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ACOUSTIC MODELING AND TESTING OF A COMPLEX CAR MUFFLER

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Abstract

Perforated mufflers are used by exhaust system manufacturers to improve the broad-band attenuation at low frequencies, with the drawback that this normally also implies an increased pressure drop. The detailed modelling of this type of muffler depends on knowledge of the perforate impedance which is influenced by hole geometry as well as the details of the flow distribution. The existing formulas for calculation of perforate impedance are semi-empirical and a number of alternatives have been published. One motivation behind this work was to review the existing formulas for perforate impedance using accurate measured data for perforated mufflers. A modified model presented by Bauer 1977 was found to be the best. A second motivation was to show that for a detailed analysis, using 3D acoustic FEM, the mean flow can be neglected except for calculating the perforate impedances.

1. INTRODUCTION

There are two basic types of muffler elements: dissipative and reactive. Dissipative elements function primarily by conversion of acoustic energy into heat by viscous action (“damping”). This can be achieved by fibrous materials or by flow separation and turbulent dissipation. There is a trend to reduce the use of fibrous materials due to problems with ageing and possible fibre emissions. An advantage with turbulent dissipation is that it can create damping at low frequencies, without increasing the volume, but at the cost of increased pressure drop. Reactive elements function primarily by wave reflection due to impedance mismatching. In this paper a complex muffler consisting of multiply connected chambers connected by perforated plates and pipes is analysed. In order to make a complete model a 3D acoustic FEM approach was chosen. The Mach-number in an exhaust pipe is normally less than 0.3 which means that inside a mufflers where the flow has expanded the average Mach-number is normally much smaller than 0.1. Therefore one can expect mean flow or convective effects on the sound propagation to be small and possible to neglect. The main effect of the flow for a complex perforated muffler is the effect on the perforate impedances. Existing models for perforate impedances subject to a mean flow are all semi-empirical. Several studies have been conducted and resulted in a number of models, see e.g. Refs. [1-11]. In spite of this large number of publications a single verified global model does not exist. So one task of this work was to test different models to determine which give the best fit with measured transmission loss data for simple through and cross flow mufflers. For the predictions the 3D FEM software FEMLAB [12] has been used. Assuming a negligible mean flow the sound pressure p will then satisfy the Helmholtz equation:

$$\nabla \cdot \left(\frac{1}{\rho_0} \nabla p - \mathbf{q} \right) + \frac{k^2 p}{\rho_0} = 0 \qquad (1)$$

where $k = 2\pi f / c_0$ is the wave number, ρ_0 is the fluid density and c_0 is the speed of sound. The **q** term is a dipole source term corresponding to acceleration/unit volume which here can be put to zero. Using this formulation one can compute the frequency response using a parametric solver to sweep over a frequency range. Through the FEMLAB software different boundary conditions are available and here continuity of normal u_n velocity combined with: $(p_1 - p_2)/Z = u_n$, where Z is the perforate impedance and 1 and 2 denotes the acoustic pressures on each side of the perforate, was used. It can be noted that the use of continuity of normal velocity is consistent with our assumption that mean flow effects are small and can be neglected.

1.1 Impedance Model and Testing Procedure

Different perforate impedance models have been tested and it was found that a model used by Peat [9] and first presented by Bauer [6], gave the best fit with our measured data as shown for instance by Figure 1. The normalized impedance of a perforate with combined grazing and through flow is according to the model from Refs. [6,9]:

$$Z = \left[\left(\frac{\sqrt{8\mu\rho_o\omega}}{\rho_0 c_0 \sigma C_D} \right) \left(1 + \frac{t_h}{d_h} \right) \right] + \frac{0.3 M_g}{\sigma} + \frac{1.15 M_b}{\sigma C_D} + \frac{jk}{\sigma C_D} \left[t_h + \delta \times F_{\text{int}} \times F_\delta \right] \qquad (2)$$

where Z is the normalized impedance of the perforate, μ is the dynamic viscosity, σ is the perforate porosity, M_g is the grazing flow Mach number, M_b is the bias (through) flow Mach number, C_D is the orifice discharge coefficient, t_h is the wall thickness and d_h is the hole diameter. The factor δ is the acoustic end correction for both side of the hole and put equal to $0.62 d_h$ and F_δ is the flow effect on acoustic reactance assumed to be 0.38 according to Rice [11]. The interaction between the holes has been taken into account according to $F_{\text{int}} = 1 - 1.47\sqrt{\sigma} + 0.47\sqrt{\sigma^3}$.

All experimental results presented here were carried out at room temperature using the muffler test facility at MWL, in which the 2-port for mufflers can be measured using the procedure described in Refs. [13-14]. Six condenser microphones (B&K 4938) flush mounted in the duct wall, three upstream and three downstream of the test object, are used to cover the plane wave range in the test duct. The flow speed is measured upstream of the test section using a small pitot-tube connected to an electronic manometer.

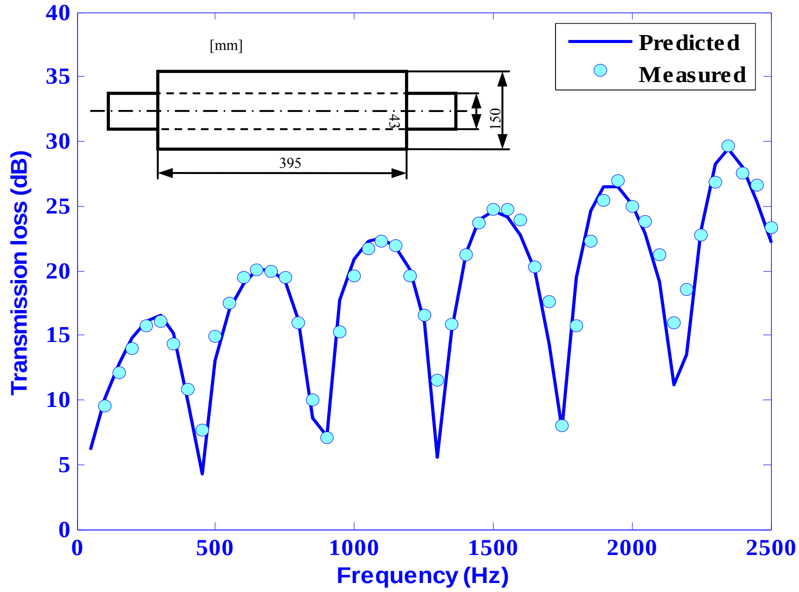

Figure 1 Example of measured and predicted transmission loss (TL) for a through flow muffler. Perforate geometry: σ =0.07, d_h =3 mm and t_h=1.5 mm. Flow conditions: M_g =0.18, M_b =0. The prediction is based on the perforate model in Eq. (2).

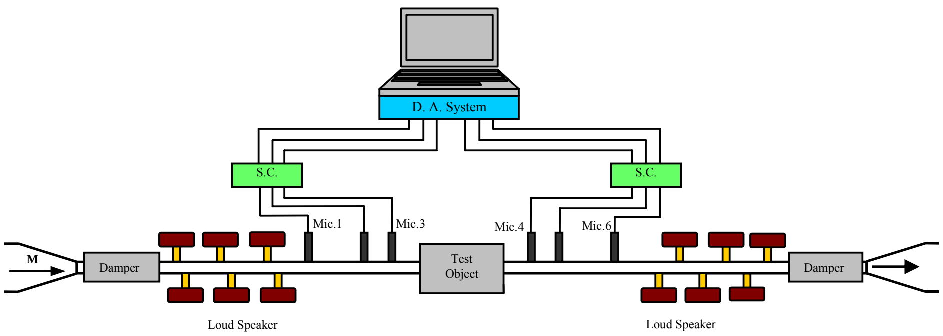

Figure 2 Muffler test rig at MWL.

2. MUFFLER DESCRIPTION

The muffler was manufactured for research purpose and was built to resemble a complex automotive muffler with multiple chambers connected via perforated pipes and plates, see Figure 3. The inlet duct (1) has 45 mm internal diameter, and 1.5 mm wall thickness. The outlet pipe (2) has 57 mm internal diameter and 1.5 mm thickness.

A stainless steel sleeve (7) of length 180 mm is located above the 288 holes perforated area in the outlet duct. It has a density of 490 kg/m³ and weight of 76 g. The layout of the muffler is shown in Figure 3, and the detailed dimensions of the inlet, outlet pipes, and baffles are shown in figures presented in Appendix A.

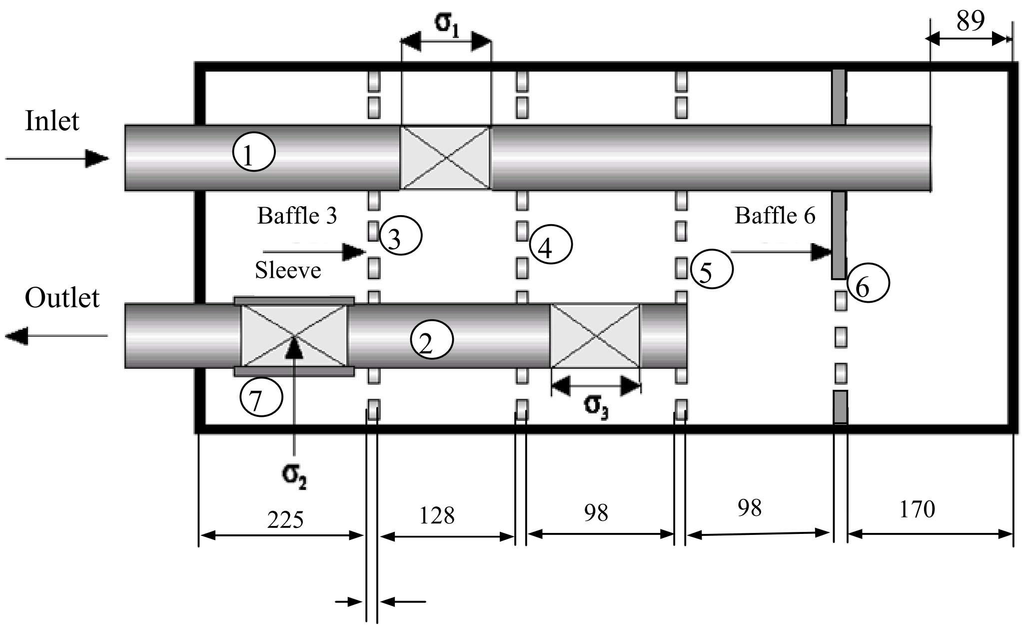

Figure 3 Sketch of the complex muffler analysed (dimensions are in mm).

As seen in the Appendix for the plates (3-6) the perforation of the plates is not uniformly distributed. This was handled by defining a perforation area or areas for each plate for which a perforation ratio was calculated. The alternative would be to try to model each hole separately, but this would result in a significant increase in the computation time.

2. RESULTS AND DISCUSSIONS

2.1 No flow Case

For this case M_g and M_b equals zero, which means that the impedance equation (2) is reduced to:

$$Z = \left[\left(\frac{\sqrt{8\mu\rho_o\omega}}{\rho_o c_a \sigma C_D} \right) \left(1 + \frac{th}{dh} \right) \right] + \frac{jk}{\sigma C_D} \left[th + \delta \times F_{\text{int}} \right] \tag{3}$$

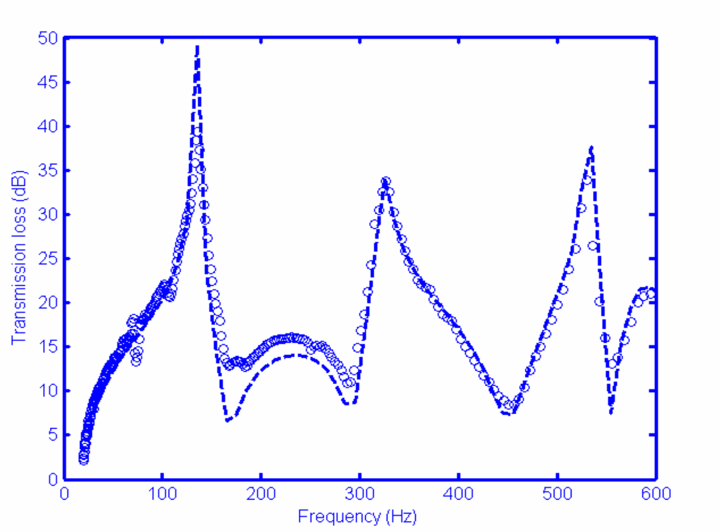

Figure 4 *Transmission loss versus frequency* at M=0.0*; ∘∘∘∘, measured; ----, predicted.*
Fine mesh and 73739 elements are used.

As can be seen from Figure 4, the model gives an excellent agreement with the measured data. To test the modeling of the perforated plates a simulation where each hole in the plates (3-6) was modeled was also performed. The result matched closely to the curve presented in Figure 4.

2.2 Case with flow

The transmission properties of the muffler are affected by the flow in three ways. The first is the convective effect through the straight pipes which is small and can be neglected. The second is the losses that occur at the open ends (area expansion/contraction) of the inlet/outlet pipes. This effect is relatively small (1-2 dB typically) and was included in the FEM model by using continuity of normal u_n velocity combined with: $(p_i - p_{ii})/R_{open} = u_n$, where i and ii denotes the acoustic pressures at each side of the open ends and R_{open} is the open end flow resistance defined below in equation (12). The third and the most important is the change of the perforate impedance, which mainly comes from the increase of the resistance with flow.

The flow distribution through the muffler elements was calculated using a 1-D incompressible flow model. The model is based on a network analysis where balance equations corresponding to Kirchhoff's laws in electrical engineering are applied. This implies that the volume flow into a junction (or node) must equal the volume flow out of the junction. Figure 5 shows a network representation of the muffler in question. Applying balance of volume flow at each node then gives:

$$Q_0 = Q_1 + Q_2 \qquad Q_2 = Q_5 + Q_7 \qquad (4), (5)$$

$$Q_4 = Q_1 - Q_3 \qquad Q_5 = Q_3 + Q_6 \qquad (6), (7)$$

where Q_m is the volume flow in path *m*.

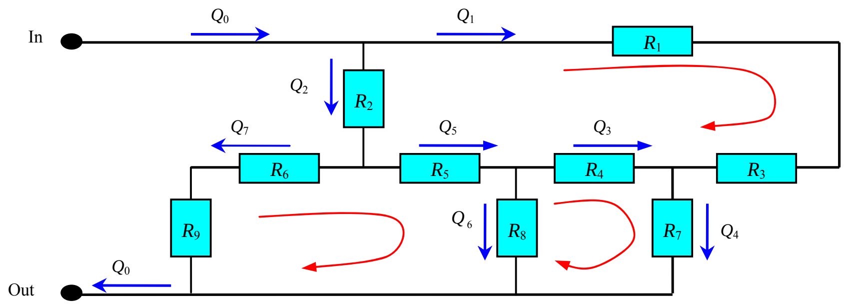

Figure 5 The equivalent network for the muffler in Figure 3.

By applying Kirchoff's second law for any closed lope, the sum of the pressure drops must equal zero which implies:

$$R_1 Q_1 |Q_1| + R_3 Q_1 |Q_1| - R_4 Q_3 |Q_3| - R_5 Q_5 |Q_5| - R_2 Q_2 |Q_2| = 0 \tag{8}$$

$$R_4 Q_3 |Q_3| + R_7 Q_4 |Q_4| - R_8 Q_6 |Q_6| = 0 \tag{9}$$

$$R_5 Q_5 |Q_5| + R_8 Q_6 |Q_6| - R_9 Q_7 |Q_7| - R_6 Q_7 |Q_7| = 0 \tag{10}$$

where, R_1, R_7 are the open end flow resistances for the inlet and outlet pipes, which have been calculated according to [15] as:

$$\Delta P = \frac{1}{2} \varepsilon \, \rho_0 \, U |U| = R_{open} Q |Q| \tag{11}$$

$$R_{open} = \frac{\varepsilon}{2} \frac{\rho_0}{A^2} \tag{12}$$

where ΔP is the pressure drop, ε is constant equal to 1, for an outlet opening and 0.5 for an inlet opening, U is he flow speed and A is the cross sectional area of the pipe. where as, R_2, R_3, R_4, R_5 R_6, R_8 and R_9 are the resistances of the perforates and have been calculated according to [15] as:

$$\Delta P = \rho c z_v U + \frac{\rho}{2\sigma^2 C_D^2} U |U| = R'_{viscous} Q + R_{perforate} Q |Q| \tag{13}$$

$$R_{perforate} = \frac{\rho}{2\sigma^2 C_D^2 A^2}, \quad R'_{viscous} = \frac{\rho c z_v}{A} \tag{14}$$

For a perforate the viscous part is neglected, for the metallic sleeve (element 7) only the viscous part is included and its value was obtained by measurement.
By solving the set of non-linear equations (4) to (10) using *Matlab*, the flow distribution in the muffler can be calculated. By combining the calculated values with the impedance model presented in equation (2) a complete FE model can be set up and used to calculate the transmission loss. An example of the results is presented in Figure 6. As can seen, the result is quite satisfactory and the deviations are believed to be related to the simple 1-D flow model used. Because the resistive part of the

perforate impedance is sensitive to the flow distribution details and a simple 1-D model is probably not sufficient to resolve this.

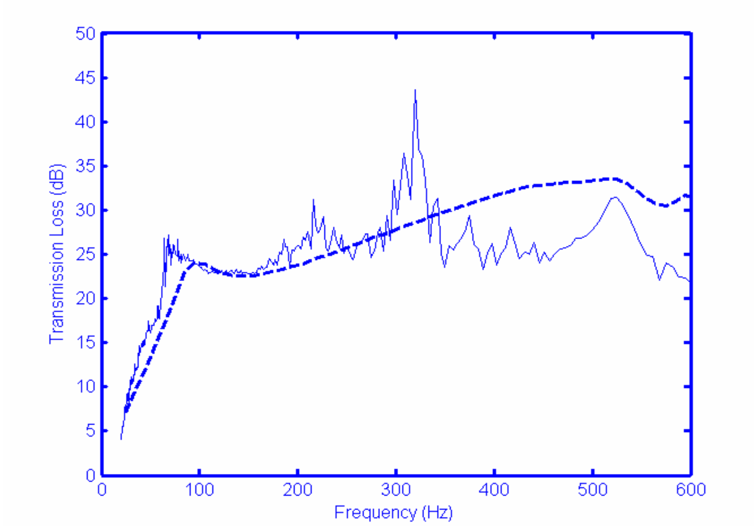

Figure 6 Transmission loss versus frequency at M=0.15*;* ––, measured; ----, predicted. Fine mesh and 73739 elements is used

3. CONCLUSION AND FUTURE WORK

In this paper it is suggested that for the modelling of complex perforated mufflers the flow effects are only included in the perforate impedances and in the inlet/outlet losses. Using this approach it is shown how a standard FEM tool can be applied for the analysis. A simple 1-D flow model was used to find the flow distribution, which is not fully satisfactory since it does not give the details of the flow distribution. This is important for accurately determining the perforate impedances. The best for the future would be to model the mean flow distribution by a 3D steady CFD simulation.

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Appendix A - *Muffler Dimensions*

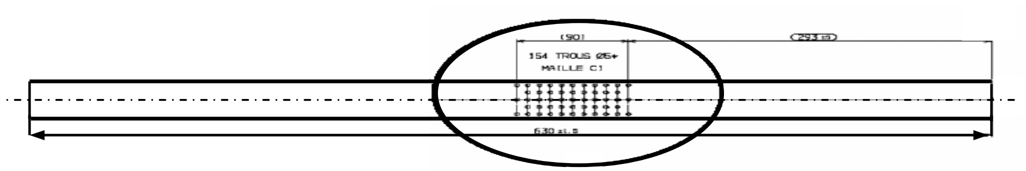

Fig.A1: Inlet pipe: σ_1=0.237

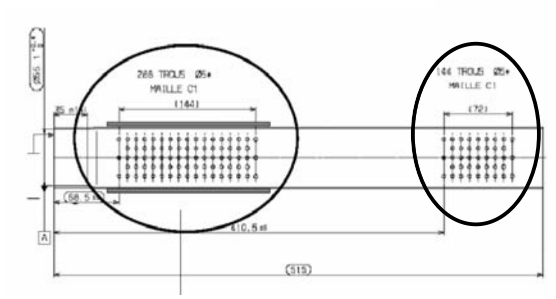

Fig.A2: Outlet pipe: σ_1=0.237 and σ_3=0.237.

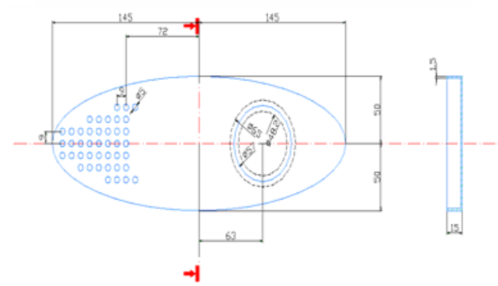

Fig.A3: Perforated baffle number 3 (50 holes).

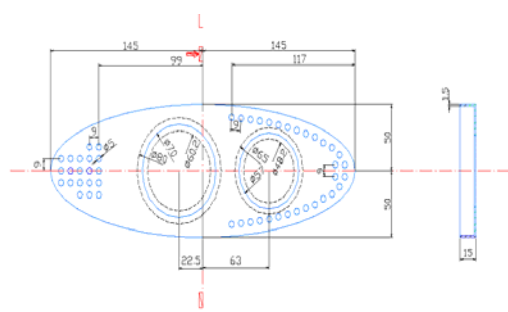

Fig.A4: Perforated baffle number 4 and 5 (72 holes).

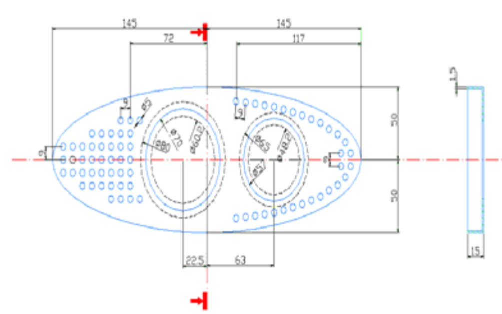

Fig.Á5: Perforated baffle number 6 (42 holes).