RESOLUTION OF MULTIPATH ARRIVALS VIA APPLICATION OF BROADBAND FREQUENCY MODULATED CARRIER SIGNALS

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Abstract
This paper shows that application of sweep-spread carrier signals with large frequency gradients can provide an opportunity for accurate resolution of multipath arrivals (reflections from channel boarders, inhomogeneities, etc). Due to the property of acoustic signals to propagate with relatively low speed, each arrival comes to receiver with its individual (rather large) time delay and has its own instant frequency, which can significantly differ from instant frequencies of other multipaths. This provides an opportunity for accurate parameters determination of individual arrivals and, thus, for accurate determination of characteristics of multipath-distorted received pulses. Obviously this can be used in digital acoustic telemetry for combating against multipath decay, as well as e.g. in echo-sounding and non-distractive control for detailed evaluation of propagation medium properties.

INTRODUCTION

In digital acoustic telemetry, echo sounders and non-distractive control systems sound and ultrasound frequencies are often used in the capacity of carriers. Every of such systems require highest accuracy for determination of received signal characteristics. However, if a transmitting pulse contains a frequency constant carrier, then owing to multiple reflections from channel inhomogeneities and boarders every receive signal consist of numerous, usually time-varying multipath arrivals, which interfere with each other so that both phase and amplitude of received signals fluctuate. This makes difficult or even impossible an accurate determination of signal parameters and its synchronisation time. Exploitation of equalizing filters, which theoretically are capable of suppressing the multipath components on the receiver side, comprises an unstable solution, especially if receivers must deal with compensation for variable and extended multipath interference. Some systems exploit the DSSS spread spectrum approach for combating the effect of multipath propagation and decreasing the influence of narrow-band noises. However, in order to achieve noticeable advantage, the frequency band (necessary for the spreading of the signal) must be unacceptably large.

Alternatively acoustic pulses can contain a frequency-modulated carrier. Such signals are usually applied in radiolocation. By means of matched filtering the sound-to-noise ratio as well as receiving stability (also in presence of narrow band noises) can be substantially improved. Concerning the multipath problem, it is assumed that the frequency-modulated signal cannot completely decay in a broad frequency band; thus at least a part of signal energy comes to receiver. However, an essential disadvantage of the systems consists in the fact that the resolution of multipath components is not executed. Besides, the practically used small values of frequency gradients (within the frequency modulated pulses) do not provide any possibility for realization of such beneficial feature.

This paper shows that, if frequency-modulated pulses have rather large frequency gradients, it becomes feasible to achieve accurate resolution of received signal into a number of multipath arrivals (separation of multipaths). Here the premise is used that, due to the property of acoustic signals to propagate with relatively low speed, each multipath arrival comes to receiver with its individual (rather large) time delay and has its own instant frequency, which can significantly differ from instant frequencies of all other multipath arrivals. Hence the multipath-induced masking effect can be effectively avoided by use of traditional filters. This provides an opportunity for accurate parameters determination of individual arrivals and, in result, for accurate determination of characteristics of multipath-distorted received pulses. Obviously this approach can be used in digital acoustic telemetry for combating against multipath decay, as well as e.g. in echo sounding for detailed evaluation of propagation medium properties or in non-destructive control (ultrasonic testing) for enhanced estimation of hidden material damages or defects.

USE OF FREQUENCY-CONSTANT CARRIER

In paper [3] it is indicated that, while working in propagation medium with high level of reverberation, an application of pulses (probe, telemetry signals, etc) with frequency-constant carriers is often connected with considerable complications during processing of such pulses on receiver side. To combat the multipath interference, usually complicated receiver structures are used, e.g. equalizers, but, even advanced equalizers do not guarantee reliable work if channel properties begin to rapidly fluctuate. The use of simple structures, like traditional filters (matched filters or band-pass filters), do not provide any enhancement of signal-to-interference ratio, and thus, do not enhance the quality of received signals.

In paper [3] it is also proven that (for pulses with frequency-constant carrier) the parameter estimation error directly depends on the sum of energies of all multipath components, and the residual of matched filtering (maximum value of the residual item in the formula of matched filtering) can be written as:

$$\max(|O_{xs}(t_i)|) = \max(|O_{xc}(t_i)|) = \sum_l \frac{\alpha_l}{\alpha_0} + \frac{\sin(\omega_0 T)}{\omega_0 T} \sum_l \frac{\alpha_l}{\alpha_0}, \qquad (1)$$

where $O_{xc}(t_i)$ and $O_{xs}(t_i)$ are normalized residuals correspondingly on in-phase and q-phase outputs of the matched filter, t_i is currently set timing (point of synchronization of the received signal with the reference), l is arrival's number, ω_0 - (angular) carrier frequency, T - pulse duration, α_0 is weighting coefficient corresponding to attenuation rate of the synchronous arrival (with number 0), and α_l is weighting coefficient corresponding to attenuation rate of the multipath arrival with number l.

In practical case (for practical frequencies used in underwater sonar applications, underwater acoustic telemetry [1, 2] and non-destructive control) the sine argument is usually much bigger than zero, and thus the second item tend to zero and can be neglected. However, the first item does not depend on carrier frequency and pulse duration, but directly depends on the energy of interfering maltipath arrivals, thus inducing a big error value in filtering of signals transmitted over reverberant channels [3].

USE OF FREQUENCY-MODULATED CARRIER

Now let a pulse to be transmitted using a frequency modulated carrier, e.g. sweep-spread carrier signal (S2C). Such pulse can be represented on the interval $iT <= t < (i+1)T$ as:

$$s_y(t) = Re\left\{\sqrt{\frac{2E}{T}}\exp[j(\omega_b t + mt^2)]\right\} \tag{2}$$

where E - is energy-flux density of acoustic wave, T - symbol duration, $m = \dfrac{\omega_b - \omega_e}{2T}$ is a frequency gradient of every section of the S2C carrier [2], ω_b and ω_e are beginning and final values of the angular frequency of the S2C carrier.

Let the signal propagates via multipath channel with paths of different length, so that multipath arrivals have various delays $\tau_0 \neq \tau_k$, where τ_0 and τ_k are correspondingly the excess propagation delays of a "useful" arrival and of an unwanted k-th multipath arrival creating reverberation. After propagation the received signal can written as

$$r_y(t) = \sqrt{\frac{2E}{T}}\sum_k \alpha_k \cos[(\omega_b - \Delta\omega_k)t + mt^2 + \theta_k] + n(t) \tag{3}$$

$$k = 0, ..., M-1$$

where $\Delta\omega_k$ is frequency resolution of multipath components coming to receiver with excess propagation delays τ_k. The resolution is defined via expression: $\Delta\omega_k = 2m\tau_k$ [2].

To process the received signal, a quadrature matched filter can be used. If the signal-to-noise ratio is large, the output of the filter will be equal to $y(t_i) = y_c(t_i) + jy_s(t_i)$, with

$$\begin{aligned} y_c(t_i) &= \sqrt{\frac{2}{T}}\int_0^T r_y(t)\cos(\omega_b t + mt^2)dt = \\ &= \alpha_0\sqrt{E}\cos(\theta_0) + \sum_l \alpha_l \frac{\sqrt{E}}{T}\int_0^T \cos(-\Delta\omega_l t + \theta_l)dt + \\ &\qquad + \sum_k \alpha_k \frac{\sqrt{E}}{T}\int_0^T \cos(2\omega_b t + 2mt^2 - \Delta\omega_k t + \theta_k)dt \end{aligned} \tag{4}$$

$$l = 1, ..., M-1; k = 0, ..., M-1$$

$$
\begin{aligned}
y_s(t_i) \quad &= \sqrt{\frac{2}{T}} \int_0^T r_y(t) \sin(\omega_b t + mt^2) dt = \\
&= \alpha_0 \sqrt{E} \sin(\theta_0) + \sum_l \alpha_l \frac{\sqrt{E}}{T} \int_0^T \sin(-\Delta\omega_l t + \theta_l) dt + \\
&\qquad + \sum_k \alpha_k \frac{\sqrt{E}}{T} \int_0^T \sin(2\omega_b t + 2mt^2 - \Delta\omega_k t + \theta_k) dt \quad (5)
\end{aligned}
$$

$$l = 1, ..., M-1; k = 0, ..., M-1$$

where t_i is currently set timing, $\Delta\omega_l$ is a frequency deviation of the l-th multipath component, which is kept synchronous with the reference, and θ_l is a phase offset of the l-th multipath component relatively to the phase of the synchronous component [2].

The right side of $y_c(t_i)$ and $y_s(t_i)$ is represented with three items. The first one is a variable containing desired values of phase and amplitude of the synchronous arrival. The second one is a variable contributing to the error in evaluation of the phase and the amplitude of the synchronous arrival induced with interference of multipath components. This is denoted as $e_{ymc}(t_i)$ in the expression for $y_c(t_i)$ and correspondingly $e_{yms}(t_i)$ in the expression for $y_s(t_i)$. Third item is a variable inducing methodical error in evaluation of the phase and amplitude connected with the method of matched filtering. Let it be $e_{yc}(t_i)$ in the expression for $y_c(t_i)$ and correspondingly $e_{ys}(t_i)$ in the expression for $y_s(t_i)$.

The errors $e_{ymc}(t_i)$ and $e_{yms}(t_i)$ are now analyzed for the case when the S2C carrier is used.

The second item of the right side in (4) and (5) can be rewritten in form:

$$e_{ymc}(t_i) = \sum_l \alpha_l \frac{\sqrt{E}}{T} \int_0^T \cos(-\Delta\omega_l t + \theta_l) dt = \sqrt{E} \sum_l \alpha_l \frac{\sin\left(\dfrac{-\Delta\omega_l T}{2}\right)}{\left(\dfrac{-\Delta\omega_l T}{2}\right)} \cos\left(\frac{-\Delta\omega_l T}{2} + \theta_l\right) \quad (6)$$

$$e_{yms}(t_i) = \sum_l \alpha_l \frac{\sqrt{E}}{T} \int_0^T \sin(-\Delta\omega_l t + \theta_l) dt = \sqrt{E} \sum_l \alpha_l \frac{\sin\left(\dfrac{-\Delta\omega_l T}{2}\right)}{\left(\dfrac{-\Delta\omega_l T}{2}\right)} \sin\left(\frac{-\Delta\omega_l T}{2} + \theta_l\right) \quad (7)$$

$$l = 1, ..., M-1$$

In the right side of expressions (6) (7) the summation is carried out along all l arrivals coming in the time interval equal to the symbol length. For accurate estimation of errors (6) and (7), one need to have exact information about amplitudes α_l and excess propagation delays τ_l in the channel. In general, detailed information about current channel properties is not available, and thus the errors $e_{ymc}(t_i)$ and $e_{yms}(t_i)$ take unpredictable values. At that, however, their maximum values are equal to

$$\max(|e_{ymc}(t_i)|) = \max(|e_{yms}(t_i)|) = \sqrt{E}\sum_l \alpha_l \frac{\sin\left(\dfrac{-\Delta\omega_l T}{2}\right)}{\left(\dfrac{-\Delta\omega_l T}{2}\right)} \tag{8}$$

The third item for expressions (4) and (5) can be represented with values $e_{yc}(t_i)$ and $e_{ys}(t_i)$, which are defined as

$$e_{yc}(t_i) = \sum_k \alpha_k \frac{\sqrt{E}}{T}\int_0^T \cos(2\omega_b t + 2mt^2 - \Delta\omega_k t + \theta_k)dt \tag{9}$$

$$e_{ys}(t_i) = \sum_k \alpha_k \frac{\sqrt{E}}{T}\int_0^T \sin(2\omega_b t + 2mt^2 - \Delta\omega_k t + \theta_k)dt \tag{10}$$

$$k = 0, ..., M-1$$

It can be shown [4] that the result of integration in (9) and (10) has a value within limits:

$$\begin{cases} \displaystyle\int_0^T \cos((2\omega_b - \Delta\omega_k)t + 2mt^2 + \theta_k)dt \le \int_0^T \cos((2\omega_b - \Delta\omega_k)t + \theta_k)dt \\ \displaystyle\int_0^T \sin((2\omega_b - \Delta\omega_k)t + 2mt^2 + \theta_k)dt \le \int_0^T \sin((2\omega_b - \Delta\omega_k)t + \theta_k)dt \end{cases} \tag{11}$$

Hence the maximum values of $e_{yc}(t_i)$ and $e_{ys}(t_i)$ are equal to:

$$\max(|e_{yc}(t_i)|) = \max(|e_{ys}(t_i)|) \le \sqrt{E}\sum_k \alpha_k \frac{\sin\left(\dfrac{(2\omega_b - \Delta\omega_k)T}{2}\right)}{\left(\dfrac{(2\omega_b - \Delta\omega_k)T}{2}\right)} \tag{12}$$

Practical values of telemetry frequencies and achievable frequency resolutions [1, 2] satisfy the inequality $\omega_b >> \Delta\omega_k$ for every k, what validates the expression $\max(|e_{yc}(t_i)|) << \max(|e_{ymc}(t_i)|)$, as well as the expression $\max(|e_{ys}(t_i)|) << \max(|e_{yms}(t_i)|)$.

Then, the normalized residuals $O_{yc}(t_i)$ and $O_{ys}(t_i)$ on in-phase and q-phase outputs of the matched filter can be rewritten as follows: $O_{ys}(t_i) = \dfrac{e_{yms}(t_i) + e_{ys}(t_i)}{\alpha_0\sqrt{E}} \approx \dfrac{e_{yms}(t_i)}{\alpha_0\sqrt{E}}$ and correspondingly $O_{yc}(t_i) = \dfrac{e_{ymc}(t_i) + e_{yc}(t_i)}{\alpha_0\sqrt{E}} \approx \dfrac{e_{ymc}(t_i)}{\alpha_0\sqrt{E}}$. Taking into account the inequality (12), the maxima of the residuals can be defined from following inequality

$$\max(|O_{ys}(t_i)|) = \max(|O_{yc}(t_i)|) \le \sum_l \frac{\alpha_l}{\alpha_0}\frac{\sin\left(\dfrac{-\Delta\omega_l T}{2}\right)}{\left(\dfrac{-\Delta\omega_l T}{2}\right)} \tag{13}$$

Comparison of (1) and (13) shows that the residual $O_{yc}(t_i)$ differs from $O_{xc}(t_i)$, and correspondingly $O_{ys}(t_i)$ from $O_{xs}(t_i)$, via such multiplier as $\frac{sin\ x}{x}$. While in practical case (for practical telemetry and sonar frequencies and achievable frequency resolutions [1, 2]) the sine argument is much bigger than zero, the inequality $O_{yc}(t_i) << O_{xc}(t_i)$ is valid. Similarly one can show that also $O_{ys}(t_i) << O_{xs}(t_i)$.

This comparison shows that the error of phase and amplitude estimates (calculated by means of matched filtering of the sweep-spread signals) depends mach less on the energy of multipath interferers than in the case of matched filtering of signals with frequency-constant carrier. Then, due to adjustment of carrier parameters such as frequency bandwidth and duration of every pulse (with S2C carrier) to current properties of the transmitting channel, the accuracy of phase and amplitude estimation can be significantly increased. At that, any additional processing of receive signals (e.g. by means of equalizing filters) becomes unnecessary. Thus, an application of matched filters for processing of S2C signals is justified and becomes practically possible.

PRACTICAL EVALUATION

While multipath propagation in underwater acoustic channels as well as in most sonography testing mediums (non-distractive control) usually comprises rather discrete profile of multipath arrivals, it becomes possible to specify a reasonable error value and using expressions above to calculate a necessary value of $\left(\frac{-\Delta\omega_l T}{2}\right)$ that would satisfy the given quality of filtering of the synchronous multipath arrival (that would correspond to an "allowed" level of distortions). At that, it is obvious that the more resolution that must be achieved, the high value of $\left(\frac{-\Delta\omega_l T}{2}\right)$ that must be provided.

Apparently, there are two ways for such adjustments. One of them consists in increasing of frequency gradient. Another one in increasing of pulse duration. However, due to limitations in all practical applications on an available frequency bandwidth, in fact, the degree of resolution of multipath arrivals will depend only on the frequency bandwidth that is occupied with pulses containing frequency modulated carrier. Combining expressions $\Delta\omega_k = 2m\tau_k$, as well as $m = \frac{\omega_b - \omega_e}{2T}$ (see [2]) and expression (13), the maxima of the residuals can be rewritten as

$$\max(|O_{ys}(t_i)|) = \max(|O_{yc}(t_i)|) \leq \sum_l \frac{\alpha_l}{\alpha_0} \frac{\sin(\tau_k(\omega_b - \omega_e))}{\tau_k(\omega_b - \omega_e)} \tag{14}$$

Expression (14) implies that, when providing a given (technically reasonable) frequency bandwidth, the multipath components of a received signal arriving with time delay τ_k can be resolved so that it will have certain (expected) level of distortions, which will not exceed an a priori known value.

It is obvious, as well, that matched filtering represents a single step in calculation of correlation function of received signal with reference [5], and the correlation function represents a multipath structure of the received signal with given degree of resolution of multipath arrivals (corresponding to the frequency bandwidth of frequency modulated carrier). At that, a maximum error value of every one of multipath components can be evaluated by means of above represented expressions.

Thus, for example, for two resolved arrivals, which are delayed by τ in relation to each other, and while the first arrival is synchronized with the reference, the outputs of the matched filter can be written as

$$\begin{aligned} y_c(t_0) &\approx \alpha_0\sqrt{E}\cos(\theta_0) + \alpha_1\frac{\sqrt{E}}{T}\int_0^T \cos(-(\omega_b-\omega_e)\frac{\tau_1}{T}t+\theta_1)dt \leq \\ &\leq \alpha_0\sqrt{E}\cos(\theta_0) + \alpha_1\sqrt{E}\frac{\sin(\tau_1(\omega_b-\omega_e))}{\tau_1(\omega_b-\omega_e)} \end{aligned} \quad (15)$$

$$\begin{aligned} y_s(t_0) &\approx \alpha_0\sqrt{E}\sin(\theta_0) + \alpha_1\frac{\sqrt{E}}{T}\int_0^T \sin(-(\omega_b-\omega_e)\frac{\tau_1}{T}t+\theta_1)dt \leq \\ &\leq \alpha_0\sqrt{E}\sin(\theta_0) + \alpha_1\sqrt{E}\frac{\sin(\tau_1(\omega_b-\omega_e))}{\tau_1(\omega_b-\omega_e)} \end{aligned} \quad (16)$$

Thus, the maximum error of phase estimation of the synchronous arrival (the first one) will be equal to:

$$\Delta\theta_0 = \max\left| arctg\frac{\alpha_0\sqrt{E}\sin(\theta_0)}{\alpha_0\sqrt{E}\cos(\theta_0)} - arctg\frac{\alpha_0\sqrt{E}\sin(\theta_0)}{\alpha_0\sqrt{E}\cos(\theta_0)+\alpha_1\sqrt{E}\frac{\sin(\tau_1(\omega_b-\omega_e))}{\tau_1(\omega_b-\omega_e)}}\right| \quad (17)$$

and maximum error in the amplitude estimation of the synchronous arrival equal to:

$$\Delta\alpha_0 = \max\left|\frac{y_c(t_0)}{\sqrt{E}\cos(\theta_0)} - \frac{y_c(t_0)}{\sqrt{E}\cos(\theta_0+\Delta\theta_0)}\right| \quad (18)$$

Adjusting the timing so that the second multipath component becomes synchronous with the reference, similar formulas can be derived also for calculation of the maximum error in estimation of phase and amplitude of the second arrival.

A quantitative evaluation of such errors in practical applications can be indicated in the following example.

Let a acoustic telemetry system to be used in a multipath channel so that multipath components come to receiver very close to each other (e.g. like surface reflections in relation to direct arrival in horizontally stratified underwater acoustic channels during long range signal transmissions). And let a frequency band of 40-80 kHz to be occupied with transmitted signals containing frequency modulated carrier.

Often in practice the surface reflection has a very small excess propagation delay e.g. in order of 50 mcs, and the amplitude of the reflection often comprises 70 percent of the direct arrival. Thus, in this example, after synchronization of the first arrival with the reference signal, the maximum phase error can be estimated as $\Delta\theta_0 = 0.0275$ rad, and the maximum phase error will be equal to $\Delta\alpha_0 = 3.8 * 10^{-4} y_s(t_0)$ V.

Similarly, after synchronization of the second (i.e. multipath) arrival with the reference signal, the maximum phase error can be estimated as $\Delta\theta_1 = 0.0562$ rad, and the maximum phase error will be equal to $\Delta\alpha_1 = 1.6 * 10^{-3} y_s(t_1)$ V.

It is obvious that, in spite of complicated conditions (presence of a very close interfere), in both cases the error comprises a small value, just in order of fractions of one percent of the measured value, what can be evaluated as entirely satisfying result e.g. for practical applications of digital acoustic telemetry or echo-sounding.

CONCLUSIONS

Comparison of residuals (1) and (13) indicates that processing of pulses with frequency modulated carrier by means of matched filters can decrease distorting influence of multipath interference. While the residual (1) directly depends on the energy and the number of multipath arrivals and does not depend on the temporal profile of the multipaths, the residual (13) rapidly decreases with increasing excess propagation delays of multipath arrivals in relation to direct arrival according to pattern $\frac{sin\ x}{x}$. Hence, by means of proper adjustment of frequency modulated carrier (such as frequency bandwidth and pulse duration), even properties of very fine structured multipath channels can be investigated, and the influence of such channel structures can be considered or, if necessary, compensated.

Every resolved multipath arrival can be evaluated for the purpose of accurate estimation of its parameters (amplitude and beginning phase).

A practical advantage of the theory here is achieved at least for improvement of underwater acoustic data telemetry systems: successful demodulation of e.g. phase encoded sweep-spread carrier symbols can be carried out without the use of such tricky and unstable units as equalizers. So, the important peculiarity of the method consists in the fact that an S2C telemetry receiver can contain only one processing element - the matched filter. So, the use of uncomplicated receiver structures in even complicated multipath channels becomes possible.

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