

# EFFECT OF BEARING CLEARANCE ON THE DYNAMIC BEHAVIOR OF A MECHANICAL PRESS WITH VARIABLE ANGULAR VELOCITY

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### Abstract

The dynamic behaviors of a mechanical press with a variable speed are more complicate than that of a press with a constant speed. In this work, the effect of the bearing clearance on the dynamic behaviors of a mechanical press with variable angular velocity was investigated. The results show that the bearing clearance may induce serious impact problem for most applications without a careful design of the velocity profile. To overcome this problem, an optimization procedure was developed in this work to design a proper velocity profile of the input in order to minimize the impact force. The results show that the impact force can be effectively reduced by the proposed method with the constraint of the specified ram speed at the working region. The results also demonstrate that the stroke velocity characteristics of a mechanical press with complex linkage drive can be realized by a simple four-bar linkage with a proper design of the input velocity by the proposed method.

## **INTRODUCTION**

The conventional mechanical presses with a constant angular velocity generally are driven by a fly-wheel powered by a small servo motor. A press for cold forging and deep drawing application generally needs a stroke characteristic with high approaching and return speeds, but with constant speed in the working zone. Generally, there are two approaches to achieve such a stroke characteristic. One is to use a complex linkage drive, for instance, with drag-link drive (six-bar) [1] or double-knuckle drive (eight-bar) [2-4]. The other approach is to use a huge servo motor instead of the fly-wheel to control the angular speed of the input crank. Although use of a servo motor to control the output of a flexible structure or a flexible mechanism [5-8] is not new, use of a servo motor to control the output of a mechanical press is a new research topic. This is due to

that a servo motor with very high capacity is generally needed by a mechanical press while servo motors with high capacity are available only recently. The advantages and disadvantages of different rotary linkage drives for mechanical presses have been investigated in detail by the research group of Shivpuri [2,3]. The main concern in the works by Shivpuri [2~4] was the load-stroke characteristics of the presses, the dynamic behaviors (i.e., vibration, impact) of the presses were not investigated. The advantage of a mechanical press with four-bar linkage (i.e., slide-crank mechanism) is the simplicity in structure and the easiness for maintenance. The disadvantage of the slide-crank mechanism with a constant angular velocity of the crank is that it lacks the characteristics of constant velocity in the working zone, and consequently is not suitable for cold forging and deep drawing. Theoretically, this disadvantage can be overcome by using a servo motor to control the angular velocity of the crank [9]. However, all the links and joints were assumed to be rigid in [9], and the dynamic behaviors of the system were not investigated.

In this work, the dynamic behaviors of a mechanical press with four-bar linkage were investigated. As mentioned, the components of a mechanical press generally are considered as rigid bodies for kinetic and kinematic analyses. In this work, the flexibility of the connecting rod and the bearing clearance were considered. The purposes of this investigation are to answer the following questions:

- (1) Is there any dynamic problem when the input velocity of a press with four-bar linkage is so designed in order to simulate the output speed of a press with a complex linkage?
- (2) How can one design the input velocity of the press in order to meeting a specified output speed, but with minimum input torque?

The above questions are the main concerns when one designs a mechanical press with a simple four-bar linkage directly driven by a servo motor.

### THEORETICAL FORMULATION

The typical mechanical press with a slider-crank mechanism is shown in Fig. 1. First, we define a global coordinate system  $\overline{OXY}$  as a fixed frame. The origin O is located at the bearing between the crank shaft and the base (see Fig. 2). Because the crank shaft is considered as a rigid body, there is only one degree of freedom in the crank shaft, as indicated as  $\theta_A$ . The kinetic energy of the crank shaft can be expressed as:

where  $m_A$  and  $\ell_A$  represent the mass and length of the crank shaft.

The connecting rod is considered as an uniform Euler beam. The standard finite beam elements were used to derive the kinetic and potential energy of the connecting rod. A moving frame  $\overline{O_B X_B Y_B}$  is attached to the connecting rod, and the  $\overline{O_B X_B}$  axis passes the two end points of the connecting rod, as shown in Fig. 2. The position of the origin  $O_B$  is represented by the vector  $R_B$  with respect to the

$$\{q\}_{B} = \{\varphi_{1}, v_{2}, \varphi_{2}, \dots, \varphi_{n_{B}}, \varphi_{n_{B+1}}\}^{T}$$

Note that  $v_1$  and  $v_{n_{B+1}}$  are equal to zero because the two ends of the rod are located exactly in the  $\overline{OX}$  axis. That is why the  $\{q\}_B$  vector does not include the  $v_1$  and  $v_{n_{B+1}}$ . The kinetic and potential energy of the connecting rod can then be expressed as:

$$T_B = \frac{1}{2} \left\langle \dot{\mathcal{Q}} \right\rangle_B^H \left[ M \right]_B \left\langle \dot{\mathcal{Q}} \right\rangle_B \dots (3)$$

where  $[M]_B$  and  $[K]_B$  represent the mass and stiffness matrices of the connecting rod. It should be noted that the  $[M]_B$  and  $[K]_B$  matrices are not constant, but are function of the generalized coordinates  $\{Q\}_B$ . In other words, it is just for convenience to express the  $T_B$  and  $U_B$  in a standard quadratic form.

The kinetic energy of the slider can be expressed as:

$$T_{c} = \frac{1}{2} m_{c} \dot{X}_{c}^{2} \dots (5)$$

Therefore, the total kinetic and potential energy of the system can be found as:  $T = T_A + T_B + T_C$ ......(6)  $U = U_B$ ......(7)

Substituting the T and U into the Lagrange's Equation,

$$\frac{d}{dt} \left[ \frac{\partial T}{\partial \{\dot{Q}\}} \right] - \frac{\partial T}{\partial \{Q\}} + \frac{\partial U}{\partial \{Q\}} = F \qquad (8)$$

One obtains the equation of motion of the system as:

$$[M] \langle \dot{\mathcal{Q}} \rangle + [D] \langle \dot{\mathcal{Q}} \rangle + [K] \langle \mathcal{Q} \rangle = \{F\} \dots$$
(9)

where

$$\{Q\} = \begin{cases} \theta_A \\ \{Q\}_B \\ X_c \end{cases}, \quad \{F\} = \begin{cases} f_{\theta A} \\ \{f\}_B \\ f_c \end{cases}$$

The  $\{Q\}$  vector represents the generalized coordinates of the system, and  $\{F\}$ 

represents the generalized force vector. Note that the [M], [D] and [K] matrices are not constant. The [D] matrix is not due to the damping of the system, but due to the Coriolis and centrifugal forces which are induced by the motion of the connecting rod. Note that the generalized force  $f_c$  includes the force acting on the joint 3 and the resistance force of the slider.

As mentioned, there are clearances on joints 2 and 3. Consequently, the forces acting on joints 2 and 3 are due to the contact forces between the shaft and bearing. The contact force can be derived from the Hertz contact theory. A simplified spring and damping model proposed by Dubowsky[10] was adopted in this work. Generally, Eq. (9) can be considered as the final equation which can be used to investigate the dynamic behavior of the system when the input torque  $M_0$  and the slider resistance force  $F_0$  are given. However, the purposes of this work are to investigate the dynamic behaviors of the press when the output speed (the crank speed) is specified. When the output speed is specified, the  $X_c$ ,  $\dot{X}_c$  and  $\ddot{X}_c$  are known, and the input torque  $M_0$  should be considered as unknown. On the contrary, when the input speed is specified the output  $X_c$  and  $M_0$  should be consider as unknowns. The non-linear equation of motion was solved by the subroutine odel15s provide by the Matlab software [11].

#### **Design of input speed**

Although the velocity profile of the ram (the slider) represents the stroke characteristic of a mechanical press, it is very difficult to use the ram speed to control the servo motor. On the contrary, the servo motor generally is control by specifying the velocity of the crank shaft. Therefore, the velocity of the crank shaft is the most important design variable. Traditionally, the Bezier curve has been widely used to design a curve by specifying some control points. The Bezier curve has also been used in [9] to design the crank velocity of a slider-crank mechanism. However, the Bezier curve is not continuous at the boundary points ( $0^0$  and  $360^0$  of the crank angle). Therefore, a periodic function which is expanded as Fourier series was used in this work to design the speed trajectory of the crank shaft. If the period of the crank shaft T is given, then the angular velocity of the crank shaft  $\dot{\theta}_A$  can be approximated by:

The parameters  $r_i$ ,  $\phi_i$  are the variables to be designed. If the control points in the speed trajectory of the crank shaft are given, then one can find the parameters  $r_i$ ,  $\phi_i$  to generate the speed trajectory of the crank shaft.

### SIMULATION RESULTS AND DISCUSSIONS

Fig. 3 shows a mechanical press with six-bar linkage drive used for deep drawing. The

stroke characteristic of the press is shown in Fig. 4. One can find that the slider velocity is constant between h/H=0.1 to 0.35. That means this press is suitable for drawing or forging. Now, this stroke characteristic should be realized by the four-bar linkage shown in Fig. 1 by varying the input velocity. If the linkage is assumed as rigid bodies, and there is no clearance in the joints, one can easily find the necessary input velocity of the crank shaft in order to produce the stroke characteristic in Fig. 4. The necessary angular velocity of the crank shaft is shown in Fig. 5. With the angular velocity, one can investigate the dynamic behaviors of the press. Fig. 6 shows the force acting on the slider. The force of the system with constant angular velocity is also shown for comparison. One can find that there are some serious impulsive forces in the press with variable speed. The impulsive forces are due to impact in the bearing clearance. The bearing clearance is unavoidable in practice. The impact force induces not only the vibration and noise problems, but also reduces the lift of the bearings. This result indicates clearly that a mechanical press with a variable speed is not realizable only by replacing the fly-wheel with a servo-controlled motor. On the contrary, the dynamic behaviors of the system should be carefully re-designed. In practical design, the rigidity of the crank shaft and the connecting rod is relatively higher than the rigidity of the bearings. Therefore, the bearing and the bearing clearance are the most critical points in designing a mechanical press with a variable speed.

Except for the bearing clearance, the design of the input velocity is also an important issue in designing a mechanical press with a variable speed. As mentioned, a constant speed in the working zone is the basic requirement of a mechanical press for deep drawing. Except for the speed in the working zone, the speed in the other region can be designed freely. In other words, there are many solutions of the input speed which can meet the requirement of a specified constant speed of the slider in the working zone. Here, an example was given to show that a slight modification of the input speed could drastically reduce the impact force and the maximum torque. Fig. 7 shows the desired output velocity of a mechanical press with four-bar linkage. One can find that the slider velocity between h/H=0.3~0.7 is constant. Therefore, it is suitable for deep drawing. The necessary input velocity is shown in Fig. 8. Note that the input velocity was expressed by Eq. (10) with n=15. With the input velocity, the dynamic behaviors of the press were investigated. The normal force acting on the slider is shown in Fig. 9. One can find that there is an impulsive force around 0.2s. This is due to an impact in the bearing clearance of joint 3. To overcome the impact force, the driving torque of the motor also has a sharp peak around 0.2s. To avoid the impact or to minimize the impact force, an optimization procedure was used to find the optimal input velocity. The optimization problem can be formulated as:

Minimum 
$$\dot{M}_0 = f(r_1, \phi_1, r_2, \phi_2, \dots, r_{15}, \phi_{15})$$
  
Subjected to  $\dot{X}_c = \dot{g}(r_1, \phi_1, r_2, \phi_2, \dots, r_{15}, \phi_{15}) = Const$  for  $t_1 \le t \le t_2$ 

The  $\widetilde{M}_0$  represents the peak value of the input torque. The  $r_1, \phi_1, \dots, r_{15}, \phi_{15}$  are the Fourier coefficients in Eq. (10), which are also the design variables of the optimization

problem. The constraint of the optimization problem is the slider velocity  $\dot{X}_c$  which should be constant in the working zone (i.e.,  $t_1 \le t \le t_2$ ). The optimization problem was solved by the sequential quadratic programming method. The result is an optimal velocity profile of the crank shaft, as shown in Fig. 10. One can find the profile of the optimal velocity is just a slight modification of the original velocity profile. However, the dynamic behaviors of the press with the optimal input velocity are improved significantly. Fig. 11 shows the normal force acting on the slider with the optimal input velocity. Comparing the results of Figs. (9) and (11), one can find that the peak of the impulsive force is reduced significantly. This example indicates that the design of the input velocity is a very important issue in the design of a mechanical press with a variable speed. In this work, the coefficients of Fourier series were used to design the input velocity. The initial values of the coefficients can be obtained by assuming that the system is rigid without clearance.

# CONCLUSIONS

The conventional crank press with a constant angular velocity is not suitable for deep drawing because it lacks the characteristics of constant velocity in the working zone. The dynamic behaviors of a crank press with a variable angular velocity were investigated in this work. The dynamic equation of the press with a flexible connecting rod and bearing clearances were derived. The input variable of the system is the velocity of crank shaft. A periodic function was proposed in this work to design the optimal input velocity. From the simulation results, the following conclusions can be drawn:

- (1) The impact within the bearings is more serious in a press with a variable speed than that in a press with a constant speed. Consequently, the rigidity and the clearance of the bearing are the most important issues in the design of a press with a variable speed.
- (2) There are many solutions of the input velocity with which the output velocity can meet a specified value in the working zone. The simulation results show that a slight modification of the input velocity may drastically affect the impact force or input torque. How to design the optimal input velocity is another important issue in the design of a press with a variable speed.

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Fig.3. Press with six-bar linkage



Fig.1. Model of the investigated system



Fig.2. Coordinates systems



Fig.4. Slider velocity of the press in Fig.3



Fig.5. Angular velocity required by the four-bar linkage



Fig.6. Normal force acting on the slider



Fig.7. Slider velocity of a four-bar linkage



Fig.8. The angular velocity required by the four-bar linkage



Fig.9. Normal force acting on the slider



Fig.10. The optimal angular velocity



Fig.11. Normal force acting on the slider with the optimal angular velocity