ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

DAMPING ACROSS THE LENGTH SCALES

Veturia Chiroiu[1], Ligia Munteanu[1]

[1]Institute of Solid Mechanics, Romanian Academy
Ctin Mille 15, P O Box 1-863, Bucharest 010141, Romania
veturiachiroiu@yahoo.com (e-mail address of lead author)

Abstract

The damping across the length scales (the nanoscopic scale $10^{-10}-10^{-9}\,\mathrm{m}$, mesoscopic scale $10^{-8}-10^{-6}\,\mathrm{m}$, and macroscopic scale $>10^{-5}\,\mathrm{m}$), is investigating by considering the simplest theoretical instrument for the study of material damping - the pendulum. The free transverse vibrations of a pendulum consisting of a bob suspended from a wire are studied in two cases: (1) a molecule suspended from a carbon nanotube with a single wall, and (2) a bob suspended from a rod with microstructure.

1.INTRODUCTION

Eliminating vibrations and controlling dynamics in macro-, meso- and nano-world are subjects must addressed to applications in macro-electromechanical, micro-electromechanical and nano-electromechanical structures. One of the major obstacles to miniaturization and to build useful micro- and nano-sized devices, involves changes to mechanical properties that can occur as the size of a system are below the macroscale toward the atomic scale.

Also, it is of special interesting the fact it is not possible to grow materials without dislocations and defects to crystalline order, such as vacancies, interstitials, or impurities at the meso and nano scales. The damaged materials exhibit interesting damping properties.

In this paper we study the damping across the nanoscopic scale $10^{-10}-10^{-9}\,\mathrm{m}$, mesoscopic scale $10^{-8}-10^{-6}\,\mathrm{m}$, and macroscopic scale $>10^{-5}\,\mathrm{m}$, by considering the simplest instrument for the study of material damping, the pendulum [8]. We refer to internal damping, which results from mechanical-energy dissipation within the material due to various microscopic and macroscopic processes.

Internal damping of materials originates from the energy dissipation associated with microstructure defects, such as grain boundaries and impurities.

2.TRANSVERSAL VIBRATIONS OF A PENDULUM

Properties of materials are the result of phenomena at various length scales and require accurate modeling to capture the mechanics and physics at each length-scale.

For nanoscopic and mesoscopic scales, the quasi-continuum method can be applied as an approximation theory to atomistic, which reduces to the exact atomistic theory when all the atomic degrees of freedom are considered [2, 9, 10]. Also, by reducing the number of the degrees of freedom for sets of identical atoms, the method is able to deal with mesoscopic structures such as cracks, grain boundaries and free surfaces in a uniform fashion.

In this formulation we determine the total potential energy of a system as a function of the degrees of freedom (atom or FE nodal locations). Static equilibrium is obtained by minimizing the total energy. In dynamic simulations, the force is used in Newton's second law to evolve the system in time.

The total atomic energy is a sum of the atoms energies

$$E_{tot} = \sum_{i=1}^{N} E_i \ .$$

Consider now the problem of free transverse vibrations of a vertical pendulum consisting of a bob suspended from a stiff wire. We study this problem in two cases:

1. a molecule suspended from a carbon nanotube with a single wall;
2. a bob suspended from a rod with microstructure.

For the first problem we apply the quasi-continuum method, and for the second the classical theory.

A carbon nanotube is a cylindrical molecule composed of carbon atoms. The diameter of nanotube is 100000 times less that the diameter of the sawing needle. The carbon nanotubes are 100 times much stronger than steel wire, are the perfect conductor, and have thermal conductivity better than diamond. The application of these nanotubes, formed with a few carbon atoms in diameter, provides the possibility to fabricate devices on an atomic and molecular scale, with lightweight and high strength properties. A typical section of a single-walled carbon nanotube is illustrated in Fig. 2.1, each node being a carbon atom and lines the chemical bonds [7]. Fig. 2.2 represents a damaged carbon nanotube with missing atoms in a region of the wall [1].

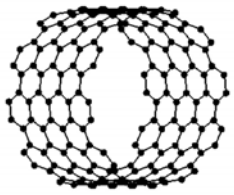

Fig. 2.1. A section through a carbon nanotube viewed from the side [1].

For the carbon nanotube the modified Morse potential function is used

$$E = E_{stretch} + E_{angle}\,, \tag{2.1}$$

where $E_{stretch}$ is the bond energy due to bond stretch

$$E_{strech} = D\{[1-\exp(-\beta(r-r_0)]^2 - 1\}, \tag{2.2}$$

and E_{angle} is the bond energy due to bond angle-bending,

$$E_{angle} = \frac{1}{2}k_\theta(\theta-\theta_0)^2[1+k_s(\theta-\theta_0)^4], \tag{2.3}$$

where r is the length of the bond, and θ is the current angle of the adjacent bond, a standard deformation measure in molecular mechanics. The modelling of vibrations of the nano pendulum is based on (2.1)-(2.3), and the quasi-continuum method with a reduced degree of freedom.

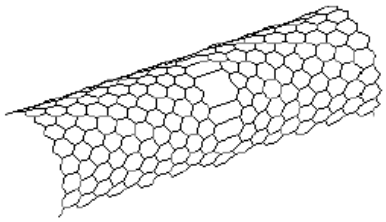

Fig. 2.2. A damaged carbon nanotube by missing atoms [1].

For the macro pendulum with mesoscopic damage, we use this motion equation in a dimensionless form

$$\chi u^{iv} - [\{\alpha(1-x)+1\}u']' + \alpha\ddot{u} = 0,\ 0 \le x \le 1,\ 0 \le t \le 1, \tag{2.4}$$

where x and t are the dimensionless space and time variables, $u(x,t)$ is the dimensionless displacement of the pendulum, χ is the dimensionless stiffness, and α is the mass ration of the wire to the bob. To represent the hysteretic damping we use a complex stiffness, consisting of a real part that correspond to the usual elastic stiffness and an imaginary part that corresponds to the hysteretic loss stiffness $\chi = \chi_1 + i\chi_2$. The boundary and initial conditions are

$$u(0,t) = u'(0,t) = u''(1,t) = u'''(1,t) = 0, \tag{2.5}$$

$$u(x,0) = f(x),\ \dot{u}(x,0) = g(x). \tag{2.6}$$

This problem (2.4)-(2.6) is investigated by employing the eigen-value approach with Laplace and Fourier transformations. The significant amounts of damping are possible at macro scale, by adding the external damping treatments. The damping levels are smaller at microscale, and may be much smaller at nanoscale. But, by knowledge of properties of nanostructures, we can control the wave motion and enhance energy dissipation.

The stress σ and strain ε relations at a point in a vibrating continuum possess a hysteresis loop. The area of the hysteresis loop gives the energy dissipation per unit

volume of the material, per stress cycle. At the nanoscale, we try to find a class of constitutive laws by applying the pseudospherical reduction method for a mechanical problem [6]. By this reduction the motion equations are associated to a pseudospherical surface Σ (with negative Gaussian curvature K). If the ratio K/d^4 is constant, where d is the distance from the origin to the tangent plane at an arbitrary point is constant, we obtain a Titeica surface [11, 12].

The Titeica surfaces are invariants under the group of centroaffine transformations, being analogues of spheres in affine differential geometry.

At present, only a few achievements have been made for determining the parametrical representation for a class of constitutive laws at the nanoscale level, for which the motion equations attached to the material system can be associated to a pseudospherical surface. We consider that this novel approach may improve the estimation of damping at different metric scales.

The Titeica function associated to Titeica surface estimates the specific damping capacity D ($D=\dfrac{\Delta U}{U_{\max}}$, where the initial energy of the system is denoted by $U_{\max}$). The ratio K/d^4 is proportional to D.

3.RESULTS

In all simulations, the dimensionless space $0\le x\le 1$ and time $0\le t\le 1$ variables are used Consider first the nanopendulum, e.g. a single-walled nanotube of $L=0.58\mu\text{m}\,(0.58\times10^{-6}\,\text{m})$ length, with a diameter of d_t satisfying $k_1=\dfrac{L}{d_t}=10$. The molecule is modeled as a rigid sphere of diameter d_b given by $k_2=\dfrac{d_b}{d_t}=5$. The mass ration of the wire to the bob is $\alpha=3$. The parameters are

$$r_0=1.39\times10^{-10}\,\text{m},\ D=6.03\times10^{-19}\,\text{Nm},\ \beta=2.63\times10^{10}\,\text{m}^{-1},\ \theta_0=2.09\text{rad},$$
$$k_\theta=0.9\times10^{-18}\,\text{Nm/rad}^2,\ k_s=0.75\text{rad}^{-4}.$$

In Fig. 3.1 it is represented the Titeica function in term of the distance d from the origin to the tangent plane at an arbitrary point of the associated Titeica surface, and k_1 ($\alpha=3$, $k_2=5$). This function characterizes the capacity of damping for nanopendulum. The ratio K/d^4 is proportional to D.

.The time variation of dimensionless displacement of the bob suspended from a single-walled carbon nanotube is represented in fig.3.2. We see that the nonlinear free decay vibrations is not a pure exponential [3-5]. For a non damaged nanotube the decay is nonlinear, and after vibrations died out, the equilibrium position remains unchanged. The decay is due only by internal damping.

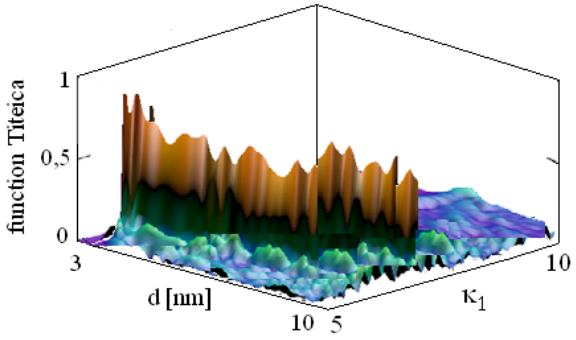

Fig.3.1. Titeica function for nanopendulum.

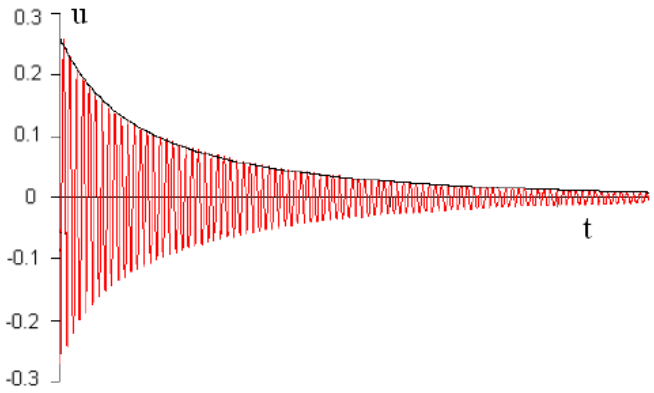

Fig. 3.2. Nonlinear free decay vibrations of a molecule suspended from a single-walled carbon nanotube.

The time history of the quality factor corresponding is obtained from Titeica function, and it is represented in fig. 3.3.

We see from fig.3.3 that in first part of the decay, the Q increases from below 4 toward the maximum of about 12. In the second part, the Q decreases from 12 toward zero.

The quality factor Q for a pure exponential decay is constant. Here, the Q becomes time-dependent. The mode coupling is significant here. When a pair of modes couple because of elastic nonlinearity, the strength of the coupling is proportional to the product of the individual amplitudes of the pair. Consequently, variability in Q can influence the evolution of vibrations.

Next we consider that the nanotube is damaged by missing of some atoms in a portion of the wall (fig. 2.2). Fig. 3.4 represents the free decay vibrations of the system in this case.

We see that the equilibrium position of the pendulum is shifting during vibrations. After vibrations died out, the equilibrium position remains shifted. We must remark that a similar behaviour was experimentally observed on different macroscopic scales [3-5].

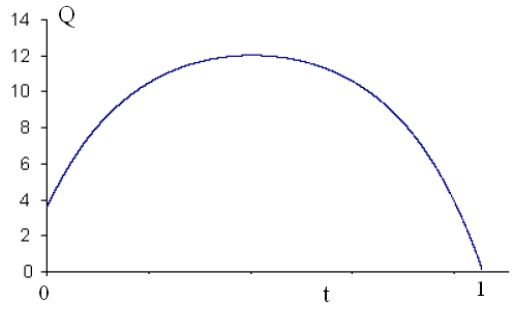

Fig. 3.3. The time variation of the quality factor for a non-damaged nano pendulum.

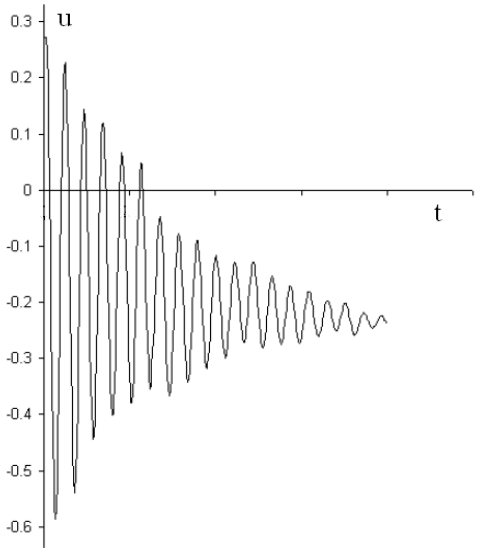

Fig.3.4. Free decay vibrations of the molecule suspended from a damaged single-walled carbon nanotube

Consider now the second problem of a macroscopic vertical pendulum consisting from an elastic circular cross section rod of $L = 5\text{cm}$ length, with a diameter of d_t satisfying $\frac{L}{d_t} = 10$. The bob is modeled as a rigid sphere of diameter d_b given by $\frac{d_b}{d_t} = 5$. The mass ration of the wire to the bob is $\alpha = 3$.

The parameters for aluminum are: Young modulus 70.3GPa, Poisson ratio 0.345, density 2690 kg/m^{-3}. The time variation of dimensionless displacement of the pendulum is represented in fig.3.4. The free decay vibrations is also nonlinear.

The shape of histories are different from the case of the nanopendulum, but the conclusion is the same - a nonlinear vibrations decay and the time-dependent quality

factor Q. The case of a macroscopic damaged macropendulum is too difficult to be treated without a coupled macro-mesoscopic theory.

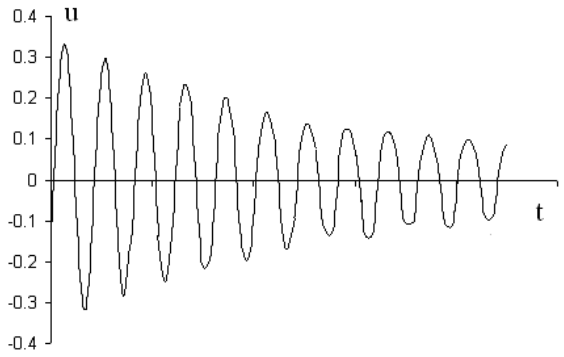

Fig. 3.4. Free decay vibrations of the macroscopic pendulum.

4.CONCLUSIONS

The free transverse vibrations of a pendulum consisting of a bob suspended from a wire are studied in two cases: (1) a nanopendulum, and (2) a macroscale pendulum with microstructure.

At the nanoscale, the damping analysis is performed by using a novel approach, e.g. the Titeica function associated to the Titeica surface corresponding to the motion equation of nanopendulum.

The Titeica function is expressed as a function on the distance d from the origin to the tangent plane at an arbitrary point of the associated Titeica surface, and the pendulum parameters $k_1 = \frac{L}{d_t}$, $k_2 = \frac{d_b}{d_t}$ and α, where L is the length of the single-walled nanotube, d_t is its diameter, d_b is the diameter of the molecule modeled as a rigid sphere, and α is the mass ration of the wire to the molecule.

This function estimates the specific damping capacity $D = \frac{\Delta U}{U_{\max}}$, where $U_{\max}$ is the initial energy of the nanopendulum.

The damping of macropendulum is analysed by classical theory. From this analysis it results that he decay of vibrations is not a pure exponential for both problems, and the quality factor Q becomes time-dependent.

An interesting result is obtained for a damaged nanopendulum. Its equilibrium position is shifting during vibrations. After vibrations died out, the equilibrium position remains shifted.

As a conclusion, vibrational responses of the pendulum can be, especially for damaged materials, very complex, and cannot be explained by classical damping

models. It is clear from here that the pendulum is still important to engineering, in a fundamental sense of understanding of properties of materials.

Acknowledgments. The authors acknowledge the financial support of the National University Research Council (NURC-CNCSIS) Romania, Grant nr. 33344/2005, code 152/2003, and Grant nr. 27664/2005, code160/2005

REFERENCES

[1] Belytschko, T., Xiao, S.P., Schatz, G.C., Ruoff, R., "Atomistic simulations of nanotube fracture", Northwestern Univ., Evanston, report. 1-27 (2000).
[2] Chiroiu, V., Ştiucă, P., Munteanu, L., Donescu, St. (2005). *Introduction to nanomechanics* (Editura Academiei, Bucharest 2005).
[3] Peters, R., "Model of internal friction damping in solids", Mercer Univ., Macon, Georgia, Dept. of Physics, internal report (2002).
[4] Peters, R., "Nonlinear damping of the linear pendulum", Mercer Univ., Macon, Georgia, Dept. of Physics, internal report (2003).
[5] Peters, R, "Damping", Mercer Univ., Macon, Georgia, Dept. of Physics, internal report (1990).
[6] Munteanu, L, Donescu, St., *Introduction to Soliton Theory: Applications to Mechanics,* Book Series "Fundamental Theories of Physics", vol.143 (Kluwer Academic Publishers, 2004).
[7] Ruoff, R.S., Qian, D., liu, W.K., "Mechanical properties of carbon nanotubes: theoretical predictions and experimental measurements", Comptes Rendus Physique, 4, 993–1008 (2003)
[8] Teodorescu, P.P., Chiroiu, V., Munteanu, L., "Damping across the length scales", Int. Symp. "Dissipation pf energy, acoustical, vibrational ans seismical processes", Bucharest (2005).
[9] Teodorescu, P.P., Chiroiu, V., Munteanu, L., "The pseudospherical reduction of an uniaxial deformation problem for the carbon nanotubes", The 5-th Conf. Balkan Society of Geometers, August 29-September 2, Mangalia (2005).
[10] Teodorescu, P.P., Chiroiu, V., Munteanu, L., "On the soliton mechanism of bending for carbon nanotubes", Int. Conf. on Advanced Materials - ICSAM 2005, 15-17 sept. 2005, Bucharest, 151-156 (2005).
[11] Titeica, G., *Géométrie projective différentielle des réseaux* (Editura Cultura Natională, Bucharest and Gauthier-Villars, Paris 1924).
[12] Titeica, G., "Sur une nouvelle classe des surfaces", C. R. Acad. Sci., Paris 150, 955–956 (1910).