

$$\begin{bmatrix} [K_{11}] & [K_{12}] & [K_{15}] \\ [K_{21}] & [K_{22}] & [K_{25}] \\ [K_{51}] & [K_{52}] & [K_{35}] \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \tag{14}$$

Now the displacement W_2, which include all internal nodes, will be condensed utilizing the second equation in the above matrix equation with f_2={0}. After some mathematical manipulations, the above equation can be written as:

$$\begin{bmatrix} \overline{K}_{11} & \overline{K}_{12} \\ \overline{K}_{21} & \overline{K}_{22} \end{bmatrix} \begin{Bmatrix} W_1 \\ W_5 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_5 \end{Bmatrix}, \quad \begin{matrix} \overline{K}_{11} = K_{11} - K_{12}K_{22}^{-1}K_{21}, & \overline{K}_{12} = K_{15} - K_{12}K_{22}^{-1}K_{25} \\ \overline{K}_{21} = K_{51} - K_{52}K_{22}^{-1}K_{21}, & \overline{K}_{22} = K_{55} - K_{52}K_{22}^{-1}K_{25} \end{matrix} \tag{15}$$

Matrix equation (15) will be converted into input/output form as described in equation (13):

$$\begin{bmatrix} -\overline{K}_{12}^{-1}\overline{K}_{11} & \overline{K}_{12}^{-1} \\ -\overline{K}_{21} + \overline{K}_{22}\overline{K}_{12}^{-1}\overline{K} & -\overline{K}_{22}\overline{K}_{12}^{-1} \end{bmatrix} \begin{Bmatrix} W_1 \\ f_1 \end{Bmatrix} = e^{\mu} \begin{Bmatrix} W_1 \\ f_1 \end{Bmatrix} \tag{16}$$

The above equation represents an eigenvalue problem. It can be proven that each of eigenvalue is reciprocal of the other. To calculate the propagation factor μ the hyperbolic relation $\mu = \cosh^{-1}\left(0.5\left(e^{\mu} + e^{-\mu}\right)\right)$ may be used. Generally speaking, the propagation factor is a complex number. Its imaginary part represents the phase shift between the input and output. While the real part identifies the stop bands boundaries and attenuation factor.

Numerical Results

In order to attenuate the vibration of beam under vibration, the shunt circuit should be tuned to one of the beam natural frequencies. Since the geometric periodicity in beam configuration provides pass-stop bands, Figure 3, which attenuate some frequency ranges, the selected tuning frequency should lie in a pass band. The RLC (resonating) shunt circuit is tuned such that the selected tuning frequency is selected to be equal to a natural frequency of the beam. To demonstrate shunt circuit tuning and the corresponding attenuation, a beam made of aluminium of mass density (ρ) 2700 kg/m^3, and Young's modulus of elasticity (E) 71 GPa is selected. The beam has length 45cm, width 3.6cm and thickness 1mm. The geometric periodicity is introduced by dividing the beam to four identical cells of length 11.25cm. The length of the thin element (to the left) is 8.25cm while the thick element (to the right) is 3cm long. The thick element consists of base material (aluminium) and two identical piezoelectric patches bonded on both sides. The patches are 3.6cm wide, 3cm long and 1mm thick each. The mechanical and electrical properties of a patch are:

Modulus of elasticity E 68 GPa, mass density ρ 7800 kg/m^3, dielectric constant ∈ 2.37×10^{-8} Farad/m and piezoelectric constant d_{31} -320×10^{-12} m/V. Despite the existence of the piezoelectric pairs, the beam is exited at root and response is calculated at its tip. The results were validated by comparison with numerical and experimental results published in reference [22].

Figure 3 shows that the numerical results obtained for the frequency response of the plain and periodic beams without rotation together with the attenuation factor. Now, the natural frequency of 1024 Hz is selected as a tuning frequency for the shunt circuits but this requires investigating the optimal tuning conditions. Since the shunted piezoelectric patch slightly lowers the beam stiffness, the selection of tuning frequency should account for this reduction. Thus the optimum resistance and tuning frequency were found 1770 Ohm and 988Hz respectively. Figure 4 shows the tip response at optimum tuning conditions. It is clear from the figure that the peak at 1024Hz disappeared and extra stop band appeared. This stop band connects the two neighbouring stop bands form one wide stop band that significantly attenuate all response in that range.

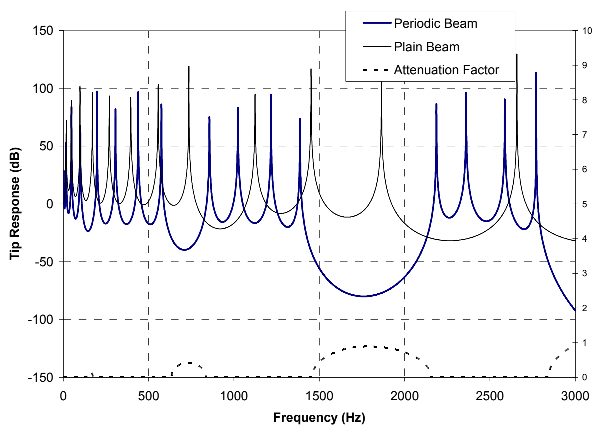

Figure 3: Periodic beam vs. plain beam (open circuit)

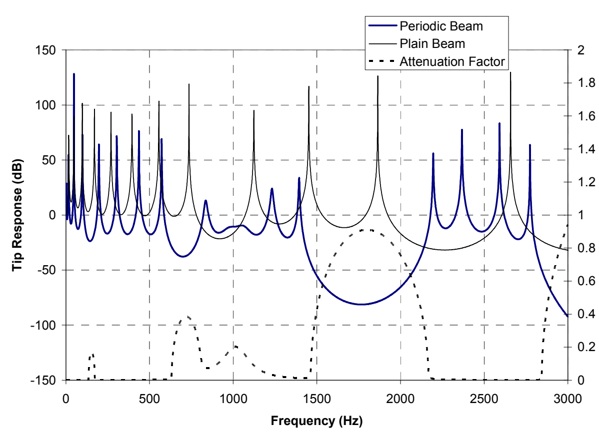

Figure 4: Periodic beam at optimum conditions (Freq.=988Hz, R=1770 Ohm , Non-rotating)

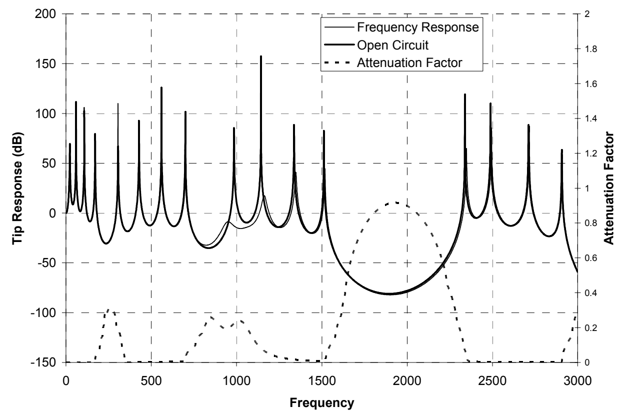

Figure 5: Periodic Beam open circuit and optimum shunt circuit modes Rotation Speed=20Hz (Freq.988Hz, R=1700 Ohm, Hub Radius=0)

At the current optimum tuning conditions, the beam rotational speed is increased, assuming zero hub-radius, to study how the rotation affects stop-pass bands. Figure 5 shows average attenuation factor, and the tip response for the beam under rotating conditions (20Hz). It is clear, from the figure, as the rotational speed increases, wider stop bands are generated with less attenuation levels. Also the stop bands, and natural frequencies moved to higher frequency values. This is due to the increase in the beam stiffness due to centrifugal force induced by rotation.

CONCLUSIONS

Form the results shown above; it is clear that geometrical periodicity has high ability to attenuate the vibration in some frequency bands (stop bands). Moreover, the passive shunting of the piezoelectric controllers with proper tuning introduces new stop band around the tuning frequency which increases vibration attenuation. Also, the increase of the rotational speed of the beam, the stop bands get wider but less attenuation factor is obtained. This leads to less attenuation level in the beam but increasing the frequency bands attenuated.

The finite element code proved good agreement with published results, for all running conditions. Also, the selected 3-node, 6-mechanical DOF proved good results.

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